MODERN MECHANICAL ENGINEERING

A PRACTICAL TREATISE
WRITTEN BY SPECIALISTS

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TRANSPORT OF HEAVY MACHINERY

BY

G. M. S. SICHEL, B.Sc.

Transport of Heavy Machinery

A.—By Road

Occasions frequently arise, not only in manufacturing countries but also in places both at home and abroad, where new industries are being started, when it is necessary to transport heavy or bulky machinery from the nearest railway station or port to the site where the plant will be installed; or the heavy machinery may have to be transported from the makers' works to docks for shipment overseas. In many cases road transport is necessary, as from its size the piece to be moved cannot be sent by rail, and the object of this article is to draw attention to various points which have to be taken into consideration when transporting heavy or bulky machinery by road.

Legal Considerations.—It wi l be readily understood that when heavy weights are taken by road, the road surface is frequently damaged, more particularly when the road is not one which is usually traversed by heavy traffic, or where the road has a special prepared surface (e.g. asphalted), or where the road has no great depth or has insufficient foundation, or is laid on boggy or clayey ground.

In Great Britain damage of this kind to any public highway, or bridge carrying the highway over a river, or canal, or depression, is taken care of by special legislation, of which the "Roads and Bridges (Scotland) Act" of 1878 may be cited as a typical example. Section 57 of this Act is headed "Extraordinary Traffic" and reads:

"Where by the certificate of their surveyor or district surveyor it appears to the authority which is liable to repair any highway that, having regard to the average expense of repairing highways in the neighbourhood, extraordinary expenses have been incurred by such authority in repairing such highway, by reason of the damage caused by excessive weight passing along the same, or by extraordinary traffic thereon, such authority may recover in a summary manner before the Sheriff (whose decision shall be final) from any person by whose order the excessive weight has been passed, or the extraordinary traffic has been conducted, the amount of such extraordinary expenses as may be proved to the satisfaction of the Sheriff to have been incurred by such authority by reason of the damage arising from such excessive weight or traffic as aforesaid.

"Provided that any person against whom expenses in respect of the passing of excessive weight or of extraordinary traffic are or may be recoverable under this section, may enter into an agreement with such authority as is mentioned in this section for the payment to them of a composition in respect of such passing of such excessive weight, or of such extraordinary traffic, and thereupon the person so paying the same shall not be subject to any proceedings under this section. This section shall have effect in every county in Scotland notwithstanding that the other provisions of this Act have not been adopted or are in force therein."

It will be observed that the Act refers only to damage to highways, and no mention is made of damage to gas- and water-pipes, sewers, &c., below the level of the roadway. When a large weight is being taken along a roadway, the vibration set up in the ground in the vicinity of the heavy weight is often so intense that gas- and water-pipes, &c., unless buried at a considerable depth below the surface, are frequently cracked or broken, although the surface of the highway may not be injured or broken by the heavy traffic. Such heavy traffic and vibration may also break through the highway into the tunnels of rail and tramways running below the street level. Though recovery for such damage may not be obtained through the Act quoted, it is quite probable that an action at common law, to recover the cost of damages which have been incurred, would succeed. This is a most important consideration, as the damage which may be done may exceed many times the cost of the piece which is being transported.

In addition to the Act quoted, special instructions are issued to the police force in all large cities, for regulating the passage of heavy transport through cities and crowded areas. These instructions appear in the Police Instruction Book, and are based on the Locomotive Act of 1863, which deals with the use of locomotives on public highways. These instructions make provision for the following:

1. In large cities, and even in smaller towns, certain streets are totally debarred from very heavy traffic; this applies especially where alternative routes are available, or where the surface of the roads is not suitable for the passage of heavy traffic, or where the vibration set up by the heavy traffic may damage important buildings or interfere with special processes.

2. In towns where the manufacture of heavy plant is carried on, and the transport of such plant by road is inevitable, the regulations state that such traffic must not proceed through certain defined areas (usually the centre or business portion of the city) between the hours of 8 a.m. and 5 p.m., so as not to interfere with the ordinary traffic which is at a maximum between the hours stated.

3. A limit is also placed on the width of a piece that may be hauled by road, so as to allow other traffic room to pass.

4. Steam-whistles are not allowed to be blown when passing along the city or town highways, nor is steam allowed to be blown off by means of drains or by safety-valves, so as not to cause horse-drawn traffic to take fright, and by bolting possibly cause serious accidents

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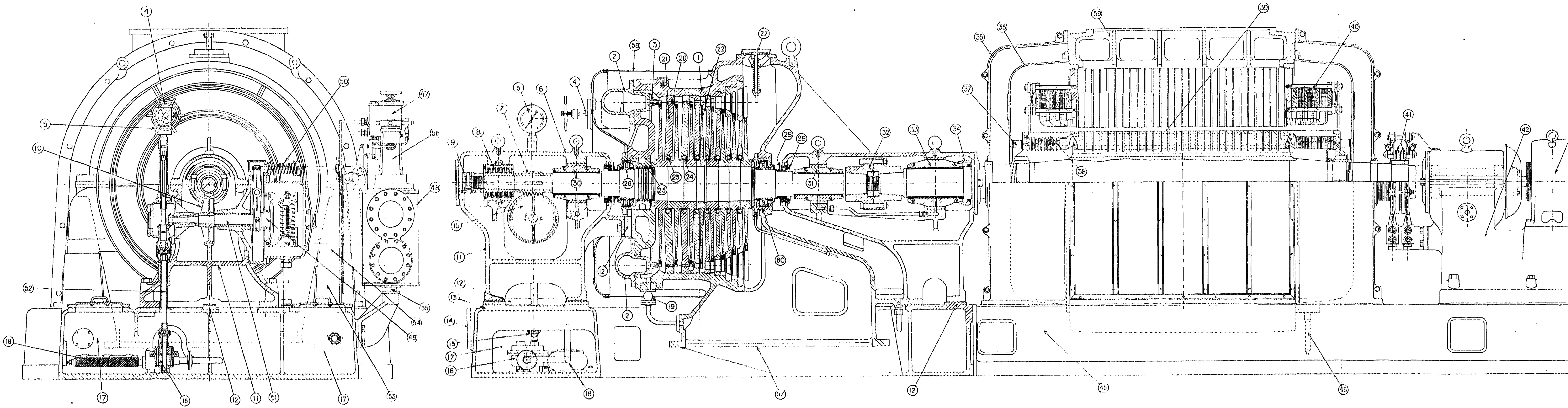
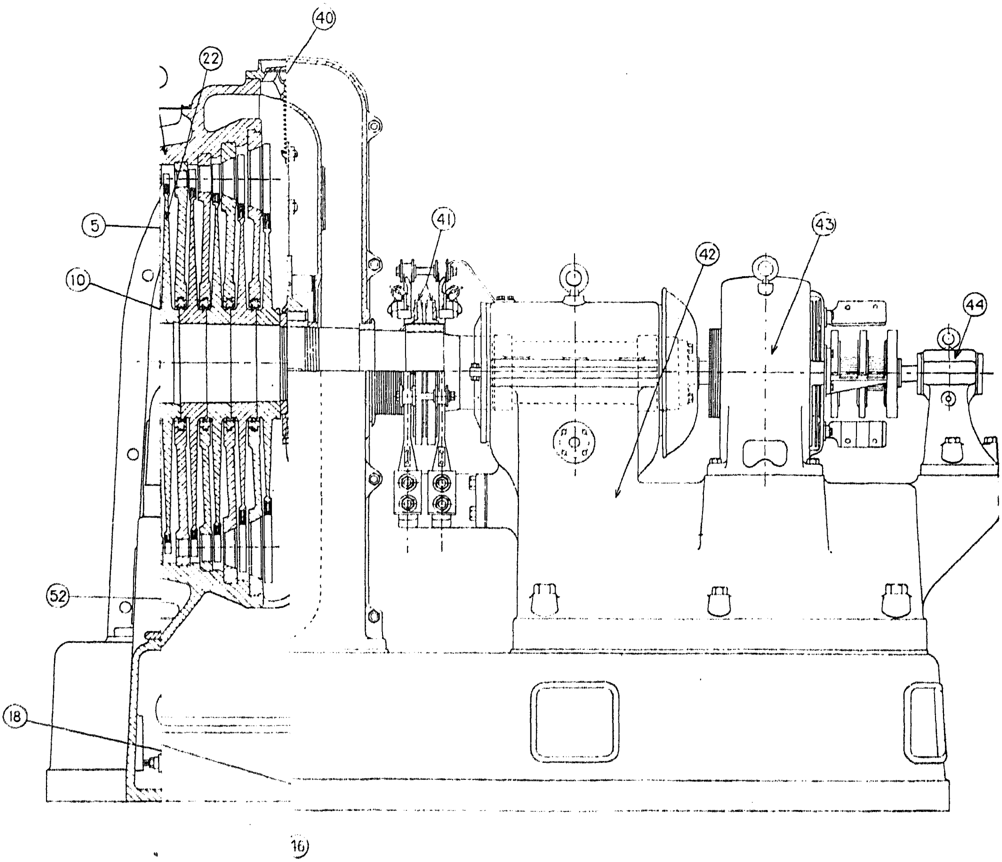


1. Turbine cylinder.
2. Nozzle box.
3. Nozzle block.
4. Overload valve.
5. Tachometer.
6. Thrust pedestal cover.
7. Worm.
8. Thrust block.
9. Emergency overspeed governor.
10. Worm wheel.
11. Thrust pedestal.
12. Guide or director key.
13. Thrust pedestal soleplate.
14. Inspection door.
15. Universal coupling.
16. Oil pump.
17. Oil tank.
18. Oil strainer box.
19. H.P. cylinder drain valve.
20. Diaphragm blades.
21. Diaphragm.
22. Turbine wheel.
23. Diaphragm collar.
24. Diaphragm labyrinth strip.
25. H.P. labyrinth gland.
26. H.P. paddle wheel.
27. Relief valve.
28. Steam and water baffle.
29. Oil thrower.
30. Turbine H.P. bearing.
31. Turbine L.P. bearing.
32. Solid coupling.
33. Generator bearing.
34. Oil catcher.
35. Outer end bell or silencer.
36. Inner end bell.
37. Rotor fan.
38. Rotor end bell.
39. Rotor.
40. Stator winding.
41. Rotor slip rings.
42. Generator outboard pedestal.
43. Exciter yoke.
44. Exciter outboard pedestal.
45. Generator soleplate.
46. Generator main cables.
47. Governor valve.
48. Steam chest.
49. Turbine governor.
50. Speeder spring.
51. Governor shaft.
52. Oil pump driving shaft.
53. Turbine soleplate support for cylinder.
54. Turbine cylinder feet.
55. Steam chest spring support.
56. Emergency stop valve.
57. Exhaust flange connected to con
58. Planished steel lagging.
59. Generator stator.
60. Turbine L.P. gland.

6000 K.W. STEAM TURBO ALTERNATOR. BY METROPOLITAN VICKERS CO.



1. .nth gland.
2. e wheel.
3. :.
4. water baffle.
5. . ort for cylinder.
6. P. bearing.

55. Steam chest spring support.
56. Emergency stop valve.
57. Exhaust flange connected to condenser.
58. Planished steel lagging.
59. Generator stator.
60. Turbine L.P. gland.

. M. TURBO

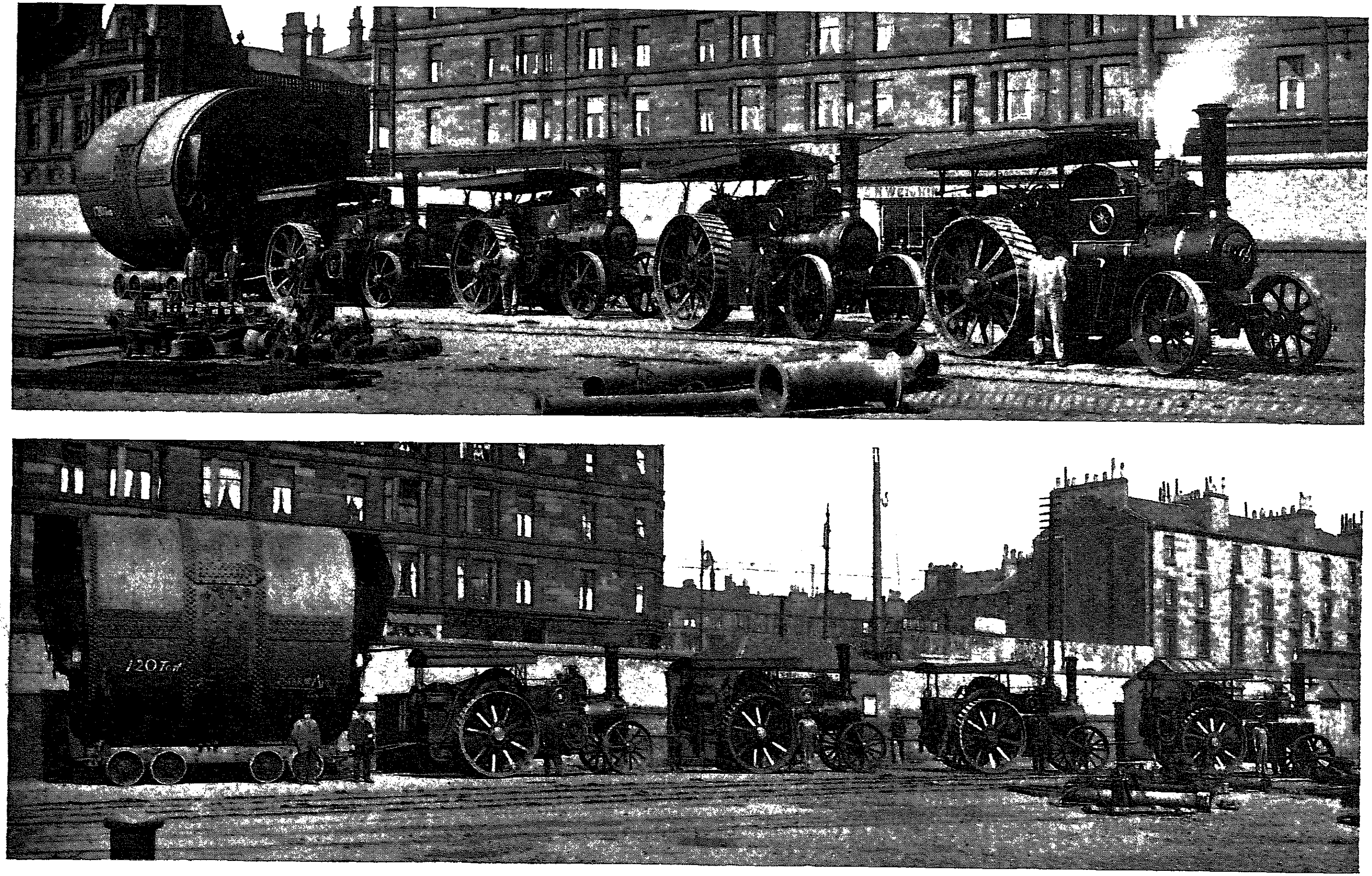


Fig. 1.—Haulage of Marine Boiler (120 tons) from Makers' Works to Ship Fitting-out Basin

5. The speed of heavy traffic is kept down to a minimum when passing through city areas, so as to not only have the heavy piece well under control, but also to limit the vibration which is set up in the ground and communicated to buildings, water- and gas-mains, &c.; in addition, a man with a red flag should walk some twenty yards in front of the traction engine, both to limit the speed and to warn people out of the way.

The above statements will serve to indicate the nature of the police regulations; these may vary in different parts of the country, and in certain districts may exist only in word and not in fact, more particularly where the manufacture of heavy plant is one of the staple industries of the district.

Mechanical Traction versus Horse Traction.—In Great Britain the alternative to mechanical traction is horse traction, though in countries overseas teams of mules or oxen are used. It will, however, suffice if horse traction is compared with mechanical traction, because of all draught animals the best is undoubtedly the type of draught horse found in Great Britain, and represented by the breed known as Clydesdales. One good Clydesdale can not only draw a load of five tons on the level, but can unaided start up this load from rest, a very different matter. When large weights have to be hauled by road and only horses are available, it becomes necessary to use a team of horses, which may number anything from six to twenty or more, depending on the load and the nature of the road or track. The main difficulty is to get all the horses to pull steadily together when starting up; invariably one or two horses in the team turn fractious when they find they are pulling at an apparently immovable load, and one such horse is often quite enough to upset the whole team. Another difficulty is that when on account of narrow space available, e.g. approaching a gateway which is only just wide enough to pass the traffic, it becomes necessary to move forward a few inches at a time, horses are difficult to handle, and either do not move the heavy weight, or "jump" at it and go forward a few feet; several serious accidents due to this cause have been narrowly averted—accidents involving damage to buildings as well as to the piece being hauled. If it is necessary to draw back a short distance, the team has to be unhitched and taken round to the back end; all of which operations take time, and are frequently attended by other local difficulties.

On the other hand, mechanical traction gives practically perfect control of the haulage; if it is desired to "inch" forward or backward, when approaching a narrow gateway or passage, this can be done with the greatest nicety without uncoupling the tractor. One good steam-tractor will do the work of a whole team of horses, and do it more quickly and with better control. If necessary, two, three, or more tractors can be coupled together when dealing with very heavy weights or when negotiating a heavy gradient.

Steam-tractors.—Some of the finest engines for road traction are manufactured by Messrs. John Fowler & Co., Ltd., Leeds, and represented by the accompanying photographs. The design and construction of these engines is the result of many years' experience in all parts of the world, and some particulars will therefore be of interest.

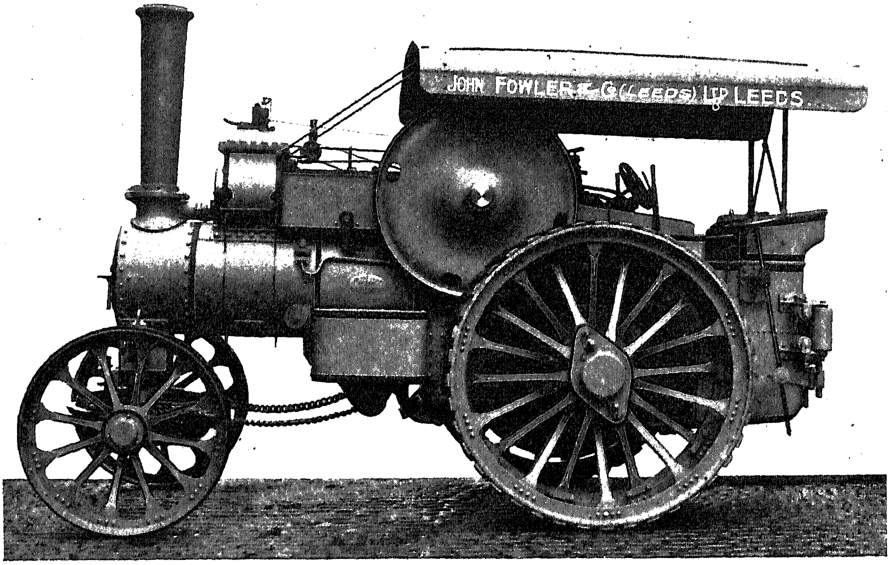


Fig. 2.—Standard Type of Traction Engine, made by Messrs. John Fowler & Co., Ltd., Leeds, showing Winding-drum on Rear Axle

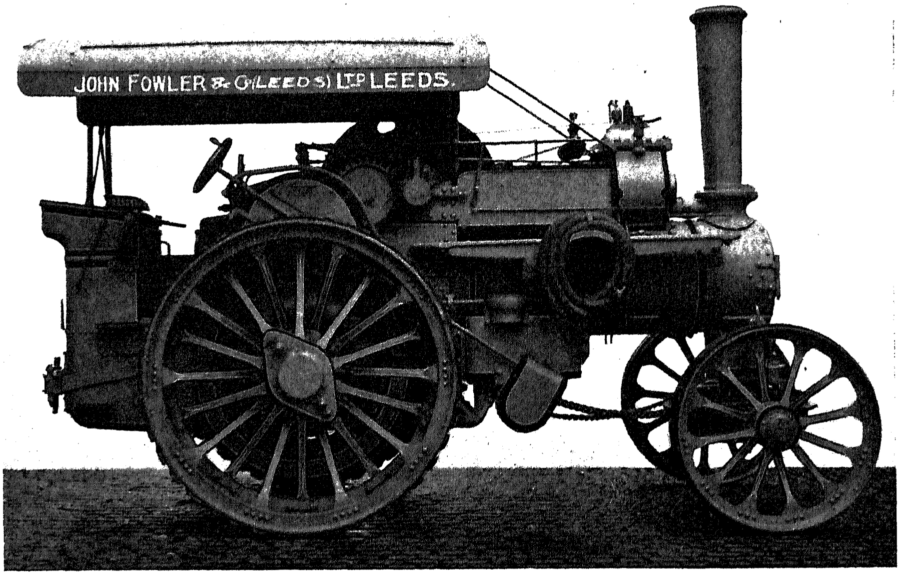


Fig. 3.—Traction Engine made by Messrs. John Fowler & Co., Ltd., Leeds, showing Steering-gear and Water-lifting Device

Of some five standard sizes, the smallest and the largest weigh approximately 9 and 14 tons respectively, the indicated horse-power developed being from 45 to 50 and 70 to 85 in the two sizes. The speeds attained on

good level roads are 2 and 4 miles per hour on slow and fast gears respectively. The steam pressure is 180 lb. per square inch, and superheaters have been tried in a large number of cases with markedly improved results, shown by an economy of 20 per cent in fuel and 30 per cent in water. On long cross-country haulages, where it may not be possible to obtain fuel and water as and when required, this is a most important consideration. The engines are supplied with a steam-operated water-lifting device for drawing up water from the roadside, and have many special details in construction which have been found from many years' experience to make for strength and reliability; next to haulage-power, probably the most important consideration is that of reliability, and reliability only comes with experience.

In the transport of heavy weights by land (i.e. road traction), the engine has to supply the whole of the power that may be required, and all sorts of difficulties, seen and unforeseen, may arise, so that if the engine is provided with means for extricating itself and its load from difficulties so much the better. On the Fowler engines, a most useful addition is a winding-drum, together with a long length of flexible steel-wire rope; this provision is extremely useful where the tractor and load have to pass over a bridge, which may not be strong enough to carry them both simultaneously. In such a case the engine crosses the bridge first, and then, by means of the winding-drum and rope, pulls the load across; when working on a downward gradient, the load can be lowered first, by means of the winding-drum and rope, and the engine then follows on. Should the engine, when manœuvring the load into position, have to go off the road or hard ground and get bogged in soft soil, the winding-drum and rope can be used to pull itself out of difficulty.

Although the weight of a large steam-tractor is as much as 14 tons, it is frequently found, when hauling exceptionally heavy weights, more particularly over roads paved with sets, that the ordinary road wheels shod with steel cross-bars do not obtain a sufficient "grip" of the road, with the result that skidding and slipping take place and the surface of the roadway may suffer very severely in consequence.

This difficulty is overcome by the adoption of Boulton or wood-block wheels for the main drivers; the arrangement of this type of wheel is shown in the accompanying photograph; the cast-steel wheel rims have square recesses, into which are fitted blocks of hard-wood strengthened on the outside by a steel casing. Each block is fitted with a spring and cushioning arrangement at the back; the cushion takes up any undue jar on the wheels, and the spring is compressed by the weight of the engine, and thus secures an excellent grip on the roadway and eliminates slipping.

Traction Wagon or Trailer for Carrying Loads.—The wagon or trailer, which carries the load and is hauled by the tractor, should be constructed with the greatest strength and stiffness, as the stresses put on these trailers are often very severe. A great many considerations have to be taken into account in the design of a wagon for carrying heavy loads.

1. The wagon must be strong enough to carry safely the maximum load for which it is designed.

2. The height of the wagon above road-level should be kept as low as possible, compatible with other considerations, so as to keep the centre of gravity of the load well down and thus secure the greatest possible stability against overturning. This is very important, as it frequently happens that both wheels on one side sink into a soft part of the road when transporting a heavy weight, and the wagon and load assume a dangerous angle; several cases are on record where the whole wagon and load has capsized sideways, and the difficulties of reloading at the roadside under such conditions can be left to the imagination.

3. The wheel rims should be as broad as possible, so as to have a large area of contact with the ground, and thus lessen the weight per square foot.

4. The wheels should be of large diameter, the larger the better, so as

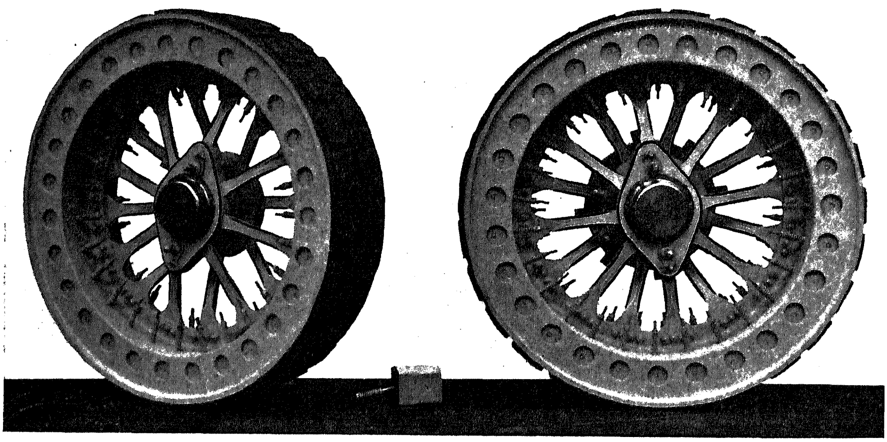


Fig. 4.—Boulton or Wood-block Wheels

to not only increase the area of contact with the road or ground but also, by easily rolling over the ground, reduce the tractive effort required. This is a very important consideration, and neglect to take it at its proper value has been the cause of an immense amount of difficulty which has been experienced, and severe damage to roads and bridges which has been done on big haulage contracts.

5. It is an advantage in some cases to have one pair of wheels swivelling, i.e. true bogie wheels. The advantages, however, are considerably discounted by the alternatives which result: (*a*) the diameter of the bogie wheels has to be reduced, (*b*) the height of the trailer has to be raised, both of which are objectionable, as pointed out above.

6. Where the trailer is not of the bogie type, arrangements should be made to attach the draw-bar to either end, and in the best practice the draw-bar should be fitted with a strong spring in order to take the shock of the heavy initial pull required to start up the trailer

Too much importance cannot be attached to paragraph 4, especially to

the statement dealing with the diameter of the wheels. Suppose we take two wheels, both of which have the same area of contact with the ground and thus support the same weight per square inch; but in one wheel the diameter is big and width small, in the other the diameter is small and width big. It will be found that the tractive effort required to pull a given weight on wheels of small diameter will be very much greater than when the wheels are of large diameter. The small wheels have the effect of working up a fold in the ground immediately in front of the wheel, so that instead of rolling over the ground the wheel has to be dragged against the fold in the ground. This trouble does not arise when dealing with roads paved with stone sets or with roads built on solid foundations, but is particularly liable to occur on softer roadways or cross-country highways.

The trailer usually has a top or platform of thick wood, which is rigidly fixed to the underframe. The latter is made of heavy baulks of timber, reinforced with steel angles and plates at the joints. In some trailers the underframe is entirely of steel, but such a frame has not the " spring " or " give " obtained with the trailer made of well-seasoned baulks of timber. The timber platform is useful in more ways than one; not only does it prevent slipping and shifting of the load on the trailer, but the platform makes it possible to nail down wood " shores " all round the load, as an additional safeguard against the load moving on the platform, particularly on long haulages or on rough roads or where the road is hilly.

It is not usual to have the trailers for the heaviest class of haulage spring-mounted, though in trailers or wagons to carry up to 10 tons springs are sometimes used. The use of springs for very heavy weights is not desirable, on account of the shocks to which such springs would be subjected when supporting a heavy weight in transit over a rough road; these shocks often are very severe, and result in breaking the springs. On the other hand, the use of springs, on trailers, cushions to a large extent vibrations which are set up in the ground, and which result, as mentioned previously, in the cracking of buildings and the fracturing of water- and gas-pipes, &c.

Loading Up.—Care should be taken to distribute the load over a number of wheels, so as to keep down to a minimum the weight on each wheel. Thus for very large weights it is customary to employ two trailers, each with four wheels, and distribute the load over the two trailers, which are coupled together by means of a draw-bar. The possibility of doing this depends almost entirely on the dimensions of the piece to be transported; for such weights as Lancashire boilers, which have considerable length, the use of two trailers is eminently suitable. On the other hand, Scotch dry-back marine boilers, which are large in diameter and short in length, are best handled on a single trailer.

In placing the weight on the trailer, care should be taken to see that the load is evenly distributed on the wheels. An exception is made, however, when loading up a bogie trailer, i.e. a trailer with swivelling front wheels. In this case the weight must always be kept towards the back of the trailer, on account of the liability of the latter to overturn when the weight is central

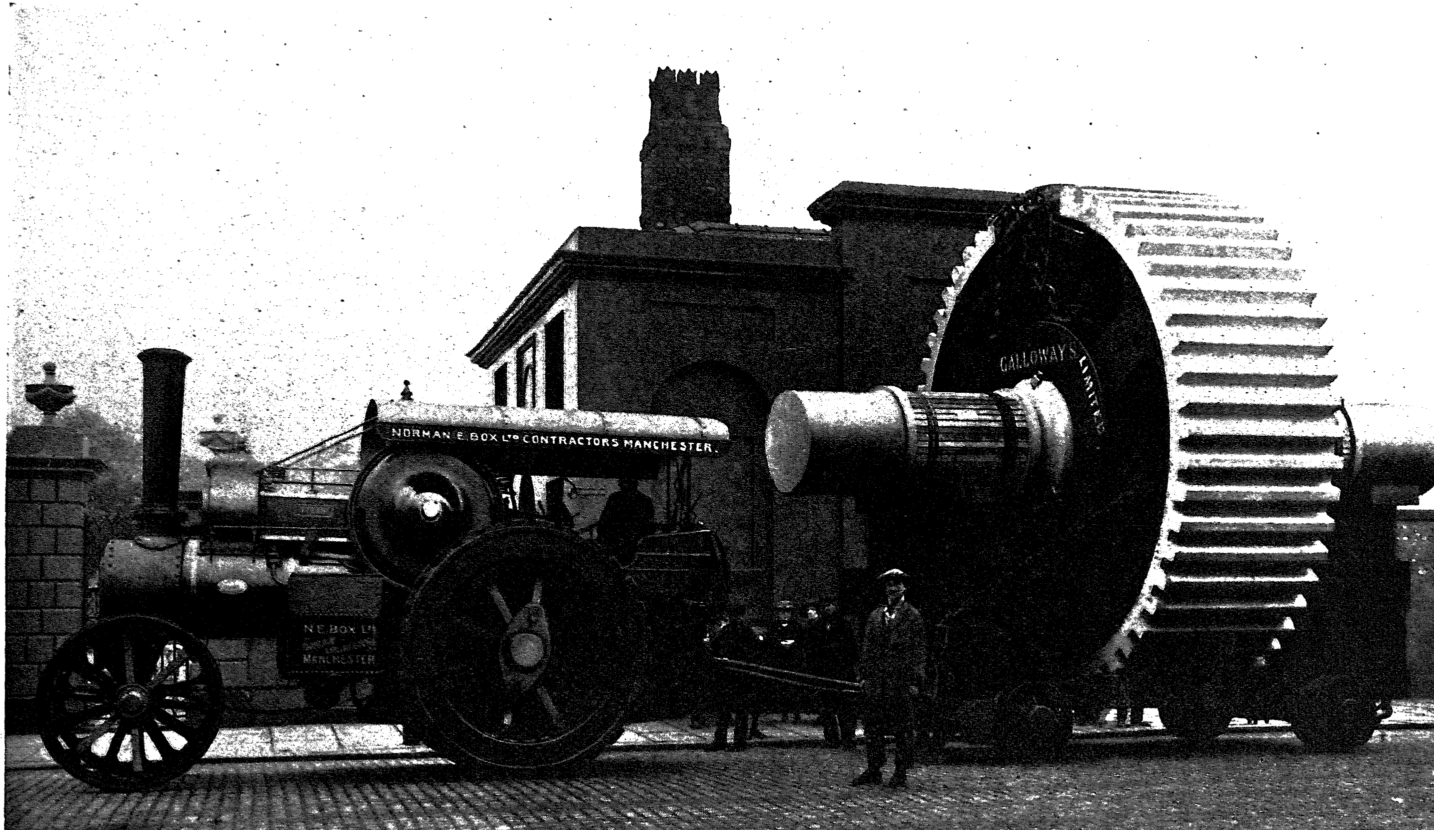


Fig. 5.—Transport of Large Spur-wheel and Axle, showing Method of Loading and Protection of Axle Journals by Casing of Wood Laths

or too far forward, and the trailer is making a sharp turn. The type of trailer to be used depends on the weight and dimensions of the piece to be transported; it is particularly in a problem of this kind that experience counts; the man who has had most experience being able to tell instinctively whether a trailer is suitable or not. In general, however, it is good practice to distribute the weight over as many wheels as possible. After the piece has been placed in position on the trailer it should be well secured against movement in any direction on the trailer; this is best accomplished by securing heavy "shores" or battens of timber to the wood platform, by means of 6-in. nails or bolts, the former being preferable as they do not require any holes drilled in the platform. In addition, the piece should be securely held down by lashing chains tightened up by right- and left-hand screw shackles, or the chains or wire ropes may be tightened up by driving in several wood battens, wedge-shaped, between the piece and the chain, or between the frame of the trailer and the chain or wire rope; care should be taken, however, to secure the wedges from coming out, and if this is impossible the wedges should be examined along the road from time to time and tightened up.

Preparation of Piece for Haulage.—Any machined faces or parts should be protected from damage; if the piece has to rest on a machined face on the trailer, and the platform of the trailer has any old nails driven into it or bolt heads or nuts, even below the surface, that may cause damage, it is advisable to nail down some clean timber $1\frac{1}{2}$ or 2 in. thick on the platform before loading up the piece. Machined parts or any highly finished surfaces, e.g. shaft journals, should be painted with tar or with a mixture of tallow and white lead, the latter being easily removable; this is done as a protection from damp and rusting. In addition, such surfaces as shaft journals should be wrapped round with several layers of sacking and then with a flexible protection of wood laths, from 1 to 2 in. thick, bound together with hoop iron and stout tacks, in order to obviate injury from chain or wire slings when the piece is being lifted, or from accidental injury from blows or collisions which would indent the finished surfaces. Machined faces usually have some bolt or stud holes in them, which provide an easy way for attaching covers of wood from $\frac{1}{2}$ to 1 in. thick or more. Electrical machinery, which may be severely damaged by rain or damp, should preferably be packed in stout wood cases, the framework made of baulks of timber 6 in. square or more, and the sides 2 in. thick, so as to offer ample protection against damp or injury. In addition, as much as possible of the metallic parts of electrical machinery containing windings should be wrapped round with two or three layers of stout brown paper securely tied on, in order to absorb any moisture that may form on the metallic surfaces by "sweating". The tendency to sweat almost invariably occurs when a body of metal, which has cooled down to the temperature of the outside air, is taken straight into an engine-room or building where the temperature of the air is considerably warmer than the temperature of the outside air. Sweating of big bodies of metal is often very severe, and results in pools

of water forming in cavities or recesses in the surfaces exposed to the warm air, and in the case of electrical machinery this free moisture may cause considerable damage by soaking into the insulation on the electrical windings.

Unloading, or Removing Piece from Trailer.—In general, while an overhead crane is usually available for loading up at the dockside, goods-yard, or makers' works from which the piece is being transported, it happens more frequently than not that no cranage facilities exist at the receiving end, and means have therefore to be improvized for unloading. A very great deal of time and trouble in unloading can often be saved if the conditions at the receiving end are known when the piece is being loaded up at the dispatching end. Where good cranage facilities exist at the receiving end, no particular provision need be made when loading up, but if the piece has to be unloaded by means of jacks, and has to be rolled into position, the operations at the receiving end can be greatly facilitated, say, by the provision of wooden skids (if necessary shod with flat iron) under the piece, or by letting some convenient part of the piece overhang so as to allow a jack to be placed underneath easily; if the dimensions of the piece will allow it to be done, it may be a considerable help at the receiving end to have the piece loaded up across the trailer instead of lengthwise on it; each problem, however, requires its own solution, and it is very much the exception rather than the rule to have two haulage problems identical in all respects.

In the case of boilers, or similar pieces which can be rolled, the easiest way is to build up, next to the trailer, a platform of beams of wood or steel girders, or beams laid on a bank of earth, so that the height of the platform is approximately the same as that of the trailer. If the steam-tractor has a winding-gear, as described earlier, the tractor is brought round to the side of the piece nearest the baulk or platform, a steel-wire rope is thrown over the top of the boiler and attached to any convenient man-hole opening, or anywhere where a good grip can be secured. The tractor then starts to wind up the rope and thus rolls the boiler off; it may be necessary to assist the tractor by means of a couple of hand-operated hydraulic jacks suitably placed under the boiler. The same process can be used if the trailer breaks down on the road and the piece has to be transferred to a new trailer; the latter is brought close alongside and the piece rolled from one to the other by means of one or more tractors, assisted by hydraulic jacks.

Where rolling cannot be employed, the piece will have to be jacked up, a little bit at a time, and packing, in the form of short ends of heavy timber, be placed under the piece on which to rest it while the jacks are being packed up for a further lift. This process is continued until sufficient room has been obtained below the bottom of the piece and the trailer to allow two or more heavy baulks of timber or steel girders to be passed through underneath. These baulks or girders must be long enough to extend well clear of the sides and wheels of the trailer, and be strong enough to support the whole weight of the piece. The girders are then securely packed up at the ends and the piece lowered slowly by means of the jacks till it rests on the girders. The trailer is then free to be drawn away, and the piece lowered

slowly by jacks down on to the ground or previously prepared way along which the piece can be taken on rollers. For weights up to 5 or 6 tons hollow steel rollers are most suitable (old boiler tubes 4 in. diameter and about $\frac{1}{4}$ or $\frac{3}{8}$ in. thick do very well); for heavier weights solid rollers should be used, otherwise the rollers may be crushed. The larger the diameter of the rollers the better, though the diameter is limited by the weight of the larger rollers, which makes them awkward to handle. As a rule, solid rollers from 2 to 3 in. diameter will be found quite big enough to handle comfortably; old line shafting does very well for this purpose. The rollers should be long enough to extend well beyond the sides of the piece; if the latter has any grooves or projections on the under side, two or three girders or baulks should be placed on the rollers first before the piece is lowered down on to them; unless this is done the rollers are apt to get locked by the groove or projection and greatly increase the difficulty of moving it. The rollers should be spaced every 2 or 3 ft., and as one roller works out at the back is should be immediately placed again in front of the piece. If it is desired to move the piece straight ahead, the rollers are placed square across the track; if it is desired to move round a corner, the rollers should be set at an angle to the piece, but always square across the track; care should, however, be taken that any rollers set at an angle to the piece are long enough to continue to offer support when the roller has worked back as the piece progresses.

To move the piece along the rollers, two or three crow-bars can be used when the track is good. Where more power is required, a quick-acting jack of the Barrett type will be found most useful, and two or more can be used. These jacks are usually fitted with a projecting foot which is part of the rising and falling column; the foot projecting is very useful for getting underneath the weight which is being rolled, and lifting it in case it is necessary to free a roller which may have got out of place, or got locked.

Alternatively, if it is possible to secure a hold on a tree or on a heavy beam placed across a doorway, or on a girder, a set of chain-blocks will draw the piece along on the rollers easily, but this is rather a slow process. Considerably more speed can be made if an overhead crane is available in an adjacent building, or if a traction engine can be obtained, with or without drum winding-gear. The crane or engine is made to pull on a long wire rope, which passes round a snatch-block fixed some distance in front of the weight; if necessary the rope is taken round one or more additional snatch-blocks in order to enable the crane or engine to exert a pull in the right direction.

As a rule, it always pays to take the piece as close up to its final position as possible by means of the traction engine and trailer, so as to cut down any rolling to a minimum. This frequently means that the trailer has to be dragged over rough or unprepared ground, or made-up ground that may be soft and yielding. It is usual, therefore, to carry with the tractor a number of steel plates about $\frac{5}{8}$ or $\frac{3}{4}$ in. thick, which are laid down on the ground first, before the trailer is dragged over. These plates, on account of their large area, distribute the weight over the soft ground and lessen

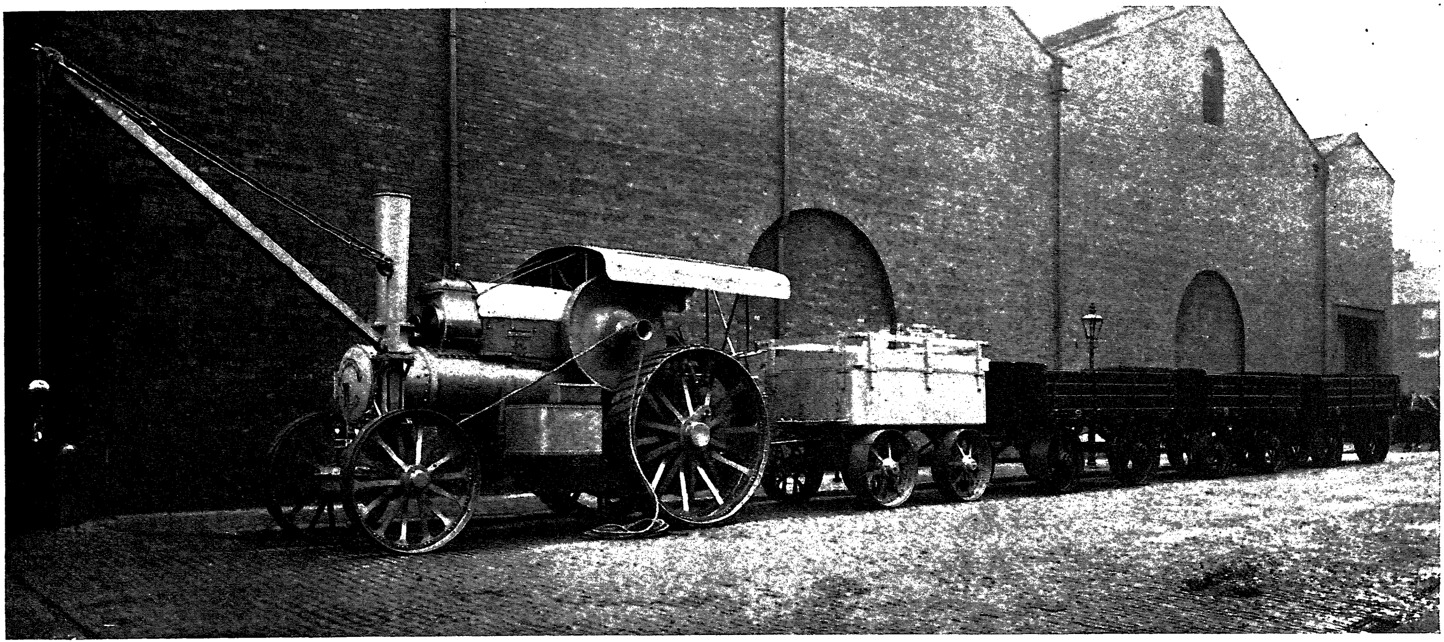


Fig. 6.—Traction Engine and Train of Spring-mounted Wagons (made by John Fowler & Co., Ltd., Leeds)

Engine provided with small jib crane for loading and unloading wagons. Tender for water and fuel behind engine, for long runs, military field service, &c.

the risk of the trailer sinking in. The most convenient size and form of these plates is a disc of about 30 or 36 in. diameter; this enables the plates to be easily rolled on edge by one man from the rear of the trailer after the latter has passed over them, and placed again in front; in laying the plates down it will be found advantageous to let them overlap a few inches.

A word of caution is necessary when moving a heavy piece, either up- or down-hill, where the angle of inclination is sufficient to cause the piece to start off on its own or to continue to move once it has been started. Despite the fact that most trailers are fitted with some form of hand screw-on brake, it is always advisable to have short ends of timber handy, say 2 to 3 ft. long and 3 × 9 in. in section, or heavier (old railway sleepers cut in halves do very well), which are placed a few feet behind the rear wheels, if the piece is going up an incline, and a few feet in front if the piece is going down-hill. This ensures that if the hauling power (be it engine, wire rope, or chain-blocks) should fail, the piece can only run a few feet before being stopped. The steeper the incline the closer to the wheels should the timber stop-blocks be kept, so as not to allow the piece enough length to get up speed; if sufficient speed be obtained, the momentum of the piece may carry it right over the wood stop-blocks, with the result that a runaway occurs, the consequences of which may be disastrous.

Further Considerations.—Before a long haulage is attempted, it is desirable to make a flying survey of the route, in order to get a good idea of the difficulties that may be encountered, particulars of road surface, hills, bridges to be crossed over as well as under, carrying capacity of bridges, headroom under bridges, narrowest part of roadway, any awkward corners or turnings, possibility of obtaining supplies of water and fuel along the route, &c. Alternative routes may have to be decided on, even if considerably longer, if the proposed route is too hilly, or the bridges are unable to support the weight of the piece; where one engine can easily manage a given load on the level, it may take two or three engines to take the same load over a hilly road. Even though the route to be followed passes through towns conveniently spaced, it is advisable to attach to the rear of the train a caravan in which the men can sleep and carry the necessary supplies of food; this enables the greatest amount of progress to be made when the conditions are favourable. On long haulages it is also necessary to attach a wagon to the train, which is designed to carry reserve supplies of fuel and water, as well as all the appliances and tools that may be required along the road; hydraulic jacks, both long and short, a length of wire rope, several stout snatch-blocks, steel plates for use in soft parts of the roadway, several short baulks of timber, short ends of lighter timber for making wedges, &c., axes, shovels, picks, &c., spare parts for the engines, &c. The foregoing remarks will simply serve to indicate the nature of the preparations that have to be made before undertaking a long haulage.

Haulage Contracts.—With regard to the costs of road haulage, these will vary in different parts of the country, due to local conditions. It is usual, however, to quote a lump sum for the haulage of any given piece,

the sum being based on a rate of so many shillings per ton. Thus, on a recent contract involving the haulage in parts of an electric generating set, weighing 60 tons, from the railway goods-station to a power-station some $2\frac{1}{2}$ miles away, the price quoted was £60, or at the rate of 20*s*. per ton. On longer haulages, of course, the price per ton is correspondingly higher. The price also will be higher if any of the parts are unusually bulky or heavy, so that special care has to be taken or special provision made.

Before any work is done a contract in writing should be properly completed, setting out exactly what the haulage contractor has to do, the price to be paid for the actual haulage, and the rates for men and engines, &c., in case demurrage or delay occurs for which one or other of the parties is responsible. In particular it should be stated which party unloads at

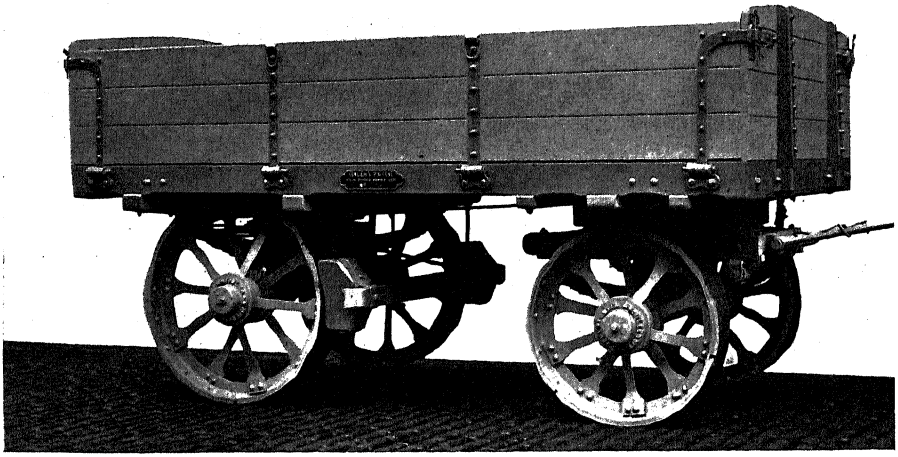


Fig. 7.—Spring-mounted Trailer for loads up to 10 tons

the destination, and precisely where the piece must be unloaded. With regard to insurance, it is the custom for haulage contractors to have a permanent policy which covers all risk of injury to men, plant, and goods. The premiums paid for such a policy will depend on the ability and reputation of the haulage contractor; a firm that has a good record can obtain such a policy for a much smaller premium than would have to be paid by a firm with less experience. Though the piece is thus covered by the haulage contractor's policy, it is advisable for the owners of the piece to take out a policy themselves, in order to provide against all possible risk of damage.

Haulage Weights and Distances.—Though there has always been a certain amount of long-distance haulage of heavy weights in Great Britain, the bulk of this work is done over comparatively short distances, say up to 5 or 10 miles. During the Great War, however, the amount of long-distance haulage increased very considerably, particularly in the later stages of the War, when the submarine campaign carried on by the enemy made transport by sea a risky business. Marine boilers weighing up to 40 tons were

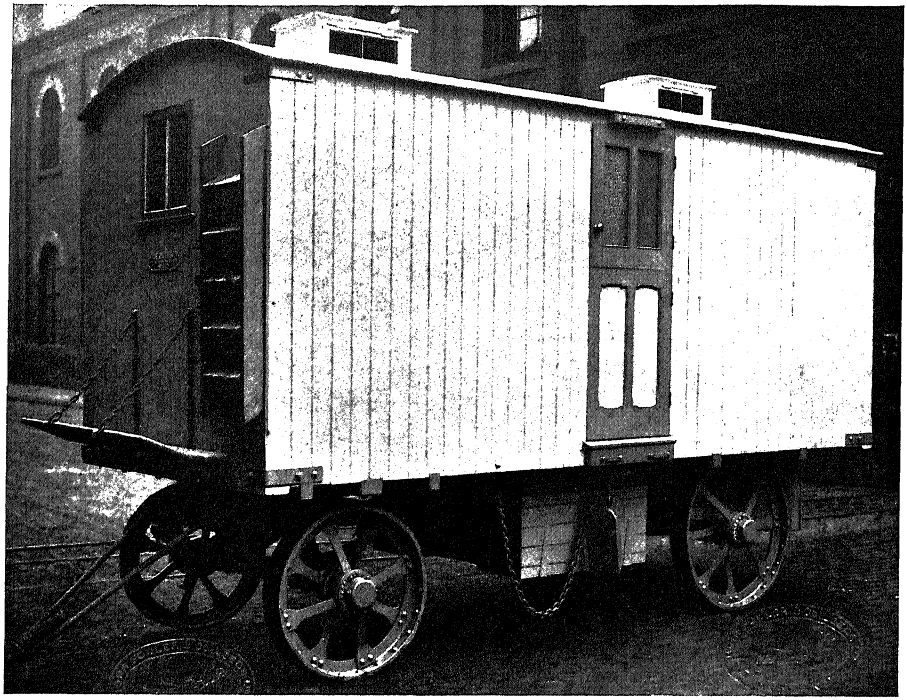


Fig. 8.—Caravan for Accommodation of Men on Long Haulage Contracts

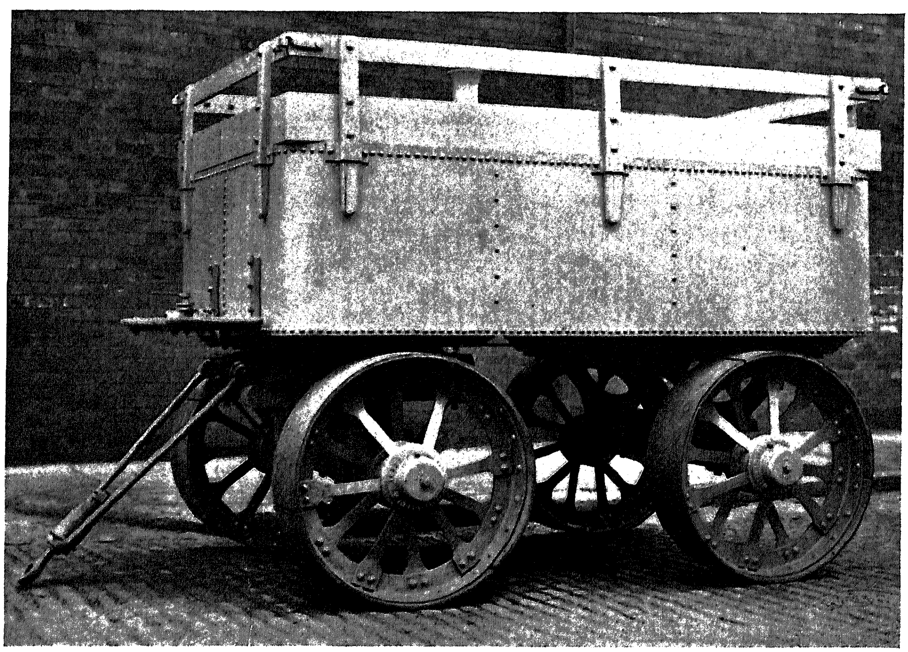


Fig. 8A.—Tender for Water and Coal or Oil Fuel

hauled over 400 miles successfully, and many smaller weights were transported by road on longer and shorter runs. Weights up to 95 tons in one piece have been transported by road, but the distance has necessarily been small, and the possibility of transporting such weights has been largely due to the road or way having good, solid foundations. In general, the main highways follow a track which is chosen on account of the easy gradients, and not on account of the ground over which it passes. The nature of the foundations, therefore, varies considerably; in some parts the road is laid on stone or conglomerate base; in other parts the road is laid over boggy or peaty ground. It is a risky business taking a heavy piece over a soft bit of road, on account of the danger of sinking in; the risk, however, can be minimized by plating the road, though even plating will not always enable very large weights to be taken over soft ground. The distance covered per day varies largely according to circumstances; where the road surface is good and the road level, 50 miles can be covered in 12 or 14 hours, but when the weight being transported is very large, the distance covered may be very much less; if the road is soft, either naturally or due to rainy weather, the progress may be no more than a mile or less per day. As an instance of the extra time required for haulage where the conditions were unfavourable, an actual haulage contract, involving the transport of a 30-ton piece for 240 miles, occupied two and a half months, instead of an estimated two to three weeks; the trouble experienced which caused the delay was due almost entirely to the bogie wheels of the trailer conveying the piece being much too small in diameter, with the result that not only was the time taken very much longer and the costs, therefore, higher, but a great deal of damage was done to the highway and to bridges over which the weight passed.

B.—By Rail

The transport of heavy machinery by rail is, from an engineering point of view, a much simpler proposition than transport by road; that this is the case will be realized when the following points are considered:

1. The tractive effort or haulage-power available in a rail locomotive, is very much greater than the haulage-power of road transport engines.

2. The railroad or permanent way is a properly constructed metal track, well supported on sleepers, which distribute the weight of rolling-stock and goods; the danger, therefore, of sinking in is reduced to a minimum.

3. The gradients over which the heavy machinery has to pass on railroads are very slight compared with road gradients, and do not exceed 1 in 40 as a maximum.

4. On account of the comparatively large diameter of wheels used on railway wagons, and also due to the wheels running on a smooth metal rail, the power required to draw a loaded wagon is much less than in road transport, and the margin of power available in the locomotive is therefore correspondingly greater.

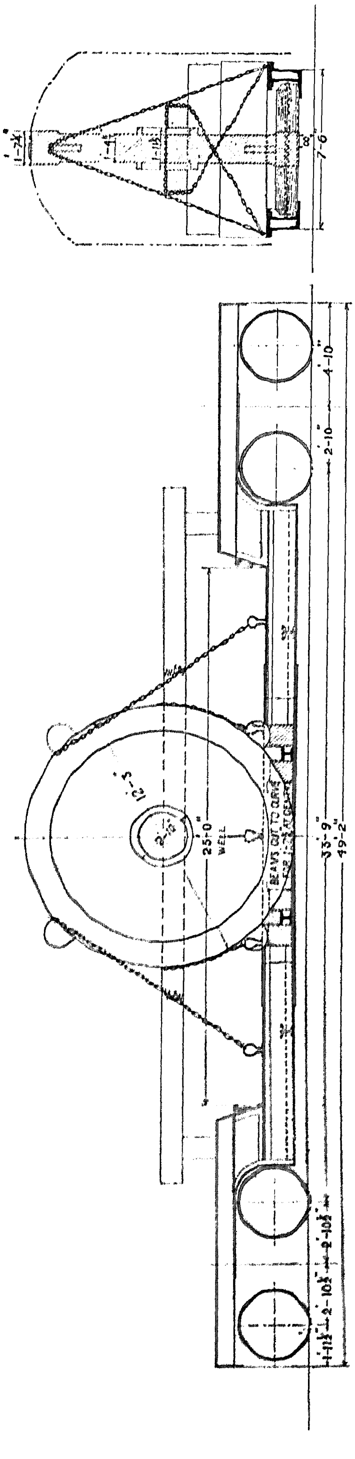


Fig. 9.—Method of Loading up Fly-wheel weighing 38¾ tons. Note method of supporting and cross-bracing

Although the two limiting factors which determine the possibility of transporting a piece of machinery are weight and size, it may be stated generally that in rail transport the only limiting factor is size or overall dimensions. It is very unusual to find machinery, which can be dismantled into its component parts, where the weight of any one part exceeds 50 tons; as weights up to 160 tons in one piece have been transported by rail, weights up to 50 tons or thereabouts do not offer any special difficulties, provided their overall dimensions are such as to bring them "within gauge" when loaded up on a railway wagon. These gauge limits are determined for width by the gauge of the track and the clearance between the tracks (where a double track is used); also by the width of tunnels or the distance between the supporting piers of bridges passing over the railway track. The vertical limit is determined by the height of tunnels or overhead bridges.

Thus in Great Britain the limit for width is 8 ft., though on the systems of some railway companies 8 ft. 6 in. is allowed. The limit for height is 12 ft. 11 in. above the top of the running rails, measured in the centre of the track, and 9 ft. 10 in. above the top of the running rail, measured at the side of the loaded wagon. Goods which are within gauge are accepted by the railway companies without special arrangement, but where the dimensions of a piece of machinery or goods are such that when loaded up the piece will be "outside the gauge", special arrangements have to be made, and each case has to be considered on its merits. As a rule, before the transport of any extra heavy or extra bulky piece is attempted, a scale drawing of the piece is sent to the railway company and dimensioned drawings are made by the railway engineers, showing how the piece

must be loaded up, if it is possible to take it at all by rail. An example of this kind is given in the accompanying drawing, which shows the method adopted for loading up a cast-steel flywheel disc, 12 ft. 3 in. in diameter, weighing $38\frac{1}{2}$ tons. The difficulty of loading up was increased by the fact that there was no hole through the centre of the flywheel hub or boss, through which a temporary shaft might have been put, resting on the heavy wooden baulks at the sides of the wheel. The weight of the baulks and packing used was approximately 4 tons, making a total load of $42\frac{1}{2}$ tons; the tare (or weight) of the crocodile or well wagon was 28 tons, thus making the total weight of wagon plus load over 70 tons; this gives a load per axle of over $17\frac{1}{2}$ tons. It is this figure, i.e. load per axle, which must be carefully considered, in view of the increased bending moment obtained on railway bridges when the concentrated load per axle, or per foot of rail, exceeds that for which the bridge has been designed.

To show how the distribution of weight is effected, a photograph is shown of a special gun-truck built to carry very heavy guns, where the gun without mounting weighs 160 tons; of this weight 107 tons is supported at the breech end and 53 tons at the muzzle end. It will be seen that the breech end is carried by 12 axles and the muzzle end by 6 axles, giving a total load per axle of about 13 tons, including the tare of the wagons. On account of the great length of the gun, the supports at each end have to be made swivelling, in order to allow the gun-truck, when loaded up, to negotiate curves on the railway track.

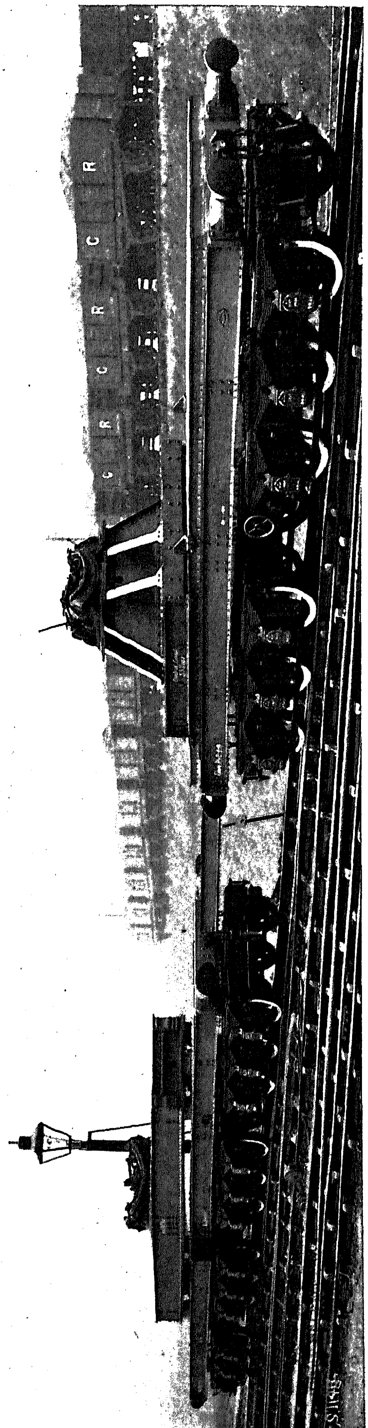


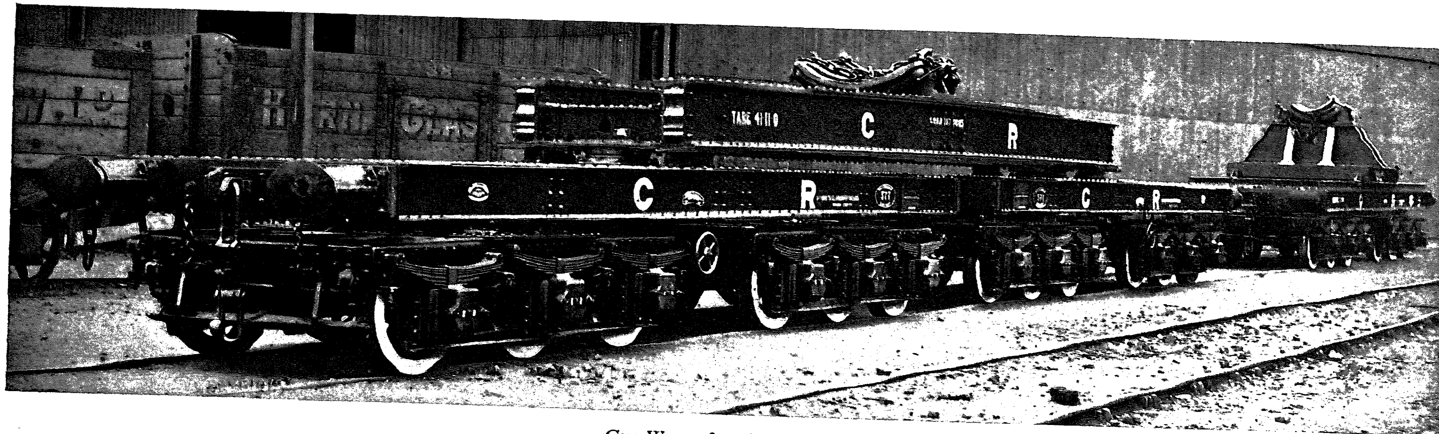
Fig. 10.—Special Gun-wagon to carry 160-ton Guns

Exceptionally Bulky Traffic.—This is accepted for transport only by special agreement, and special terms are charged on account of the extra precautions that have to be taken and arrangements that have to be made. The vertical gauge dimensions, given previously, allow for a certain clearance below the crown of a tunnel or an overhead bridge, and under certain circumstances the gauge dimensions for height can be exceeded, but only by a few inches. On the other hand, the gauge dimensions or width can be greatly exceeded in case of necessity, and many cases are on record where a piece, loaded on a wagon on the "up" line, has hung over the side of the wagon and fouled the traffic on the "down" line. The most notable case of this kind occurred during the Great War, when a cast-steel propeller-shaft bracket, weighing over 30 tons and having two or three very long arms or ribs, was taken by rail over 100 miles of main-line railway in Great Britain. The arms were so long that when the wagon carrying the piece was travelling on the down line, they overhung not only the up line but even extended 12 in. over the platforms on the up line. Such traffic can be handled only on Sundays, when both tracks can be entirely cleared, and as a rule is made up into a special train; drawings to scale are made for each part of the track where the clearances may be on the "tight" side, and special instructions are printed and handed to each signalman and stationmaster along the line where the over-gauge traffic will pass, several days in advance; these instructions set out in detail exactly what precautions are to be taken for the safe working of the special traffic.

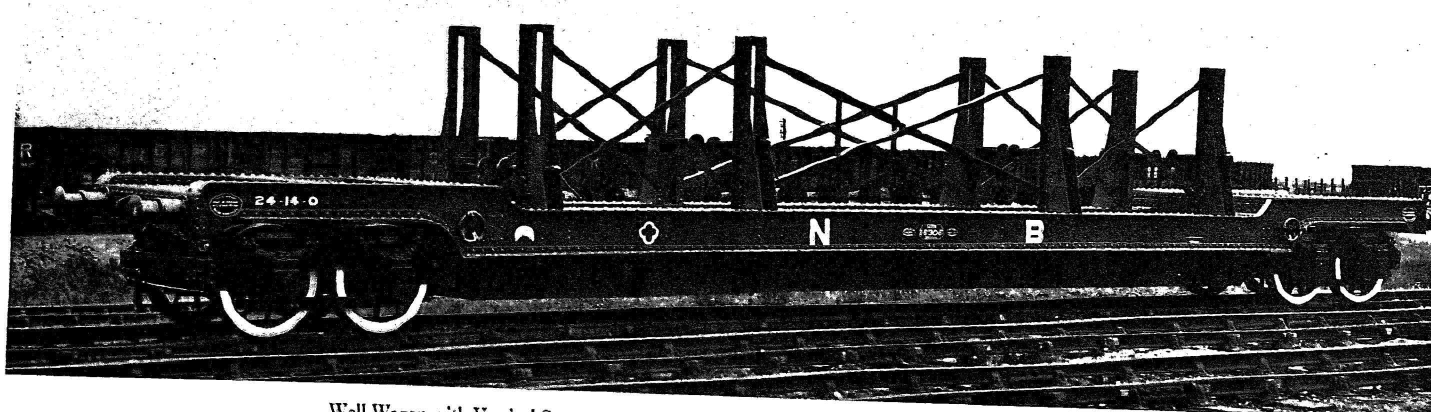
Where the height of the railway wagon and the dimensions of the piece are such that no overhanging part of the piece is below platform level, the overhang can be greater than would otherwise be the case, as the projecting parts will then clear the platforms along the route.

Special care is taken by the railway companies when transporting extra-heavy or bulky traffic; an inspector travels all the way with the traffic, and whenever the train stops an examination is made of the railway wagon, axle-boxes, couplings, &c., as well as of the supports, wedges, and chain lashings securing the piece to the wagon. Care must be exercised in shunting such traffic, and nothing in the way of "flying shunting" must be attempted; by flying shunting is meant the practice of dealing with empty or lightly loaded wagons by getting them up to a certain speed and then letting them run on by their own momentum till they strike the train of wagons to which they are to be attached. In the case of a wagon loaded with a heavy piece of machinery, the shock which results when the wagon strikes a train is very severe, and has frequently resulted in carrying away the packing supporting the piece and caused considerable damage to the wagon, as well as to the piece itself.

For the conveyance of heavy machinery special wagons are used, of which examples are shown in the accompanying photographs. Where the dimensions of the piece are "within gauge", flat-topped wagons are used, and if necessary two or more of these wagons are coupled together for the purpose. When the dimensions of the piece are such that it can be brought



Gun Wagon for 160-ton Guns



Well Wagon with Vertical Supports and Adjustible Clamps for carrying large Boiler and Ship Plates

Fig. 11

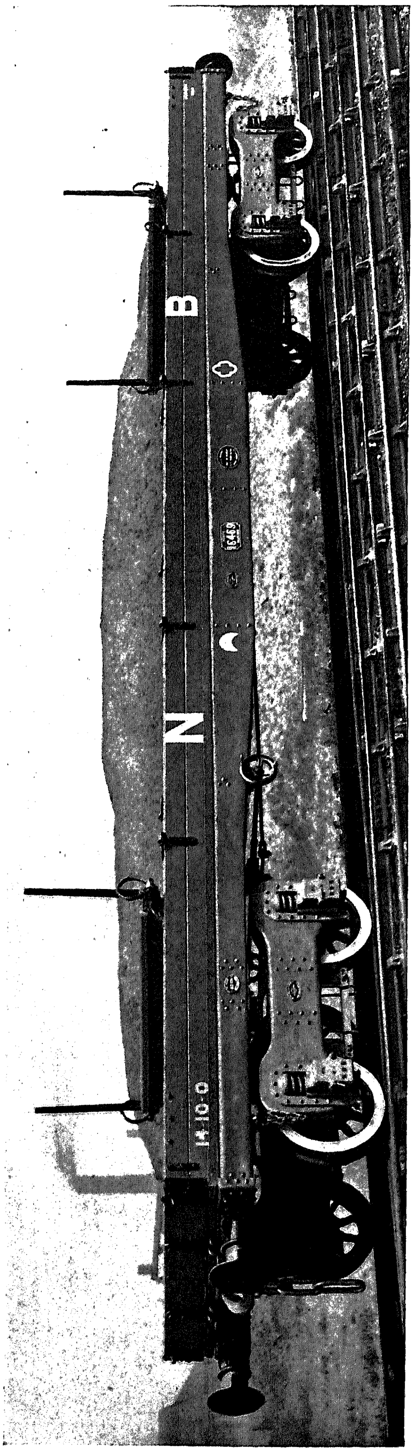
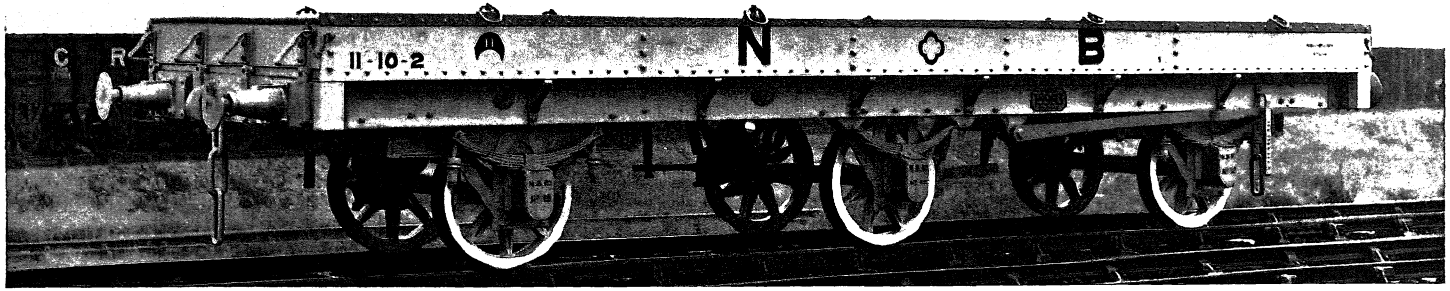


Fig. 12.—Flat-topped Wagon for Carrying Steel Girders, Wooden Baulks, &c.

within gauge by using a crocodile or well wagon, the latter type is used; as a rule they are of considerable length, and are supported at each end on a bogie, or swivelling wheels, in order to get round curves. While the amount of swivelling provided is quite sufficient to suit any curves on the railway companies' tracks, trouble is sometimes experienced when these wagons are shunted on to the private sidings belonging to a power-station or works, where the curves may be very much sharper in order to suit local conditions, and result in the wheels "locking" on the wagon frame. In a case of this kind the piece has to be taken to the nearest goods-yard where suitable cranage may be obtained, and the piece either transferred to a flat-topped wagon, or else unloaded and hauled to site by road.

Loading and Unloading.—With regard to loading and unloading heavy or bulky goods, this will be carried out by the railway companies and at their risk so long as the piece does not exceed in weight the capacity of the cranes available at the loading and unloading stations. At the same time they may refuse to handle any pieces exceeding 3 tons in weight; but as a rule the limiting conditions are the facilities existing at the loading and unloading stations. Before lifting any piece which, on account of its size or peculiar design, may present unusual difficulties in slinging, the railway company may send for representatives of the senders or of the consignees and request them to advise how the piece should be slung. Slings and lifting tackle will



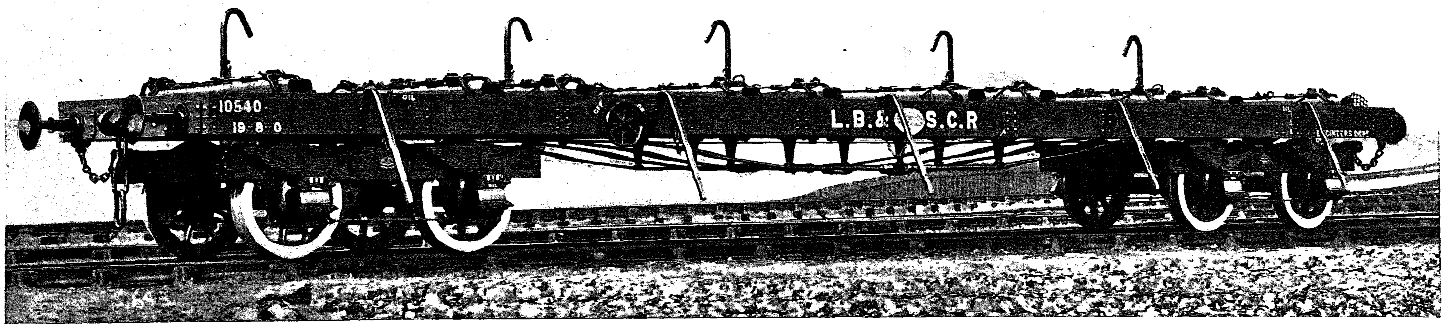


Fig. 13.—Flat-topped Wagons for Heavy Loads

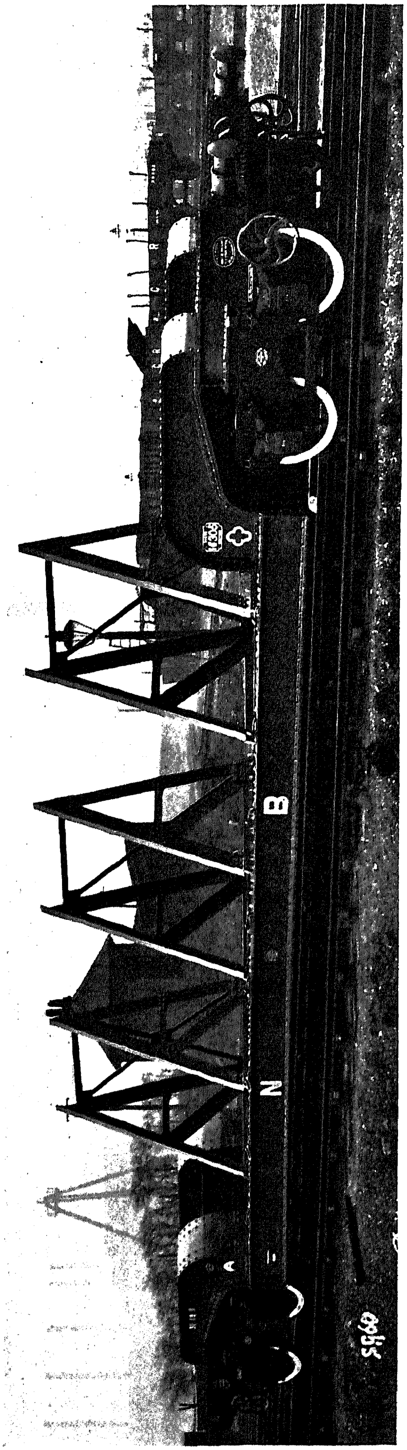


Fig. 14.—Crocodile Wagon with Special Framework for supporting large Steel Boiler Plates

be provided by the railway companies, but as a rule these will be found to be chains, which are open to objection on account of their tendency to press into and damage any machined parts of the piece being lifted, and also on account of their liability to snap off short, due to crystallization of the links, especially in cold or frosty weather. Hence the consignor or consignee may find it advantageous to supply his own wire-rope slings.

Where the weight of a piece is beyond the lifting-power of the railway companies' cranes, the consignor or consignee will have to find and pay for the special cranes required for loading and unloading. In certain cases, however, it may be more convenient to load or unload the heavy piece by means of hydraulic or screw jacks; this is particularly the case in power-station work, where large pieces of machinery may have to be unloaded by means of jacks, on account of the power-station crane not being ready for use, and also to obviate paying heavy demurrage charges on special wagons; for special wagons (30 to 50 ton, flat-topped wagons, crocodile or well wagons, plate wagons, &c.) the charges for demurrage are about double the demurrage charges for ordinary wagons; this is done in order to compel the consignee to unload and release the wagon at once, as most railway companies do not possess more than a few special wagons.

If no suitable cranes are available at the sending or the receiving end, and it would be too risky to use jacks, the railway companies will hire out their breakdown cranes, which are used in case of break-

downs or collisions. Breakdown cranes vary in lifting capacity up to about 50 or 60 tons, the larger sizes generally being steam driven; the charges for the use of these cranes va y with the distance from the depot where the crane is kept to the station or site where the lift has to be made; in a recent case, where a lift of 25 tons had to be made, some 30 miles from the crane depot, the charge was £10 per day or part of a day. This charge is reasonable when it is pointed out that the charge includes making up a special train of loco, crane, and van, men to work the crane, men's wages, and cost of fuel for the loco. This type of crane is also used in case a wagon, loaded with a heavy piece of machinery, breaks down *en route*; in a case of this kind, the breakdown crane and a new wagon are taken to the place where the breakdown has occurred and the piece transferred to the new wagon, or the breakdown crane may be used for lifting or supporting the wagon carrying the heavy piece, while new parts, axles, wheels, &c., are being fitted.

C.—By Sea

In the transport of heavy machinery by sea the only difficulties of an engineering nature occur in the loading at the port of shipment and unloading at the receiving port. The transport of goods by sea, however, is complicated by the procedure which has to be gone through before they can be sent from one country to another, and by the regulations of the receiving country dealing with the import of various classes of goods, particularly of machinery. Thus in some countries there is such a heavy import duty on finished machinery that it is sometimes dispatched to these countries in an unfinished or incomplete condition, and the completion of the parts carried out on site by men sent out specially for the purpose. Transport of machinery by sea is also largely influenced by the handling facilities existing at the port to which the goods have to be consigned; this question also influences the general design of the plant to be sent overseas. Thus it has happened not infrequently that a ship has arrived at its port of destination and been unable to unload a heavy piece of goods, due to the cranes at the port not being large enough to handle such weights. When a case of this kind occurs, the ship has to proceed to the nearest port, which may be hundreds of miles away, where cranes of sufficient sizes are obtainable; the piece is then unloaded and transferred either to a vessel having special lifting facilities or to a barge, which is towed to the port of destination and unloaded by means of a floating crane, or the piece can be placed on the deck of the floating crane. Recently a piece of machinery, weighing approximately 50 tons, was dispatched by rail to a power-station in London, on the south of the River Thames. Due to local difficulties in London, on account of railway bridges, &c., it was taken to the docks on the north bank, and special arrangements were made with the port authorities to lift it from the railway wagon by means of a floating crane; it was placed in a steel barge and towed behind the floating crane across the river

and unloaded in the power-station yard by the floating crane; it was then lowered on to rollers and dragged into the power-station by means of the station crane pulling on a long wire rope, led round a snatch-block, as previously described.

Besides considerations of an engineering nature, the transport of machinery by sea often involves questions of international finance. Probably the best way of indicating the points which have to be considered will be to take a hypothetical case where a manufacturer in Great Britain has accepted an order for machinery from a customer abroad, the order including for delivery c.i.f. (i.e. cost, insurance, freight) at the overseas port indicated by the customer in his order.

Packing.—When the machinery is completed, the manufacturer must pay particular attention to the packing. All parts of electric machines that are very easily damaged by damp must be wrapped round with several layers of thick paper as a preventive against " sweating ", and the parts should then be securely packed in strong wooden cases. As a further safeguard against damage by damp, some manufacturers line the inside of cases containing electrical goods with sheet zinc, and solder up all the seams of the lining. It may be taken for granted that the packing of goods for transport by sea has to be very much stronger and more secure than in the case of transport by land, on account of the greater risks run. The protection of finished surfaces of castings and bright parts should be carried out as described previously, but more thoroughly. The customer will indicate on his order what shipping mark and lettering should be painted on each piece or case. This mark usually consists of a diamond or other conventional figure, with certain distinctive letters marked in it, and the name of the port to which the goods are consigned printed below; the size of letters used should not be less than 6 in. high. In addition to these shipping marks, each piece or case should be numbered consecutively; this is important, as will be seen later.

Shipping Agent.—The shipping agent acts at the loading port for the manufacturer. When the manufacturer knows that he will have the goods ready by a certain date, he sends the shipping agent an approximate specification, in which particulars are given of the plant to be shipped, and detailing pieces exceeding 2 tons in weight or 15 ft. in length. The shipping agent is requested to obtain rates from the companies whose ships run to the port of destination, and particulars of sailings approximately at the required date. In quoting the rates, the first one is the basic rate, and is for pieces or packages weighing up to 2 tons; progressive rates are also quoted, e.g. from 2 to 4 tons, 4 to 6 tons, &c., according to the weight of the pieces set out in the manufacturer's specification.

Having obtained the necessary particulars from the shipping agent, the manufacturer either " opens an insurance " or instructs his shipping agent to do so, that is, an insurance company that accepts this class of business is notified, and an insurance opened for a sum slightly exceeding the total value of the shipment, plus insurance and freight, with a margin

of say 15 per cent added to the total, to cover any probable rise in the cost of replacement in case of loss. It is usual to insure goods from manufacturer's warehouse to warehouse at the receiving port. After the total value of the shipment cost plus freight and other charges, with the covering margin added, has been determined, the insurance policy is " closed ". In shipping machinery it is always advisable to take out an " all risk " policy, to cover loss, breakage, or damage from any cause whatever. There are many types of insurance policies for goods transported by sea, and the whole subject of marine insurance is a large and complicated study in itself, which it is not intended to take up in this work. The cheaper policies exclude certain risks, but as a rule it will be found the wisest course to take out an all-risk policy.

Having decided to ship the consignment by a certain vessel, the manufacturer dispatches, by road or rail, the plant, consigned to the steamer at the loading berth specified by the shipping company to the shipping agent and passed on by him to the manufacturer; thus the plant may be consigned to S.S. *London*, at Queen's Dock, Glasgow. At the time the manufacturer dispatches the goods he notifies the shipping agent, so that the latter may be on the lookout for the goods on their arrival at the docks; the shipping agent acts for and on behalf of the manufacturer while the goods are at the docks. If the goods arrive in an undamaged condition at the docks, the shipping company's wharfinger gives the railway company or road haulage contractor (if the goods have arrived by rail or road) a " clean signature ", i.e. a signature free from any comments, e.g. " not examined ", &c. The remuneration of the shipping agent is usually $2\frac{1}{2}$ per cent of the freight charges, though in the case of consignments involving special difficulties, on account of their weight or bulk, this figure may be somewhat higher. In Great Britain freight rates include loading charge, but on the Continent, as a rule, they do not. At nearly all large home ports there are special cranes available for loading and unloading very heavy pieces; the capacities of these cranes vary up to 150 tons or more; they are usually owned by the harbour or port authority, and special charges are made for their use. If special cranes are required for lifting any parts of the consignment under consideration, the shipping agent notifies the shipping company, who in turn make arrangements with the port authority. If special cranes are in frequent demand, the port authority arranges the time when the special lifts have to be made, and it is the duty of the shipping agent to see that the pieces are delivered at the special crane before the given time; the ship also will have to be transferred from its ordinary loading berth to the wharf on which the special crane is placed, and immediately the pieces have been loaded the ship will have to be removed. The pieces forming the consignment are lifted on board by the shipping company, who engage stevedores, or employ their own stevedores, to do the slinging and supervise the actual lifting. When the shipping agent hands the pieces over to the shipping company, or their representatives, for lifting on board, the shipping agent has finished with the pieces, and the responsibility for seeing that they are

properly and securely loaded on board rests with the shipping company. The shipping agent pays the harbour dues to the harbour trustees, or port authority, and also has to pass a Customs form within seven days of the vessel's clearance (or departure), stating briefly particulars of the shipment; this information is required for the Customs statistical purposes.

Documents.—When the manufacturer has decided to send the goods by a particular ship, he sends a detailed specification to the shipping agent of the consignment. This specification shows clearly the marks, number, weight (both with and without packing), measurement, and description of each package. The shipping agent takes out bills of lading, in triplicate, each copy of which is stamped, and one or two unstamped copies for filing purposes. The bills of lading are usually made out in the manufacturer's own name as shipper, and consigned " to order " or " to order for account of Messrs. ——— " (consignees). This set of bills of lading, when signed, is the " title to the goods "; the shipping agent sends them to the shipping company for signature, but the latter will not sign them until the goods are on board the ship. When the shipping agent gets them back, he forwards the bills of lading to the manufacturer, together with account for freight, harbour dues, &c. The manufacturer then makes up his invoice and includes therein the freight, insurance, and all other charges made by the shipping agent. He draws a bill (1st, 2nd, and 3rd of exchange) on his customer, and forwards the following documents to his banker:

Bill of exchange in triplicate,
Bill of lading in triplicate,
Stamped insurance policy,
Invoice for goods in triplicate,

these documents being duly endorsed by the manufacturer. The latter also advises his customer that he has made the shipment, and sends him invoices and shipping specification with particulars, shipping marks, weight, size, and number of each package; he also intimates to his customer that he has drawn a bill on him at the agreed term (i.e. sight, 30 days, 90 days, or whatever may be the customary usance in the trade to the port of destination), and advises that the shipping documents are attached to the bill of exchange, and will be delivered to him on his acceptance or payment (whichever has been arranged) of the bill of exchange. The manufacturer's banker forwards the documents to the bankers at the port of destination, who are his agents, and, after the customer has either " accepted " or paid the bill, the shipping documents are handed to the customer, and when payment has been made the bank's agents advise the manufacturer's banker, who in turn notifies the manufacturer that his account has been credited with the proceeds of the bill, i.e. the amount of the bill less charges.

On receipt of the documents, the consignee endorses the bill of lading and hands it to the ship's agent, who, in return, hands him a delivery order. After paying the Customs duties and receiving a receipt, the consignee pre-

sents both documents to the stevedores who unload the ship, and thus obtains possession of the consignment. If the consignee is situated some way inland, the duties mentioned above may be carried out by a shipping agent.

The unloading of the pieces at the receiving port is usually carried out by the shipping company, but the harbour and other dues are paid by the consignee.

Rates and Tariffs.—All freight rates are quoted so much per ton "at ship's option ", i.e. the shipping company reserve the right to charge for the goods by weight (per ton of 20 cwt.) or by volume (per ocean ton of 40 c. ft.), whichever is the more remunerative to them. For the purposes of comparison, the rates for 1922 for machinery from Glasgow to a number of representative ports are given:

1. Alexandria ..	60/–, plus 10 per cent primage.
2. Buenos Aires	45/–, plus 75 per cent surcharge, plus 10 per cent primage.
3. Calcutta ..	50/–, plus 10 per cent primage.
4. Cape Town ..	70/– net.
5. Durban ..	75/– net.
6. Hong-Kong ..	82/6, plus 10 per cent primage.
7. St. John ..	40/– net.
8. New York ..	50/– net.
9. Rio de Janeiro	65/–, plus 75 per cent surcharge, plus 10 per cent primage.
10. Sydney ..	90/–, plus 10 per cent primage.
11. Wellington ..	90/–, plus 10 per cent primage.
12. Yokohama ..	82/6, plus 10 per cent primage.

The rates quoted are basic rates for pieces or packages up to 2 tons weight, shipped direct to the port indicated.

"Primage" is a surcharge made by "rings" or "conferences" of shipping lines trading to the same ports, in order to induce shippers to send their goods by the steamers belonging to the companies in the ring or conference; the primage is returnable at certain periods (usually every six or twelve months) provided the shippers have confined their shipments, during the period in question, to the steamers of the lines forming the conference. The intention is to prevent a shipper taking advantage of an " outside " steamer offering cheaper rates of freight, as doing so would entail forfeiture of the accrued primages. Where the goods are shipped to a port to reach which transhipment is required, the rates are much higher, due to the transhipment charges.

Exceptionally bulky or very heavy pieces, that either cannot be taken down into the ship's hold or will give the ship a better trim if not placed in the hold, are carried on deck and are known as " deck cargo "; special rates are quoted for deck cargo, and the words " deck cargo " are noted on the bill of lading against those pieces that will be carried on deck, and such pieces must be specially insured. It is obvious that such pieces, if liable to damage by weather or sea, must be exceptionally well packed; such goods

as a rule will not be accepted by an insurance company on an all-risk policy and must be separately insured. Certain shipping companies, in addition to the ordinary freight rate, charge an *ad valorem* rate of 1 per cent on the value of all pieces or packages which have great value, but are of relatively small weight or bulk.

Certificates of Origin.—In the case of shipments to countries which give a preferential tariff to goods manufactured in whole or part in Great Britain, a declaration to that effect must be written, typed, or printed on the back of the manufacturer's invoices and signed by him.

Declarations of this kind differ according to the country to which the goods are consigned; copies of the forms required by each country can be obtained at offices of the British Chambers of Commerce. In shipping goods to most European countries, it is necessary for the manufacturer to include amongst the shipping documents a "certificate of origin" for the goods. Some countries require a consular certificate of origin, i.e. a certificate viséd by their consul at the loading port; other countries insist on a Customs certificate of origin. Particulars of the requirements of each country can be obtained from a shipping agent; it may be remarked in conclusion that the requirements of this kind for South American countries are much greater and more precise than for any other part of the World, and the penalties for inaccuracy of statements on the consular forms are extremely heavy.

PIPE-WORK

BY

E. G. WEEKS, A.M.Inst.C.E.

Pipe-work

CHAPTER I

Pipes: Ferrous

Pipe-work has during recent years become a much more important section of mechanical engineering than formerly, particularly in connection with large modern electric power stations, where the increased steam pressures and temperatures used by engineers in the constant endeavour to obtain increased fuel economy call for most careful design and manufacture.

Considerable developments in this country have also been made in the use of pipes of great length for conveying surplus high- or low-pressure steam or gas from industrial plants to neighbouring power stations, where it is used in turbines or gas-engines for generating power direct or, in the case of gas, for firing gas-fired boilers for steam raising. The object of this section is to put before the reader information regarding the manufacture and design of piping, such as described above, and to give general information on pipe-work in a handy form for reference.

Pipes used in mechanical engineering may be divided into two main sections:

(*a*) Ferrous pipes.
(*b*) Non-ferrous pipes.

The former section includes (1) cast-iron and cast-steel pipes and (2) wrought-iron and mild-steel pipes which may be either riveted, welded, or solid-drawn. The latter section includes brass and copper pipes, and flexible metallic pipes.

Cast-iron Pipes

Cast-iron pipes are very largely used for low-pressure steam-, gas-, and water-pipes where weight is of no great importance. Cast-iron pipes are generally slightly cheaper, when completed, than steel pipes, but the question of cost is largely influenced by conditions regarding transport, nature of work, and labour available, and each individual case must be dealt with separately.

For awkwardly-shaped pipes and for bends, elbows, and tees or junction pieces, cast iron has considerable advantages, but at the same time it should

be noted that recent advances made in the building up of special mild-steel pipes by welding processes have brought into the field a serious rival to cast iron. The built-up mild-steel welded pipe referred to has the advantage that in many cases it can be obtained in a shorter time than the cast-iron pipe, and often this is of great importance, as usually the awkwardly-shaped pipe is a closing pipe required to complete an installation which is otherwise ready for commission. Cast iron should never be used for high-pressure steam-pipes, or for bends, tees, or junction pieces in steam mains, owing to its liability to failure, either due to excessive expansion stresses, faulty casting, or deterioration under the action of heat. The use of cast-iron pipes with steam pressures above 20 lb. per square inch should be discouraged.

With superheated steam particularly, it is imperative that cast iron should not be used, as the continued exposure to superheated steam causes a gradual growth of the metal, with its ultimate disintegration and subsequent failure.

Cast iron is, of course, largely used for gas and water mains, exhaust-steam mains and drains, &c., and for these purposes is excellent.

It is also particularly suitable for laying below ground, especially in ground which is corrosive, as is often the case with made ground. For the purpose of preventing corrosion, pipes should be treated with Dr. Angus Smith's solution. This well-known and tried process consists of dipping the pipes, after testing by hydraulic pressure, into hot baths composed of asphalt or bitumen, coal-tar, and resin in the following proportions:

Best crude Trinidad asphalt or bitumen ..	about 44	per cent.
Coal tar freed from water and naphtha ..	,, 55	,,
Resin	,, 1	,,

The bath of the above mixture must be maintained at a temperature of 350° to 400° F., and the pipes must be immersed for a sufficient time to allow them to acquire the full temperature of the bath. They are then slowly withdrawn, and the surplus coating allowed to drain off. In cases where the pipes may be exposed to warm atmosphere it is advisable to sprinkle dry sand plentifully on the covering, to render it less liable to run in hot weather and to protect the coating from injury in transit.

When laying pipes underground in made ground, additional protection from external corrosion can be obtained by casing the pipes round with a jacket of clay about three or four inches thick.

In most modern foundries cast-iron pipes are cast vertically, and this practice has much to recommend it, a more uniform thickness of pipe wall being obtained than when they are cast horizontally and the cores are liable to float or move.

When cast vertically, the metal is more uniform in quality, and bad or porous places are avoided, as any bad metal or scum passes to the top of the mould, and provision can be made for removing this by casting a portion of metal on the top end of the pipe, which will contain all the impurities, and which can be cut off after cooling.

Cast-iron pipes below 3-in. bore are generally cast in 6-ft. lengths as a

standard, and above 3-in. bore in 9-ft. lengths, while some foundries have facilities for casting the larger pipes in 12-ft. lengths.

The thickness of the walls of cast-iron pipes is controlled largely by working conditions, practical casting considerations, and probable variations of wall thickness. In practice the stress in the metal due to internal pressure rarely exceeds 2000 to 2400 lb. per square inch under test pressures, which should be twice the working pressure.

Cast iron used in the construction of pipes should be cast from best grey pig and scrap iron.

The following tests should be carried out with samples cut from the castings:

(1) Test pieces 0·798 in. diameter with a gauge length of 3 in. should have an ultimate tensile strength of 11 tons per square inch.

(2) Test pieces machined to 1 in. square section × 14 in. long should sustain without fracture a load of 2500 lb. concentrated at the centre when supported on knife-edges 12 in. apart.

For use in exhaust-steam or hot-water services, where expansion of the pipes will increase the loads on them, lower stresses than 2000 lb. are usual.

Cast-steel Pipes

As already mentioned, cast-iron pipes are unsuitable for use in high-pressure steam services, especially when highly superheated steam is used. When designing steam-pipes for such working conditions, it is advisable to dispense with castings as far as possible, and use wrought steel only by welding or riveting branches on to the pipes for connections and junctions. It is, however, not always possible to do this, and, under these circumstances, cast-steel tees and bends should be used.

The proportions of cast-steel bends and tees should be in accordance with the British standards.

All steel castings should comply with the British Standard Specification for steel castings for marine purposes, to which the reader is referred.

Cast-steel pipes and fittings should be clearly stamped or marked during manufacture with the words "Cast Steel", so that no confusion with cast-iron pipes may arise. This rule should be rigidly enforced, so that no risks may be run of installing by mistake a cast-iron pipe in a high-pressure steam service.

Wrought-iron and Steel Pipes

Steel pipes are manufactured in three ways, known as riveted, welded and rolled, or solid-drawn, depending on the internal diameter of the pipes and the purpose for which they are intended.

Riveted Pipes.—Before the advent of the welded and the solid-drawn pipes, all wrought-iron and steel pipes were made from plates

rolled into shape and riveted together. This form of pipe is still largely used, but the improvements in welded pipes have led to the adoption of the latter in many cases where riveted pipes had hitherto been used.

The jointing of riveted pipes is always carried out by means of flanges at each end of the pipe formed by riveting angle rings on to the pipe.

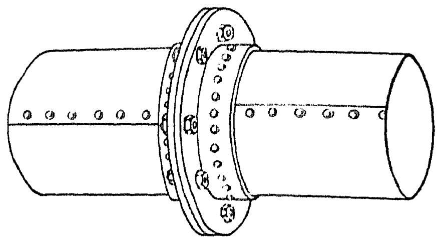


Fig. 1.—Riveted Steel Pipe

The flanges should, as a general rule, be proportioned in accordance with the British Standard Table applicable to the case.

Fig. 1 shows a typical light riveted steel pipe with angle ring-flanges, as manufactured by Messrs. Babcock & Wilcox, for use in exhaust steam-pipe ranges, and for low-pressure water-pipes.

By using this type of pipe instead of cast iron a great saving of weight is effected, and consequently the pipe supports can be proportionately lighter. In addition, the saving in the cost of transport and erection is considerable, while the risk of breakage is practically eliminated.

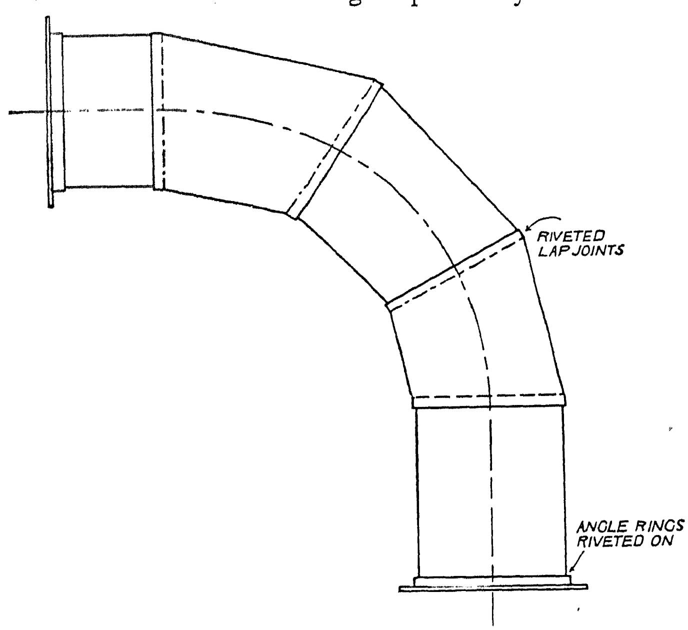


Fig. 2.—Lobster-backed Bend

Riveted pipes are not usually made with an internal diameter of less than 6 in., as below this size it is difficult to rivet the longitudinal and circumferential seams on a long pipe satisfactorily, owing to the small space available for holding up the rivet head on the inside of the pipe.

Riveted pipes may be used with advantage for the largest sizes of pipes, and for very high pressures.

The strength of a riveted pipe is calculated in the same manner as the shell of a boiler drum, with suitable allowance for the strength of the riveted joints adopted.

Bends for inserting in riveted pipe arrangements are made in two ways, known as (1) the lobster-backed bend and (2) the smooth bend.

Fig. 2 shows a typical lobster-backed bend, while fig. 3 shows a smooth bend as manufactured by Messrs. Brown & Hood, of Wallsend. The former is a cheaper pipe than the latter, but it should be remembered that, in cases where frictional resistance is of great importance, the losses with the lobster-backed bend are greater than with the smooth bend.

Welded Pipes.—The welded pipe has invaded with considerable success all the fields previously held by the cast-iron and riveted pipes.

Its advantages are that it weighs less than a riveted pipe of equal strength, and can be made of thinner material if required, while the absence of rivet-heads obviously reduces the frictional loss when passing steam, gases, or liquids.

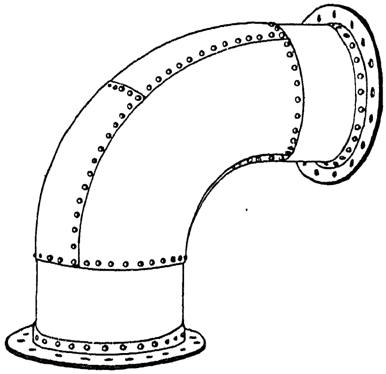


Fig. 3.—Smooth Bend

Welded pipes are made by bending steel plates, previously carefully cut to the correct size, into the form of the pipe, and welding the longitudinal seam either by a butt-weld or a lap-weld.

The butt-welded method is used for small welded pipes up to about $1\frac{1}{2}$-in. bore. This weld is made by heating the complete pipe up to welding-heat, and passing the pipe rapidly through rollers to press the weld edges together.

Another butt-weld method is also used in which the pipe is first rolled into shape, and afterwards the edges of the weld are heated with oxy-hydrogen flames, and pressed together with rollers at the sides of the pipes. This method of welding can be used for pipes between 1 in. and 36 in. diameter.

The lap-weld, as its name implies, is formed by lapping the scarfed edges of the plate when rolled into pipe form, and welding them on a mandrel in between rollers. Alternatively, the lap-weld may be made by heating the scarfed edges with an oxy-hydrogen flame, and welding the joint while hot with pneumatic hammers.

Steel pipes are usually made from Siemens-Martin open-hearth steel having a tensile strength of between 24 and 28 tons per square inch of section. Welded pipes can be manufactured in all sizes up to 48 in. diameter, and, if required, in lengths up to 40 ft., depending on the diameter and largest size of plate obtainable. In cases where pipes are subjected to rough handling, the welded pipe has an advantage over the riveted pipe, as it is very much

less liable to damage and leaks through straining, either in transport or under working conditions. In cases where the weight of pipes is of importance, it is found that welded pipes are at least 12 to 18 per cent lighter than riveted pipes.

For protective purposes, it is usual to coat steel pipes with Dr. Angus Smith's solution in the same manner as described for cast-iron pipes, and in special cases, to wrap the pipes after coating with jute hessian soaked in the Angus Smith solution. Sand is usually coated on to the finished pipe as in the case of cast-iron pipes. When gas-pipes are to be treated with Dr. Angus Smith's solution, they should not be coated inside, as the gas dissolves the inside coating, and this will probably block up the drains with Angus Smith compound. The gas has no deleterious effect on the inside of the pipes, and therefore the absence of protective coating is immaterial.

Welded-steel pipes are used successfully for air-, water-, gas-, and steam-pipes.

Lap-welded steel pipes for gas-, air-, and water-services are generally made to the following specification, which is worked to by Messrs. Stewarts & Lloyds:

Material.—The pipes to be made from open-hearth steel made by the acid or basic process, having a tensile strength of 24 to 28 tons per square inch of section.

Mechanical Tests.—Tensile tests of the material to show an elongation of not less than 20 per cent in 8 in., with a reduction in area at the fracture of not less than 45 per cent.

Strips 1½ in. wide to withstand being doubled over cold until the internal radius is equal to 1½ times the thickness of the test piece and the sides are parallel, without fracture.

Chemical Analysis.—The material from which the pipes are made to show the following analysis:

Phosphorus, not to exceed	..	..	·06 per cent
Sulphur,	,,	.. ..	·06 ,,
Silicon,	,,	.. ..	·035 ,,
Manganese,	,,	.. ..	·54 ,,
Carbon,	,,	.. ..	·125 ,,

Manufacture.—Pipes not exceeding 12 in. internal diameter to be made by the lap-welding rolling process, and to be as round and straight as it is practicable to make them by that process.

Pipes exceeding 12 in. internal diameter to be made from plates bent in bending rolls and welded by the water-gas process, and to be annealed throughout after welding.

All pipes to be smooth inside and outside, and free from scale, blisters, and other surface defects.

Tolerances.—The following tolerances to be allowed:

Pipes not exceeding 12 in. internal diameter.

5 per cent over or under the specified weight, 1 per cent over the external diameter, and a reduction in the internal diameter such as will enable disks of the following sizes to pass freely through the pipes:

Inside Diameter of Pipe.	*Diameter of Disk.*
$1\frac{1}{2}$ in. to 3 in.	$\frac{1}{8}$ in. less than inside diameter of pipe.
$3\frac{1}{2}$ in. ,, 6 ,,	$\frac{5}{32}$,, ,, ,, ,,
$6\frac{1}{2}$ in. ,, 12 ,,	$\frac{3}{16}$,, ,, ,, ,,

Pipes exceeding 12 in. inside diameter.

$2\frac{1}{2}$ per cent under or $7\frac{1}{2}$ per cent over specified weight.

$\frac{1}{4}$ in. over or under specified diameter, for sizes 13 in. to 30 in. internal diameter.

$\frac{3}{8}$ in. over or under specified diameter, for sizes above 30 in. internal diameter.

$\frac{1}{8}$ in. over or under the specified diameter of the socket in the case of pipes with inserted joint for lead and yarn.

Sizes, Thickness, Working, and Test Pressures.—Pipes to conform to the thickness, working, and test pressures specified in the following table:

Nominal Bore of Tube.	Minimum Thickness (Lap Tube).	Maximum Working Pressure.	Hydraulic Test Pressure.
Inches.		Pounds.	Pounds.
2	12 W.G.	500	1000
3	12 ,,	500	1000
4	11 ,,	450	900
5	10 ,,	450	900
6	10 ,,	450	900
7	9 ,,	450	800
8	8 ,,	400	800
9 to 12	$\frac{3}{16}$ in.	350	700
13 ,, 18	$\frac{3}{16}$,,	300	500
20 ,, 21	$\frac{1}{4}$,,	250	500
24 ,, 27	$\frac{5}{16}$,,	250	500
30	$\frac{3}{8}$,,	250	500
33	$\frac{7}{16}$,,	250	500
36	$\frac{1}{2}$,,	250	500

Solid-drawn Steel Pipes.—Solid-drawn pipes, as the name implies, are drawn down from a suitably shaped ingot to the required size and thickness. The pipes are first hot-rolled to nearly the required size by means of special rollers, and the final operations consist of cold-drawing the rolled tube through dies flooded with oil and over a mandrel to the finished dimensions.

This process gives an exceedingly fine finish, but its cost is great. For steam-pipes and boiler-tubes the final drawing of the tube is sometimes dispensed with, and the tubes are used as finished by the hot rolls.

As the tubes produced by either of the above processes are formed of one solid piece of metal without a joint of any kind, they are certainly the ideal pipes for the highest steam pressures and temperatures now in vogue.

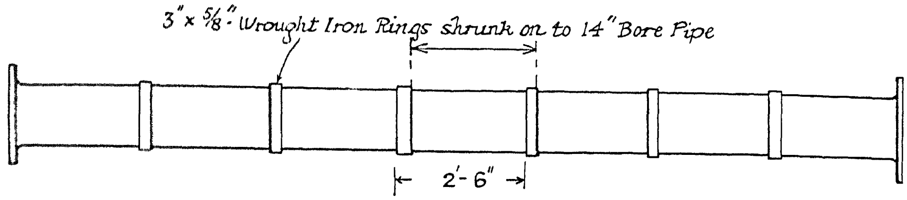


Fig. 4.—Method of Reinforcing Welded Steam-pipes

Solid-drawn or hot-rolled tubes can be obtained in any size up to 12-in. bore, and generally in lengths up to about 25 to 30 ft.

It will be seen from the above that when it is necessary to use high-pressure steam-pipes above 12-in. bore, welded pipes must be used unless two or more weldless pipes of the required aggregate area are installed.

In cases where large extra-high-pressure *welded* steam-pipes are used, it is often the practice to shrink on wrought-iron rings or hoops at intervals along the length of the pipes to prevent ripping of the pipe in the event of a weld failure. Fig. 4 shows the arrangement of reinforcing welded steam-pipes by shrinking hoops on to the outside of the pipe.

CHAPTER II

Pipes: Non-ferrous

In mechanical engineering, copper and brass tubes are used for a number of purposes, notably for steam-pipes, pressure-gauge pipes, and lubricating pipes.

Flexible Metallic Tubing

For temporary work, and for cases where steam, air, or gas is required at different points periodically, flexible metallic piping can with advantage be used. Flexible metallic piping is thus used for steam-sooting services, for boiler-tube cleaning-plants, for compressed air, for cleaning the air passages in the windings of electrical machinery, for pneumatic drills and riveting hammers, and many other similar installations.

The tubing is formed of doubled or folded strips of metal coiled up and interlocked on a flexible packing of asbestos or other fibrous material.

Connections for the ends of flexible pipes are made as shown in fig. 5, which can be rapidly attached simply by inserting asbestos packing in the stuffing-box, and screwing down the gland after the fitting is placed in position on the pipe. For use with steam, the flexible pipes are usually made of bronze.

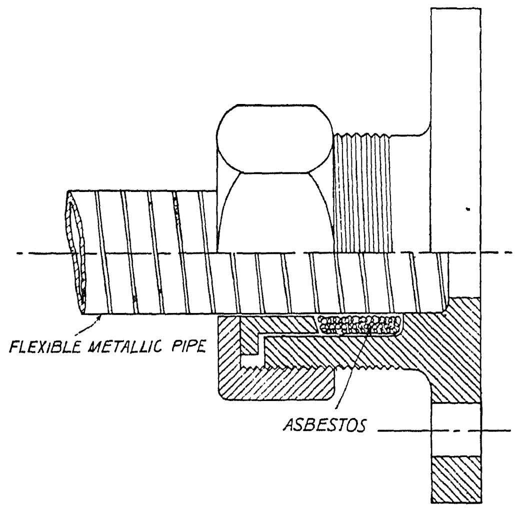


Fig. 5.—Connections for Ends of Flexible Pipes

Copper Pipes

In certain cases copper tubes are used for high-pressure steam-pipe connections, notably in locomotive work and in the navy and mercantile marine when saturated steam is used. In the case of modern steam-power plant, however, where highly-superheated steam is almost always used, the copper steam-pipe is never installed, as under high temperatures copper rapidly loses its strength.

The following table shows the reduction in tensile strength of copper when subjected to various temperatures:

Temperature, Degrees Fahrenheit.	Percentage Loss of Tensile Strength.	Temperature, Degrees Fahrenheit.	Percentage Loss of Tensile Strength.
70	2	640	35
140	5	750	45
250	10	800	50
330	15	840	55
420	20	940	66
440	22	970	68
500	25	1168	88

Wherever possible, solid-drawn copper pipes should be used, and these are obtainable in many thicknesses up to a diameter of 9 in.

Brazed built-up pipes should not be used, as the brazing must be regarded with suspicion, and under the action of superheated steam it deteriorates rapidly.

Flanges for copper pipes are brazed on, and great care should be taken to ensure that the brazing operation does not ruin the copper pipe.

If the brazing is carried out in an open hearth, the hot gases coming into contact with the copper are likely to be of a reducing or deoxidizing nature and leave the surface of the tube rough and covered with small cracks, which may cause ultimate failure under working conditions.

A copper pipe which has been subjected to a reducing flame and fails is often erroneously referred to as being burnt, but under brazing conditions the metal is seldom heated sufficiently to cause burning, and the failure is almost always due to deoxidation.

In order to make a thoroughly sound brazed joint the flame used should contain a large excess of oxygen in order to prevent any risk of deoxidation.

Copper pipes may be used for many purposes other than steam-pipes, such as water connections to pumps, and other apparatus where great flexibility is desired.

The ultimate tenacity of copper may be taken as 13 to 14 tons per square inch, and for general purposes, when the tubes are not subjected to high temperatures, a factor of safety of 6 may be used, the strength of the tube being calculated on the working pressure by the usual formula for the strength of a cylinder subjected to internal pressure.

The Board of Trade rule for the thickness of copper pipes is:

$$\text{For brazed pipes, } T = \frac{P \times D}{6000} + \tfrac{1}{16} \text{ in.;}$$

$$\text{for seamless pipes not over 8 in. diameter, } T = \frac{P \times D}{6000} + \tfrac{1}{32} \text{ in.;}$$

where T = thickness of pipe in inches,
P = working pressure in pounds per square inch,
D = inside diameter in inches.

Brass tubes can be obtained in all thicknesses up to 8 in. internal diameter, and in lengths up to 20 ft. The strength of brass tubes may be taken to be from 17 to 19 tons per square inch, and for general purposes a factor of safety of 6 should be used.

Brass tubes are usually made from what is known as 60/40 brass, i.e. Muntz metal, or from 70/30 brass, the first figure referring to the percentage of copper present in the metal and the last figure indicating the percentage of zinc.

Many other alloys are used for special purposes which cannot be fully described in the present article.

For condenser tubes the Admiralty use a tube containing 70 per cent copper, 29 per cent zinc, and 1 per cent tin.

CHAPTER III

Pipe Joints

There are many forms of pipe joints for connecting pipes together, though generally these may be classified under four main headings as follows:

1. Spigot-and-socket.
2. Screwed and socketed.
3. Flanged.
4. Welded.

Spigot-and-socket Joints.—These joints should not be used for steam or hot-water services where the expansion stresses are large, due to the great changes of temperature to which the pipes are subjected.

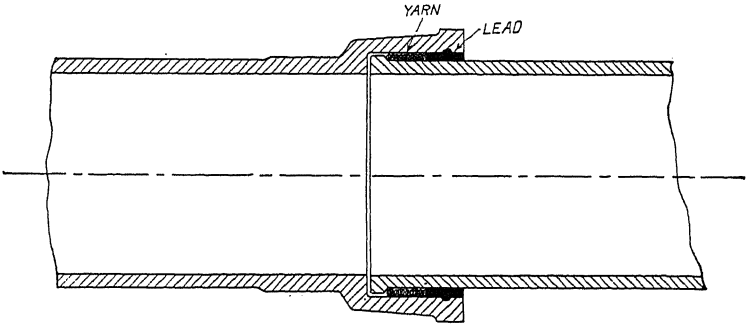


Fig. 6.—Spigot-and-socket Joint

In other cases where the temperature changes are small, such as in water, air, and gas mains, the spigot-and-socket joint offers considerable advantages over the other types mentioned.

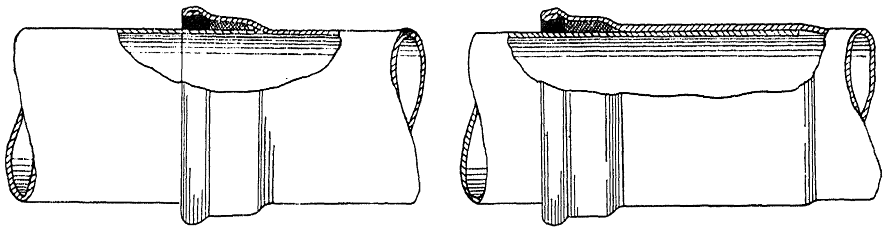


Fig. 7 Forms of Joint for Steel Pipes Fig. 8

The spigot-and-socket joint can be used with both cast-iron and mild-steel pipes. Fig. 6 shows a typical form of joint for cast-iron pipes, and figs. 7 and 8 show two forms of joint for steel pipes, as manufactured by Messrs. Stewarts & Lloyds. The long sleeve joint is used on underground pipes where there is a danger of subsidence, either due to the nature of the ground, or by reason of traffic overhead as in the case of pipes laid under highways. These joints are made tight by first caulking in several turns of hemp, and afterwards filling up the remaining annular spacing with lead, and finally caulking the lead until the joint is thoroughly tight. The lead can be poured in a molten state into the joint by placing a temporary strap round the spigot pipe, and when the lead is cooled the strap is removed and the lead caulked.

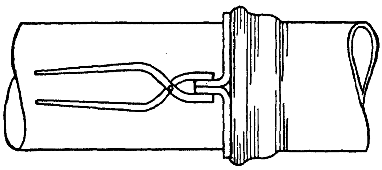


Fig. 9.—Rubber Jointing Ring

Fig. 9 shows a rubber jointing ring held in place by tongs, while fig. 10 shows a wrought-iron jointing clamp for the same purpose. The rubber ring, which should be coated with clay to prevent it being injured by the molten lead, can be used for pipes up to about 12-in. bore, while for the larger sizes the wrought-iron clamp should be used. When setting either of the above rings in position on the pipe, care should be taken to leave a small space outside the end of the socket for lead to allow for caulking.

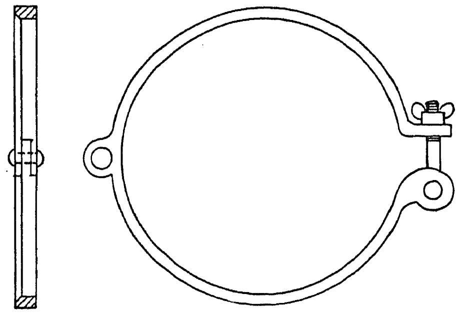


Fig. 10.—Wrought-iron Jointing Clamp

Lead wool for jointing spigot-and-socket pipes is now largely used, and has many advantages over the older method of jointing with lead poured into the joint as described above. In the newer process the lead is purchased in the form of a loosely-coiled fibrous lead rope, which can be caulked into the socket turn by turn until the joint is completed.

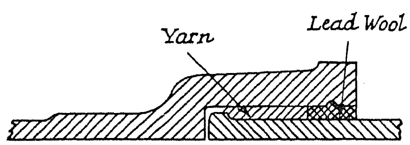


Fig. 11.—Single-collar Joint

The obvious advantage of this method is that it entirely does away with the use of the lead melting-pot, and the danger from pouring molten lead into the joints unless special precautions are taken to ensure that the pipes are quite dry. This difficulty with cast-lead joints is particularly pronounced when repairing joints, especially when mains are laid below ground and it is practically impossible to drain away any water. Cast-lead joints also have a tendency to run through into the pipes, unless great care is taken to well caulk the spun yarn at the back of the joint. Lead wool can be used for all sizes of pipes up to the largest made, and it will be appreciated that the pouring of molten-lead joints for pipes over 36 in. diameter is undesirable.

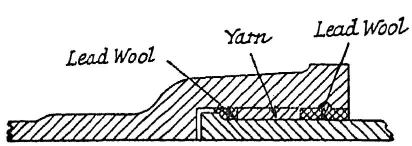


Fig. 12.—Double-collar Joint

Two main types of lead-wool joints are used, known as the single- and double-collar joint. The single-collar joint is shown in fig. 11, and it will be seen that roughly two-thirds of the joint is made with jute yarn (8 ply) tightly caulked in, and finished with lead wool at the open end of the socket. This joint is usually used with pressures up to 70 lb. per square inch. Fig. 12 shows the double-collar joint, in which a portion of lead-wool packing is inserted at the back of the joint before the yarn is caulked in and the final caulking completed with lead, as in the single-collar joint.

The double-collar joint should be used for high pressures, and as the spun yarn is protected from contact with the contents of the pipe, no rotting or contamination can take place, as the back turns of lead effectively prevent leakage into the joint.

Lead wool can be used for spigot-and-socket joints on wrought-steel pipes as shown in figs. 7 and 8. When using lead wool on very light steel pipes, it is sometimes found that the caulking of the jointing material causes slight collapse or giving of the spigot end of the tube, thereby preventing a satisfactory tight joint being made. This difficulty can, however, be overcome by the use of a temporary expanding mandrel inserted into the joint during caulking. This method of working has been evolved by the Lead Wool Company, who hold the patent rights for the apparatus required.

When making lead-wool joints special curved caulking-tools to fit the annulus of the joint should be used, and the caulking carried out with a hammer of not less than 4 lb. weight.

Where pipes are laid in trenches in the ground, it is common practice to joint up several lengths of pipe before lowering same into the trench, as this procedure reduces the number of joints to be made in the trench. Care should be taken to have ample joint-holes made in the trench where joints must be made after the pipes are in position, in order to give sufficient room for the jointer to get easily at all portions of the joint, and at the same time have ample room to swing the hammer.

On large installations it is an advantage to use compressed-air caulking-hammers, and suitable portable compressors for this work can be obtained. Fig. 13 shows a portable compressor at work, and lead wool being caulked into a joint in a 14-in. pipe.

When laying straight lengths of pipe in a trench round a curve, the joints should be yarned up first and then pulled into position and securely wedged against the side of the trench to relieve the joints and leave them free for caulking with lead wool.

All pipes laid in trenches should be tested before the ground is filled in, and in cases where the whole length of trench cannot be left uncovered until the complete pipe is finished, it may be necessary temporarily to blank up the open end of the portion of main already laid, and test it in sections as the work proceeds. When satisfactorily tested, this length of main can be filled in, the trench continued, more pipes laid and tested, and so on to the end of the main.

For testing purposes, water pressure is the most convenient method, as it immediately shows any leaks, which can be remedied at once.

It is not always possible, however, to obtain water for testing purposes, and under these circumstances the main, either as a whole or in sections as indicated above, should be tested with compressed air or gas to the working pressure. Small leakage cannot be readily detected by means of air or gas, but if the joints are painted round with soap solution while under pressure, bubbles will indicate any leakage. The soap solution should be mixed with water some time before use, so that the solution may be free from bubbles

when applied. The portable compressor shown in fig. 13 is a ready means of testing mains with compressed air.

When laying underground steel mains made up of random lengths, it is advisable to make a record of the position of joints either by a post or marker in the ground, or by keeping records of the distance of joints from some permanent object, such as telegraph poles or buildings, so that in the event of any trouble being experienced with leaks, excavation can be carried out with accuracy at the joints for repairing purposes.

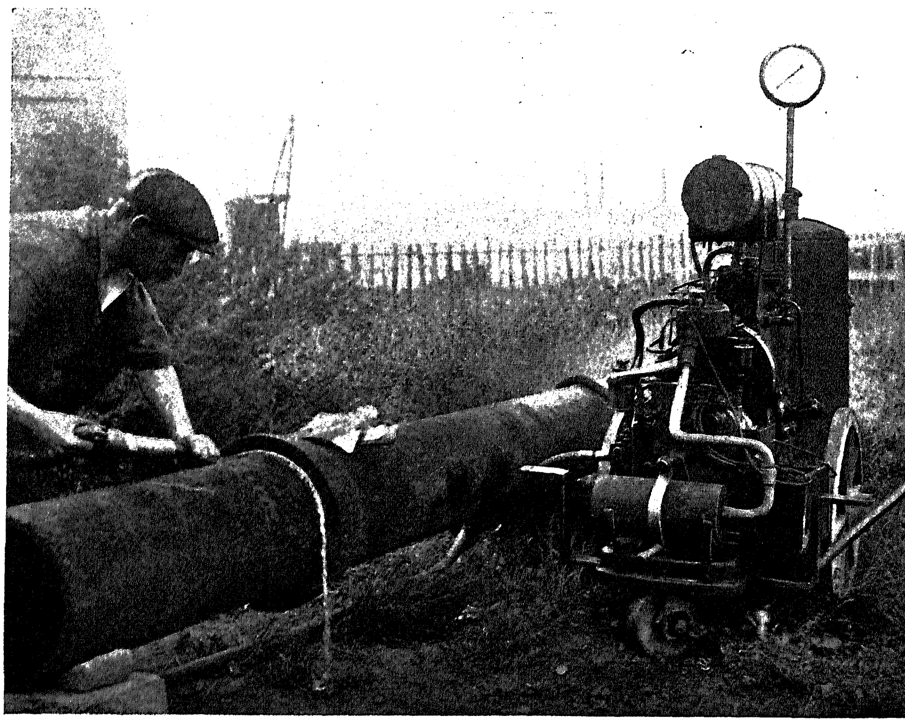


Fig. 13.—Portable Compressor for Testing Pipes

Screwed and Socketed Joints.—This form of joint is almost universally used for small gas-pipe installations, and for the smaller sizes of water-piping for connecting steel pipes. The screwed joint is not to be recommended for steam-piping or for piping above 4 in. diameter, in which cases flanged joints should be used. The sockets are usually screwed on to the end of each length of pipe as tightly as possible, the threads being first smeared with white lead, or preferably with graphite paste.

The following length of pipe is then screwed into the open socket, and so on to the end of the run of piping. The last connection must be made with a flange or running joint, as it is obvious that the last length of pipe cannot be screwed into two sockets at either end. Figs. 14, 15, 16, 17, and 18 show typical screwed connections.

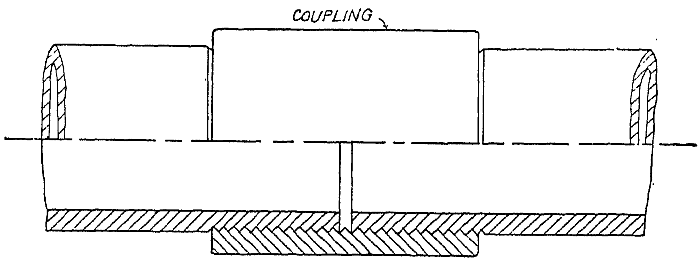


Fig. 14

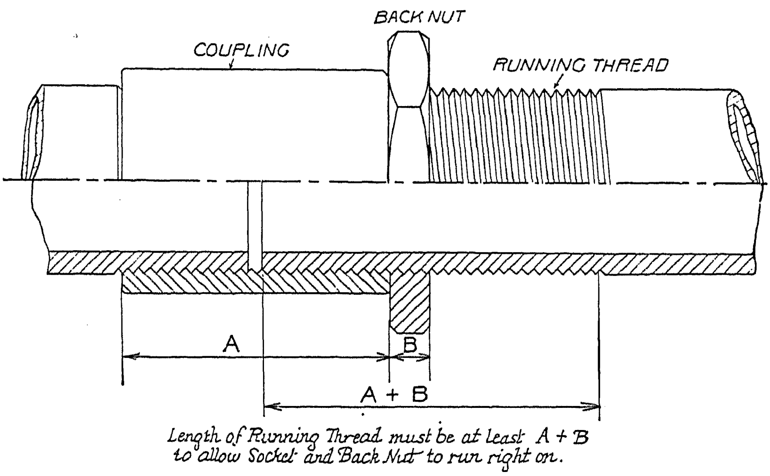


Fig. 15

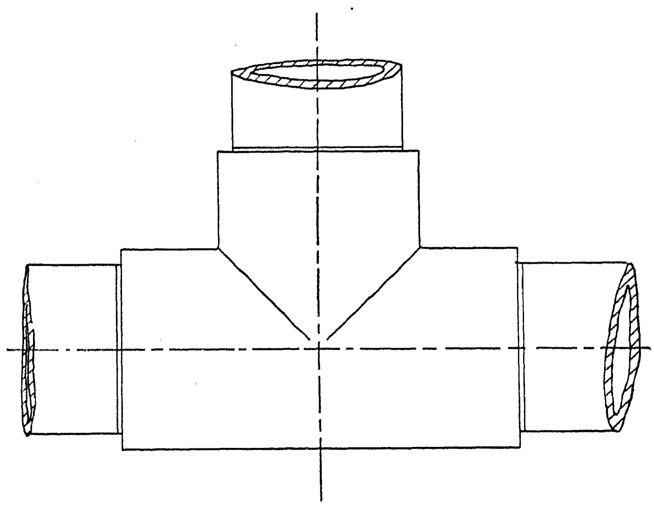


Fig. 16

Referring to fig. 15, it will be noted that the thread on one pipe is of sufficient length to allow the back nut and socket to be completely screwed on to the pipe by hand. The joint is made by screwing the socket forward from one pipe on to the other until it tightens, and then making all secure by bringing up the lock nut to the back of the socket, first inserting a turn or two of spun yarn between the nut and socket for jointing purposes. Another method of jointing screwed pipes is by means of unions screwed to the pipes. A typical union is shown in fig. 19, from which it will be seen the joint is made by a copper washer between two machined surfaces drawn together by a gland nut. Running joints are obviously not required with this arrangement.

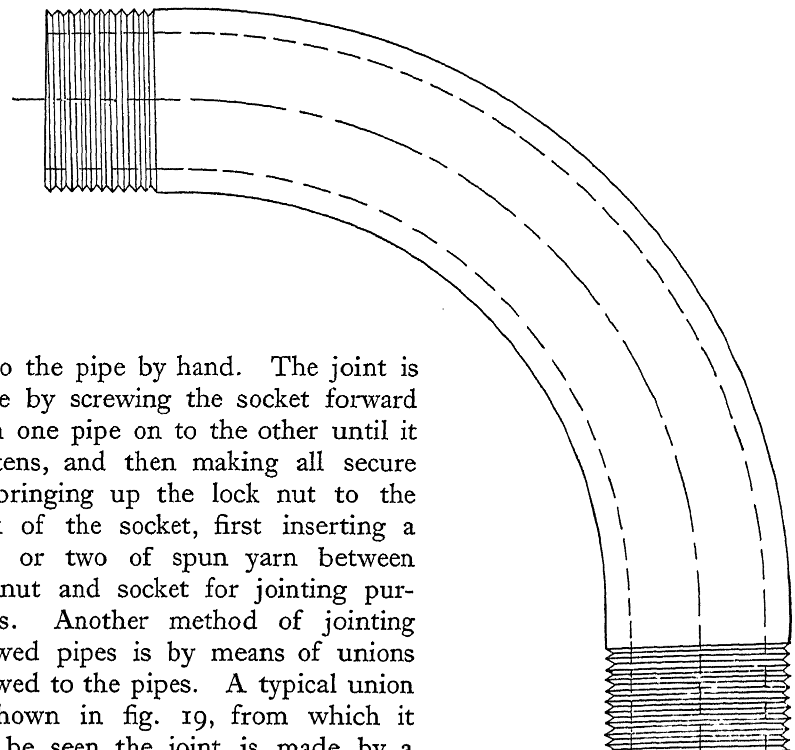


Fig. 17

Flanged Joints.—The flanged joint is the most satisfactory joint to use for high pressures, especially in cases where temperature changes are great, and is therefore universally used for steam-pipe joints.

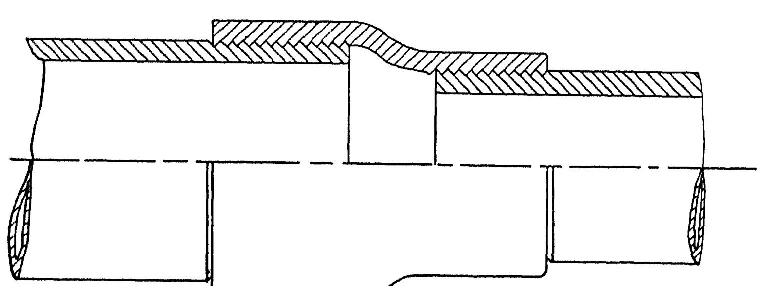


Fig. 18

The question of the proper proportions of flanged joints received, in 1890, the attention of the British Engineering Standards Committee, and as a result

a complete set of tables has been drawn up giving full particulars of flanges for all types for use with different pressures. Prior to this, each manufacturer had designed standard flanges to which his pipe flanges were made, and considerable trouble and confusion occurred on any large plant which had several different standards of flanges in use. When considering the question of flange proportions, therefore, it is desirable to adhere to the British standards, unless any valid reason exists for continuing the use of existing standards, as, for instance, when extending a plant which is already provided throughout with a satisfactory design for flanged joints. In the case of cast-iron pipes, the flanges are cast integrally with the pipe, and should be proportioned in accordance with the British standards applicable to the case. Flanges should be faced across the joint, and the bolt-holes should be drilled and cutter-barred or faced at the back, so that the nuts may bed down satisfactorily.

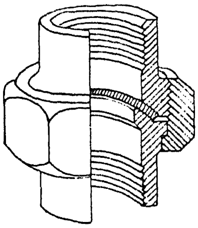


Fig. 19

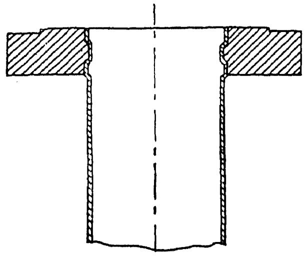


Fig. 20

Flanges for wrought-iron and steel pipes can be fixed to the pipes in several ways, e.g. expanded on, screwed on, riveted on, and welded on. For low pressures such as in exhaust-steam services the form of flange shown in fig. 20 may be used for pipes between 2-in. and 8-in. bore.

The flange indicated is manufactured by Messrs. Babcock & Wilcox, and is made of cast iron. The bored part of the flange is recessed, and the tube is expanded into the flange and the recess to hold the flange securely in position.

The following table shows the thickness of the above flanges:

Pipe Bore.	Flange Thickness.	Pipe Bore.	Flange Thickness.
Inches.	Inches.	Inches.	Inches.
2	1	6	$1\frac{3}{4}$
3	$1\frac{3}{8}$	7	2
4	$1\frac{1}{2}$	8	2
5	$1\frac{3}{4}$	—	—

For exhaust pipes below 2-in. bore it is usual to screw flanges on to the pipes and finally to expand the tubes into the flanges, while for sizes above 8-in. bore, angle-flanges riveted on are used, as shown in fig. 1.

For high-pressure steam-pipes riveted flanges are used on pipes above 6-in. bore, and screwed and expanded flanges for pipes below 6-in. bore. In certain cases welded-on flanges are used for all sizes of steam-pipes, and these will be described later, but for the highest pressures, together with high superheat, the combination of the solid-drawn pipes with

riveted-on flanges constitutes the most satisfactory design Figs. 21 and 22 show sections of screwed and riveted-on flanges as manufactured by Messrs. Babcock & Wilcox.

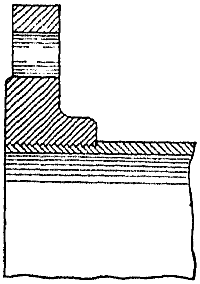


Fig. 21.—Screwed Flange

When fixing screwed flanges the pipes are screwed with a fine thread of about 11 threads per inch, and after the flanges are screwed on, the pipes are expanded. In special cases, such as an oil-pipe line with high pressures of, say, 750 to 1000 lb. per square inch, the Briggs' American Standard thread is preferred for screwing the pipes. The salient feature of the Briggs thread is that the crest of the thread is very sharp, and it is claimed that this type of thread makes a very tight joint.

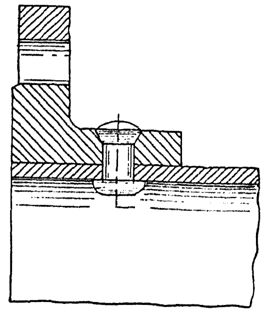


Fig. 22.—Riveted-on Flange

Welded-on Flanges.—Flanges may be welded on to solid-drawn or welded steel pipes, either by fire welding or electrical welding. This form of joint is certainly the neatest flange joint made, and the Engineering Standards Committee when considering flange proportions devoted a special table to the welded-on flange.

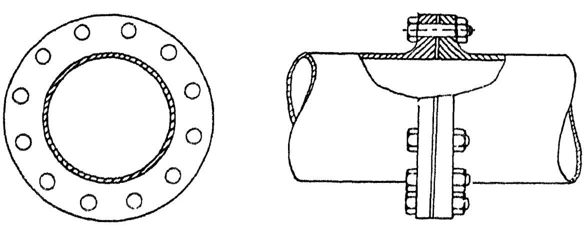


Fig. 23.—Welded-on Flange

Fig. 23 shows a typical welded-on flange for a steam-pipe as made by Messrs. Stewarts & Lloyds.

Loose Flanges.—Loose-flange joints are used chiefly in mining engineering and general contractors' work, where the pipes frequently need re-erection to suit changed conditions. Two forms of loose-flange joint are shown in figs. 24 and 25. The former, known as Stewarts' No. 1 loose-flange

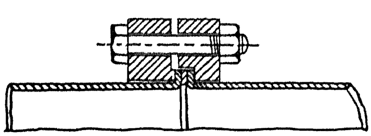


Fig. 24

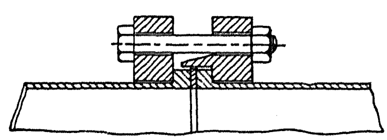


Fig. 25

Forms of Loose-flange Joint

joint, is made by flanging over the ends of the pipe after slipping on loose flanges of cast iron, cast steel, or wrought steel. The joint is made by drawing the ends of the pipes together by means of the flanges on to a joint ring in between the faces of the small flanges on the pipe. The modified form of joint shown in fig. 25 is of stronger construction than Stewarts' No. 1,

and should be used where the pipes may be subjected to excessive stresses due to subsidence or expansion.

The above joints are suitable for water pressures up to 300 lb. per square inch, and air pressures up to 150 lb. per square inch.

Welded Joints.—A development in the design of joints for water, air, and gas mains has recently been made, consisting of jointing a long sleeve spigot-and-socket joint of special design by welding, either by oxy-acetylene or oxy-hydrogen blowpipe.

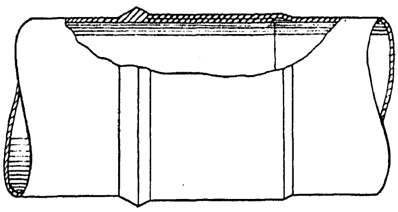


Fig. 26.—Welded Joint

This type of joint is manufactured by Messrs. Stewarts & Lloyds, and is shown in fig. 26.

This joint allows the pipes to be made of the thinnest possible metal, as there are no caulking stresses as in the lead-and-yarn joint, and the limit depends only on the requirements for pipe manufacture. The pipes can easily be inserted into one another, and the long sleeve ensures true alignment

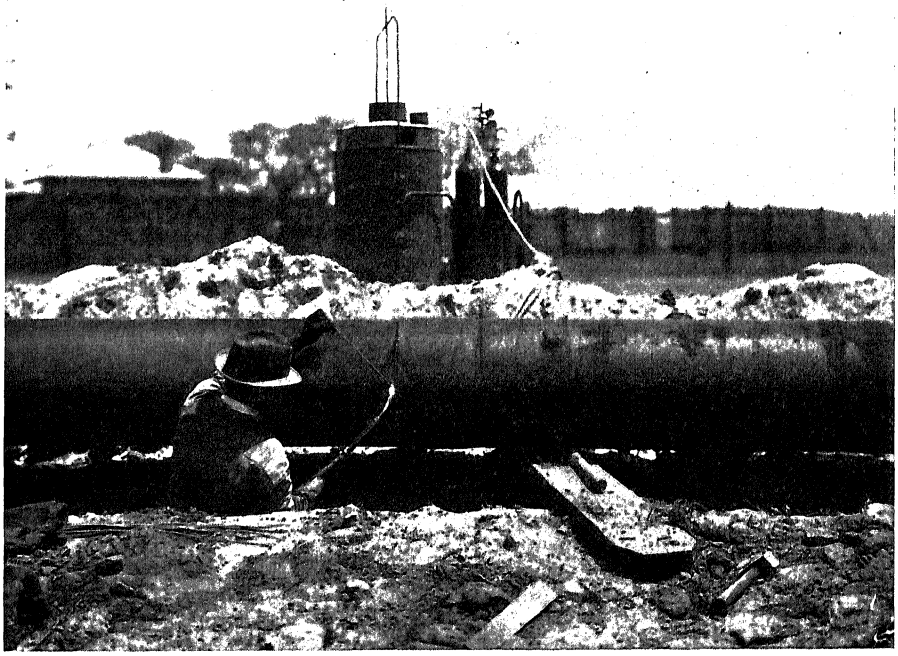


Fig. 27.—Welding 18-in.-diameter Steel Gas Main at Newcastle, N.S.W.

of the pipes automatically. As in the case of the ordinary steel spigot-and-socket pipes, it is convenient to joint up several lengths of pipe before laying them in the trench, thus reducing the number of joints to be made below ground-level. When carrying pipes round curves, the complete length of pipe for the curve can be welded together, and bent or sprung

into position without in any way damaging the soundness of the joints.

Fig. 27 shows a joint being welded in an 18-in.-diameter steel gas main, prior to lowering the pipe into the trench.

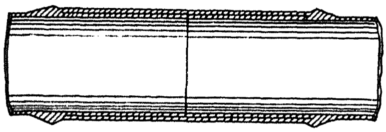


Fig. 28.—Welded-steel Slip-collar

In the event of it being necessary to connect two spigot ends of pipe together, a special slip-collar can be used as shown in fig. 28.

Jointing Material for Flanged Joints.—Flanged joints are made tight by inserting a ring of some jointing material between the flanges as

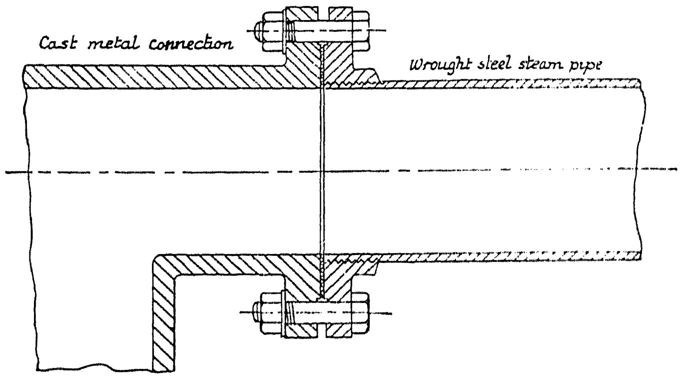


Fig. 29.—Method of Tightening a Flanged Joint

shown in fig. 29, and the most satisfactory material for jointing pipes for steam pressures up to 275 lb. per square inch is a corrugated nickel ring as shown in fig. 30. Before insertion these rings are smeared with thin graphite and boiled oil, and on being squeezed between the flange faces, the ridges of the corrugations conform to any irregularities in the surfaces of the flanges, and make a thoroughly tight joint.

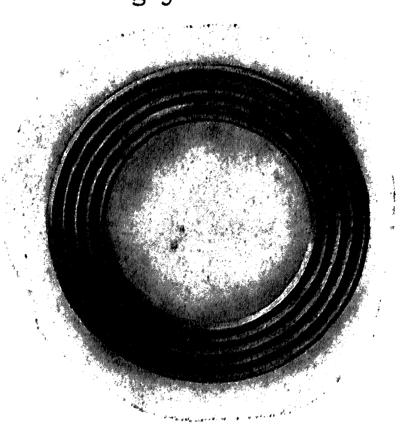


Fig. 30.—Corrugated Nickel Ring

For the highest steam pressures it is now the practice to rivet up the joints with a soft-iron caulking ring inserted between the flanges, as shown in fig. 31.

In cases where flanges join on to valves or fittings where riveting is impracticable, a serrated iron joint ring about $\frac{1}{4}$ in. thick is inserted, and the joint tightened up by means of bolts.

Expansion Joints.—Many forms of expansion joint are used, but, generally speaking, these may be divided into three sections:

1. Bends or loops. 2. Sliding expansion joints.
3. Concertina, corrugated, or bellows expansion joints.

For high-pressure steam ranges bends or loops should be used, while for low-pressure steam ranges concertina pipes are the best. Sliding expansion joints should be avoided if possible, as, while quite satisfactory when in working order, there is always a danger of the sliding portions becoming

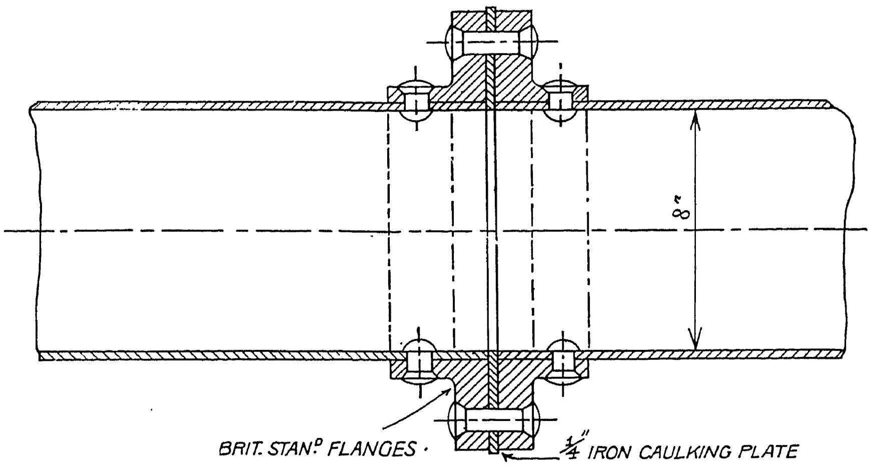


Fig. 31.—Joint made with Soft-iron Caulking

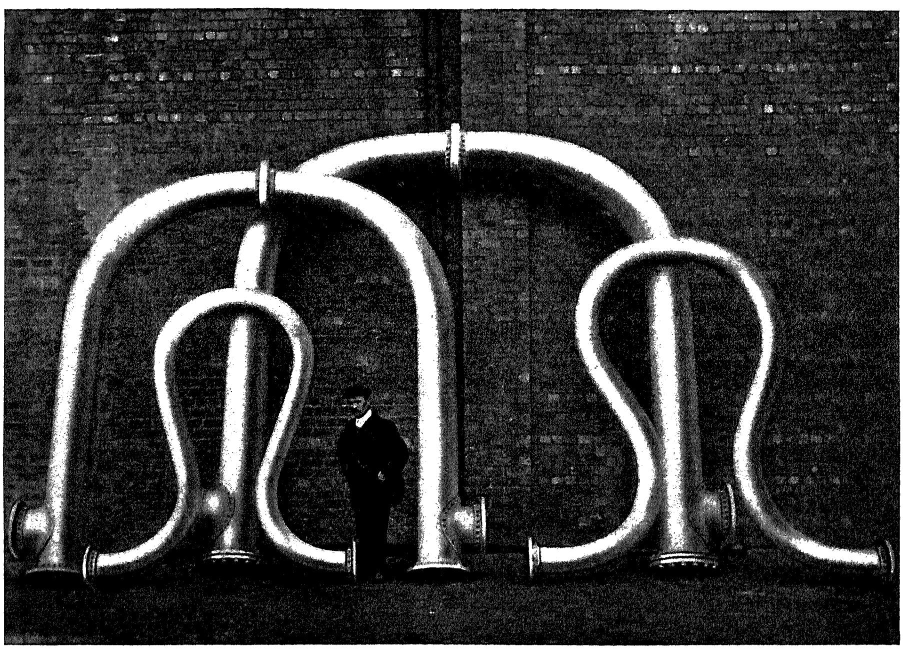


Fig. 32.—6-in., 8-in., and 14-in. Expansion Bends for H.P. Steam

jammed by corrosion, deterioration of the packing in the stuffing-box through want of proper attention, or by settlement of the pipe supports, whereby the

sliding expansion pipe becomes out of line and does not work freely. Sliding expansion joints formed by special loose flanges at several points in the line of pipes are satisfactory for water, or air, and gas mains, when subjected to expansion and contraction.

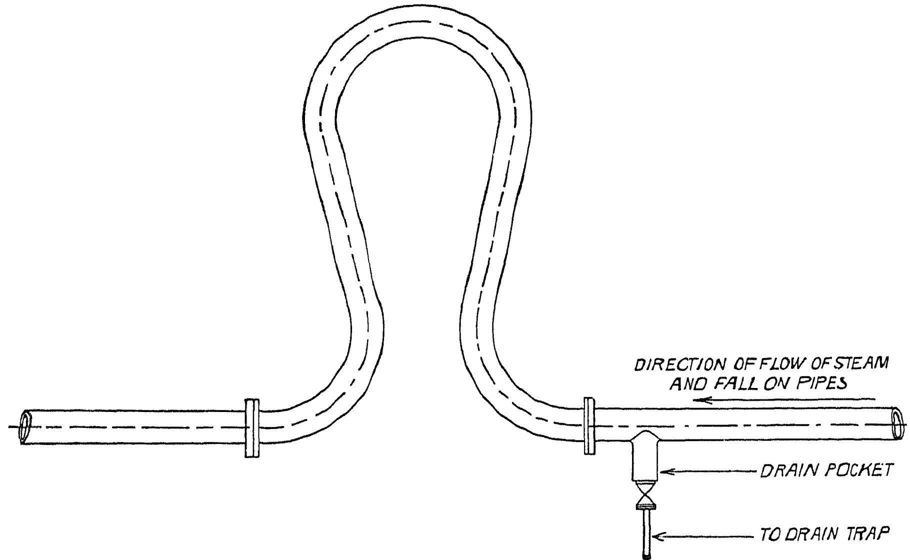


Fig. 33.—Loop with Drain Connection

Bends and Loops.—As far as possible expansion in high-pressure steam ranges should be provided for by arranging the pipes so that bends may take up the movements due to expansion and contraction.

It is not always possible to arrange this, and in these cases expansion loops should be used.

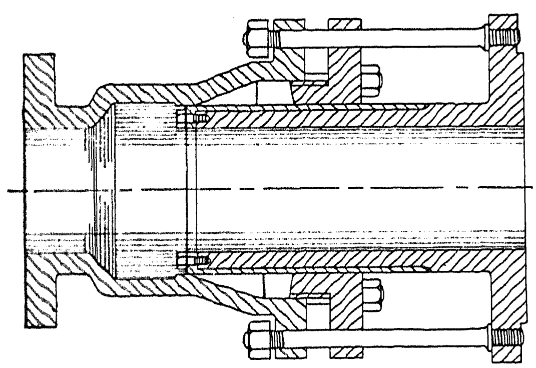


Fig. 34.—Typical Sliding Expansion Joint

Typical expansion loops, as manufactured by Babcock & Wilcox, are shown in fig. 32.

Expansion loops should always be placed with the loop vertical or horizontal, never with the loop below the pipe, as this forms a pocket in which water may collect. In cases where the loop is placed vertically, it is necessary to insert a drain connection on one side of the loop, as in fig. 33.

The above arrangement applies only to pipes of 8 in. diameter and under; for pipes above this size the loops may be formed with branches at the lower ends to form drain pockets, as will be seen in the case of the two largest pipes in fig. 32.

Sliding Expansion Joints.—A typical expansion joint of the sliding type is shown in fig. 34. This joint is designed for use in straight pipes where bends or loops cannot be fitted. The sliding portion and the gland are fitted with a bronze casing to resist corrosion.

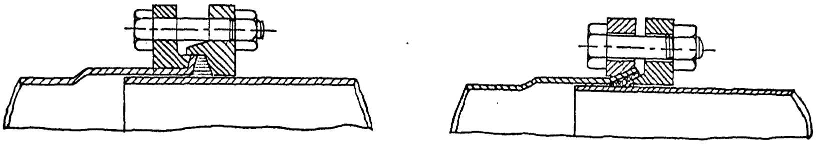


Fig. 35 Loose-flange Expansion Joints Fig. 36

It will be noted that the joint is provided with anchor-bolts which, while they allow the joint to move within safe limits, prevent any danger of the joint pulling apart, and at the same time indicate the in- and out-positions of the joint. These joints are made in standard sizes from 2½-in. to 15-in. bore.

The other type of sliding expansion joint used for air, gas, and water mains, is shown by two forms of loose-flange expansion joints manufactured by Messrs. Stewarts & Lloyds, and indicated in figs. 35 and 36. The former is known as Stewarts' patent, and the latter as Dree's patent. These joints are suitable for high-pressure water mains, and especially for hydro-electric installations.

Concertina or Bellows Pipes.

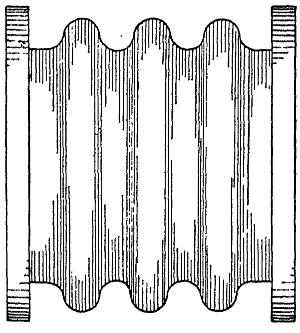


Fig. 37.—Corrugated Copper Pipe with Flanges

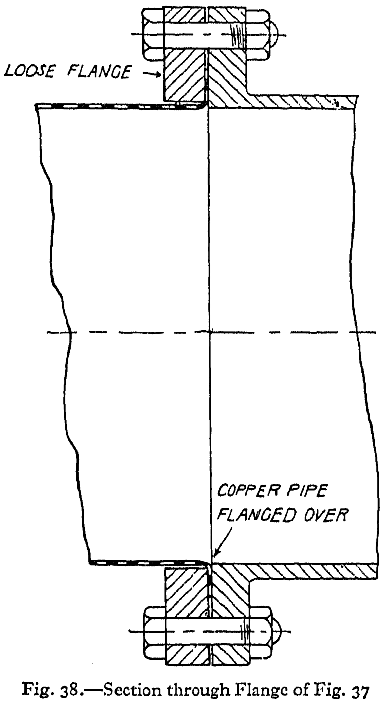


Fig. 38.—Section through Flange of Fig. 37

—One form of concertina pipe for dealing with expansion in exhaust-steam mains, and for low-pressure water connections to condensers, &c., consists of a corrugated copper pipe provided with flanges, as shown in fig. 37.

This pipe is provided with loose flanges which bear on the flanged ends of the copper pipe, thereby making a joint with the adjacent pipe flanges. Fig. 38 illustrates a section through one flange of the above pipe.

These pipes should be tested with a hydraulic pressure of 50 lb. per square inch.

Messrs. Babcock & Wilcox make copper concertina pipes as above in all sizes between 6 in. diameter and 36 in. diameter. In all cases, the pipes are 18 in. long over the flanges. These copper pipes should not be used where there is any considerable movement due to expansion or other cause. For cases where the expansion movement per expansion pipe exceeds $\frac{1}{4}$ in., the form of bellows pipe shown in fig. 39 should be used. This pipe is usually made from light-gauge steel plates joined together by riveting at the inner end of the bellows, and by rolling over and welding at the outer end. The flanges are usually formed by riveting on suitable angle-iron rings. The pipes should be galvanized after testing with a hydraulic pressure of 50 lb.

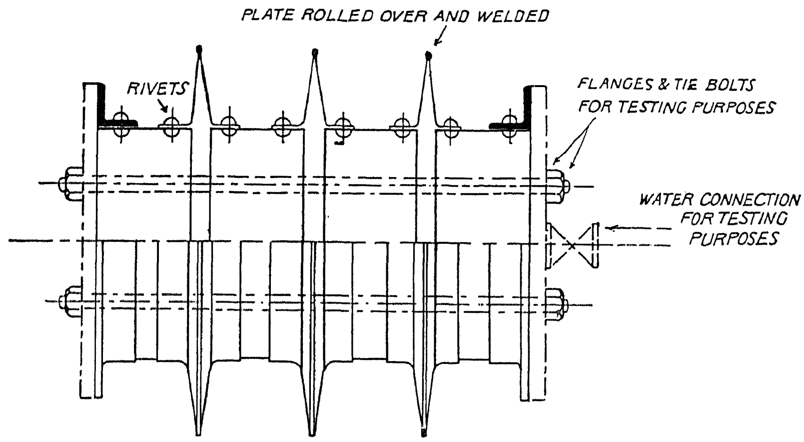


Fig. 39.—Light-gauge Steel Expansion Pipe

per square inch. Each bellows can be allowed to close in $\frac{5}{8}$ in., and therefore the amount of compression in inches that the pipe can safely withstand is the number of bellows multiplied by $\frac{5}{8}$ in. These pipes can be made with any number of bellows up to 5. It is not advisable to make the pipes with more than 5 bellows, as when made with more than that number they are very liable to sag, owing to the extreme flexibility of the pipe.

It should be noted that when testing these pipes with hydraulic pressure, the blank flanges covering the ends of the pipe should be securely tied together with tie-bolts, to prevent the water-pressure expanding the pipe beyond safe limits, which it would do if the expansion of the pipe were not prevented. The tie-bolts, shown dotted in fig. 39, are used for this purpose during test.

Expansion of Pipes.—The expansion of steam-pipes is a matter which must receive the serious attention of the designer, as unless adequate provision is made for it, the stresses imposed on the pipes, and more particularly on the boiler and engine stop-valves, will be excessive. In the case of large steam turbines, which of necessity have large diameter steam-pipes connected to their steam chests, it is of vital importance to ensure that no

stresses are imposed on the turbine casing which would distort the alignment of the turbine, thereby causing fouling of the blading, and probably stripping the blades from the shaft or casing.

The expansion of cast iron, wrought iron, and steel under the action of heat is given in the following table:

Coefficient of Expansion per 1° Fahrenheit.		
Metal.	Between 32° F.–212° F.	Between 32° F.–700° F.
Cast iron ..	·00000618	·0000087
Wrought iron ..	·00000656	·00000895
Steel	·00000600	·00000800

From the above, the expansion in inches of any length of pipe can be calculated by multiplying the length in inches by the temperature rise (i.e. the difference between the temperature of the pipes when out of commission, and the temperature under working conditions), and by the above coefficient of expansion. The mean lower temperature can be taken at 60° F. in most cases, while obviously the higher temperature is the maximum steam, air, gas, or water temperature obtaining during working conditions.

For gas- and water-pipes laid underground the temperature variation does not exceed 20° F., unless, of course, hot gas or water is being conveyed through the pipes.

In order to make allowance for the expansion of a range of pipes, the general lay-out should, wherever possible, be arranged so that the pipes may spring without putting undue stress on the rigid connections. For example, in designing a steam range for a power plant, in which four boilers deliver steam to four turbo-generators, the arrangement of steam mains may conveniently be as in fig. 40.

It will be seen that the expansion of the pipes will not be restricted, and the arrangement is very flexible. The greatest amount of expansion takes place in the long length of steam main behind the boilers, but by anchoring this main at the point A to some part of the building, the movement of the junctions of the branch steam-pipes is negligible.

The stresses in a steam range due to expansion may be considerably reduced by giving counter-stresses during erection. For example, if it is assumed that the total calculated expansion which the bends B must take up is 2 in., then, by springing out or extending the bend B 1 in. from its normal shape during erection, it is clear that when the pipes are fully heated the bend B will only be compressed 1 in. from its normal shape, instead of 2 in., which would be the case if no initial straining had been adopted.

This practice of initially straining the pipes should always be carried out, as it practically halves the loads on the pipe anchors and final points of connection to plant.

It is desirable for the purpose of checking vibration, and to control expansion in the desired direction, to anchor steam ranges to some fixed point, and the choice of the position of anchors should receive careful con-

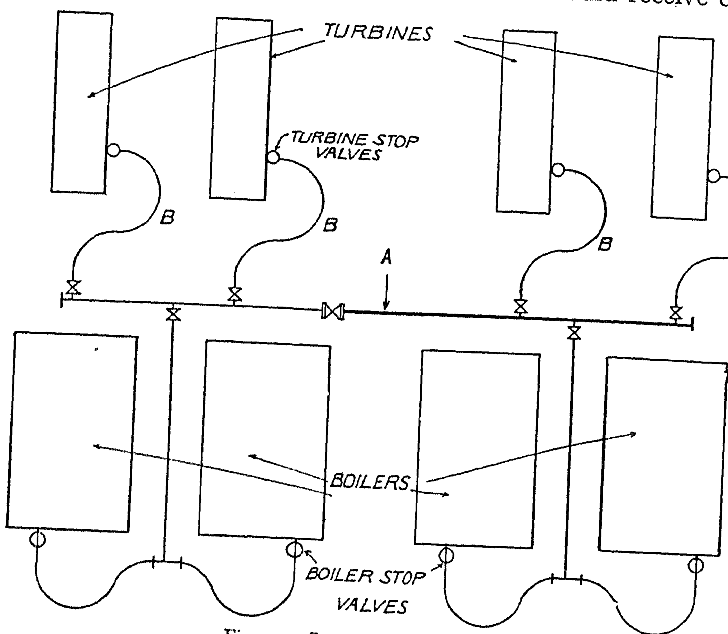


Fig. 40.—Steam Range for Power Plant

sideration, so as to ensure that perfect freedom for expansion of the pipes is allowed.

CHAPTER IV

Flow in Pipes

Steam-pipes.—When designing relatively short high-pressure steam-pipes, say 60 to 100 ft. in length, it is not usually necessary to make any calculations regarding the drop in pressure between the ends of the pipe, as under these circumstances this is practically negligible, provided the following mean steam velocities are not exceeded:

	Feet per Second.
For pipes below 2-in. bore the velocity should be	80
,, over 2-in. bore to 6-in. bore the velocity should be	85
,, ,, 6 ,, 12 ,, ,, ,,	90
,, ,, 12 ,, 18 ,, ,, ,,	95
,, ,, 18 ,, 24 ,, ,, ,,	100

In the case of long steam-pipes of over, say, 150 ft. in length, the question of drop in pressure becomes of increasing importance with the length

of the main, and in these cases the drop in pressure must be carefully considered when determining the size of pipe to be installed. It should be remembered, however, that it is not economical to provide unduly large pipes, as while a large pipe will give a small pressure drop, the increased surface of the larger pipe causes increased radiation losses.

Many formulæ for the drop in pressure in pipes have been evolved, but the following formula appears to give the most satisfactory results in practice.

It is given in *Steam* (Babcock & Wilcox, Ltd.), and in several other books, and is as follows:

$$W = 87\sqrt{\frac{D\,(P_1 - P_2)d^5}{L\left(1 + \frac{3\cdot6}{d}\right)}}.$$

W = weight of steam in pounds per minute.
D = weight per cubic foot of steam at the working pressure.
P_1 and P_2 = the initial and final pressures.
L = the pipe length in feet.
d = diameter of pipes in inches.

The above formula is based on the assumption that the length of pipe under consideration is straight, and in order to obtain accurately the drop in pressure in any system of pipes which includes valves or bends or elbows, it is necessary to make corrections to provide for the increased pipe friction due to the bends, &c.

In order to make this correction, it is usual to increase the length L in the above formula by an amount which is equivalent to the length of straight pipe, which would have the same frictional resistance as the total number of bends or valves in the service.

The resistance of a globe or mushroom valve, when passing high-pressure steam, is generally taken as being equal to the resistance of a straight pipe having a length of $\dfrac{114}{\left(1 + \frac{3\cdot6}{d}\right)}$ diameters.

This length in feet works out for various sizes of pipe as follows:

Pipe diameter.	¾″,	1″,	1½″,	2″,	3″,	4″,	5″,	6″,	8″,	10″,	12″,	15″,	18″.
Equivalent length in feet.	1·23,	2·07,	4·2,	6·8,	13·0,	20·0,	27·6,	35·6,	52·4,	70·0,	88·0,	115·0,	142·5.

The frictional loss due to an elbow or short bend is about two-thirds of that due to a mushroom valve, and therefore two-thirds of the above equivalent lengths must be added to the actual length of pipe for each elbow or bend, to give a correct result for the pressure drop.

For low-pressure steam-pipe installations, such as are used for conveying saturated or superheated exhaust steam over considerable distances

from industrial works to power plant in the vicinity, it is necessary to provide a pipe of ample cross-sectional area, so as to reduce the drop in steam-pressure to a very small amount, as frequently the pressure at the inlet end of the steam main does not exceed 16 lb. per square inch absolute, and it is necessary to transmit the steam through the exhaust-steam main with a loss of pressure of only 1 to $1\frac{1}{4}$ lb. per square inch If a superheater is installed to heat the steam immediately it leaves the engines supplying the exhaust steam, the inlet pressure to the steam main will not exceed 15 lb. absolute, leaving a drop in pressure of $\frac{3}{8}$ to $\frac{1}{4}$ lb. to be accounted for by the steam main.

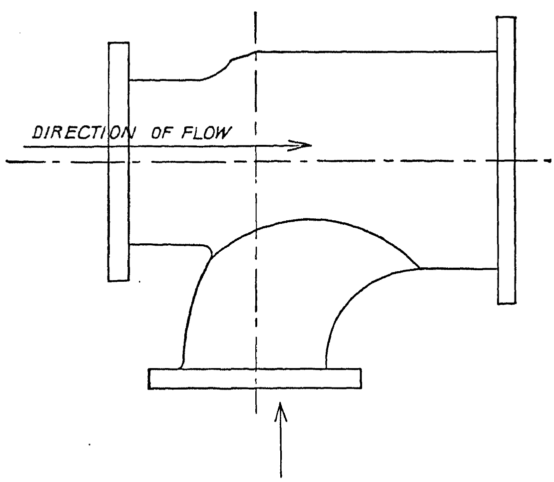


Fig. 41.—Curved Branch-pipe Connection

In order to keep the pipe diameters in a low-pressure steam main within reasonable limits, it is usually necessary to work with comparatively high velocities of flow, and it becomes specially important, in such mains, to reduce all subsidiary losses to a minimum. Full-way valves should be used wherever possible, and all bends should have a generous radius of curvature. Where it is necessary to lead in a branch pipe at the side of a main pipe, the junction should preferably be arranged as in fig. 41.

Flow of Air and Gas in Pipes.—Since the difference of pressure in the average supply system conveying gas for lighting purposes is only a few inches of water, changes of density in the system may be neglected, and for cast-iron mains under normal conditions the drop of pressure, h, in inches, of water, is given, very approximately, by the formula

$$h = \frac{\cdot 000052 l v^2 \rho}{d}\left\{1 + \frac{1}{7d}\right\},$$

where l and d are in feet; v is the mean velocity in feet per second; and ρ is the weight of the gas in pounds per cubic foot.

Dr. Pole's formula is often adopted for such mains. This is:

$$Q = 1350\sqrt{\frac{pd^5}{sl}},$$

where Q = the delivery in cubic feet per hour.
,, d = the diameter in inches.
,, p = the pressure of the gas in inches of water.
,, s = the specific gravity of the gas (air = 1).
,, l = the length of the pipe in yards.

In long air mains, supplying compressed air, the drop in pressure and the change in density may be considerable. In such a case, if the air is at atmospheric temperature, the drop in pressure is given very approximately, for pipes between 3 in. and 12 in. diameter, by the formula

$$p_1 - p_2 = \cdot 000000042 \frac{p_m^{\cdot 8} v_m^{1 \cdot 8}}{d^{1 \cdot 2}} l \text{ lb. per square inch,}$$

where l and d are in feet; p_m is the mean pressure in the pipe in pounds per square inch; and v_m is the mean velocity in feet per second.

Flow of Water in Pipes.—The resistance to the flow of water in pipes depends on the roughness of the walls and on the diameter of the pipes. Over the range of velocities usual in practice, it is proportional to v^n, where n has a value between 1·7 and 2·0, increasing with the roughness of the pipe. For purposes of calculation it is convenient to assume that in all cases the loss is proportional to v^2, in which case the Chezy formula may be used. Here

$$v = C\sqrt{mi},$$

where $m = d \div 4$ and i = loss of head per foot length of pipe, expressed in feet of water. The factor C now varies with the diameter and roughness of the pipe and with the velocity of flow v. If d is in feet, and v in feet per second, the following are typical values of C.

Material.	Velocity. Ft.-sec.	Diameter (Inches).					
		6.	12.	18.	24.	36.	48.
New cast iron.	2	100	107	111	115	120	124
	4	104	111	115	119	124	128
	6	106	113	117	121	126	130
	8	107	114	118	122	127	131
Clean asphalted or smoothly finished concrete pipes.	2	—	103	108	113	120	126
	4	—	108	113	118	126	132
	6	—	112	117	122	131	137
	8	—	115	120	125	134	141
Single riveted steel or wrought-iron pipes.	2	—	97	103	108	114	119
	4	—	103	109	114	120	125
	6	—	107	113	118	125	129
	8	—	109	115	121	128	133

In order to prevent rapid deterioration of the interior surface of a steel or cast-iron pipe line conveying water, a protective coating should be applied. One method, Dr. Angus Smith's, is given on p. 36. Also a mixture of 9 parts of pitch to 2 of boiled linseed oil gives good results, as does a mixture of natural asphalt with sufficient liquid asphalt to fill the voids in the dry material. This should be applied hot. If the pipe is not too large it is best

applied by dipping it in a bath of the mixture at a temperature of about 350° F

In this case the pipe should be preheated to about 250° F., and should be left in the bath long enough for it to attain the temperature of the mixture. It is then left to drain, and should set hard within an hour. A second coating is advisable, the time of immersion now only being sufficient to soften the outside of the first layer. Such a coating should be free from all pinholes.

In the case of pipes intended for gas the coating is usually only applied to the outer surface of the pipe, and to the inner surface of the socket in a spigot and socket joint. In pipes which are not to be completely coated, the composition may be applied hot by a brush or other suitable means.

CHAPTER V

Drainage of Pipes

Steam-pipes.—The design of a steam range should be such that any condensed steam which may collect, either when the main is in commission or when being cooled down or started up, can be automatically discharged from as few points as possible.

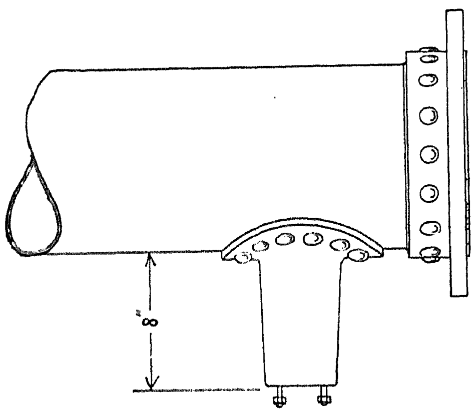


Fig. 42.—Drain Pocket

Pipes should be given a fall to the drainage points, and the direction of the flow of steam should, where possible, be such as to assist any water to travel towards the draining point.

The necessity of draining a steam main immediately in front of a vertical expansion loop has already been referred to and illustrated in fig. 33.

It is important to ensure that when starting up the whole or any part of a steam range, it may be drained so that no water can collect against any closed valve, as it will be appreciated that, when the valve is opened, any water so collected will be driven forward with high velocity to the farther end of the main. The energy stored in the moving column of water is so great, especially with a long length of pipe, that on being brought to rest against the first obstruction, such as another valve or a right-angled bend or tee piece, there is very great danger of a pipe rupture. In fact, numerous accidents have been proved to have been caused by the pipes not being drained before starting up.

These precautions as regards drainage should be considered not only

from the point of view of the initial starting up of the complete range, but for any conditions of starting up a portion of the main while the other part is already in commission.

When draining a horizontal steam-pipe, a drain pocket should be attached to the under side of the pipe as shown in fig. 42, which illustrates Messrs. Babcock & Wilcox's standard practice.

The pocket shown is riveted on, but, if preferred, the pocket can be electrically welded on.

The lower end of the pocket is arranged for the attachment of a $\frac{3}{4}$-in. or 1-in. flanged drain valve.

For automatically removing the water from a drain pocket as described above, an automatic steam trap should be connected by means of a length of $\frac{3}{4}$-in. or 1-in. pipe to the drain valve on the pocket.

A typical steam trap is shown in fig. 43, known as the " Birkby ", and manufactured by Messrs. Hopkinson.

Referring to the illustration, the body of the trap A forms a water chamber in which the water from the steam-pipe drain collects. A bucket B with central guide and a valve C is centrally supported by the outlet pipe. The valve remains on its seat until the bucket is sunk by the rising water and strikes the collar on the valve spindle. The valve then opens and the trap begins to discharge the accumulated water. This continues until the bucket is nearly empty, when its buoyancy causes it to rise and to shut the valve, thereby stopping the discharge of water.

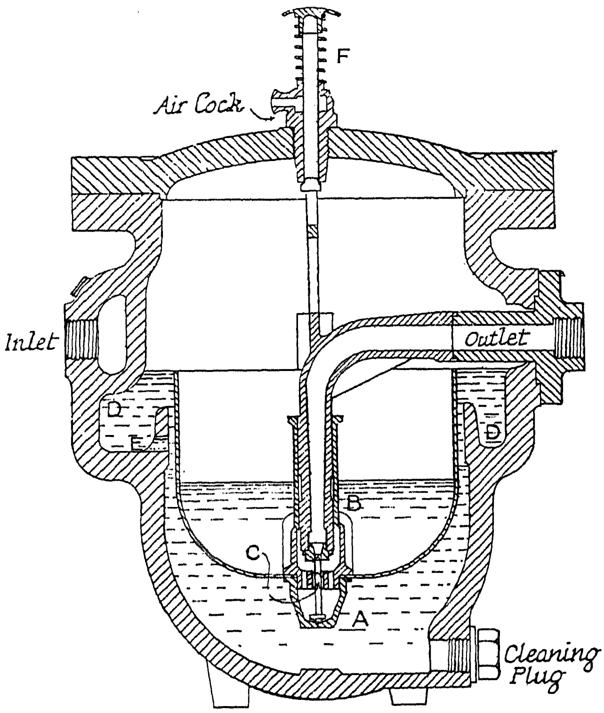


Fig. 43.—Typical Steam Trap

The action of the trap is therefore intermittent and depends on the rate at which water is delivered to it.

In the particular trap illustrated, a water gallery D is provided into which the water from the drain pocket is first discharged. The water then passes through the hole E into the body of the trap. After the trap has discharged, a " reservoir " of water is left in the gallery D which runs into the chamber A, and ensures a rapid lift of the bucket independent of the rate at which water

is entering the trap. In order to periodically test the trap by hand to ascertain that the bucket and valve are free, and to test whether the trap is working, a testing lever F is provided at the top of the cover.

For the purpose of draining low-pressure steam-pipes, drain pockets should be fitted as in the case of high-pressure pipes, but it is not necessary to provide steam traps as illustrated in fig. 43, as by arranging a suitable seal pot and overflow, as shown in fig. 44, the water collecting is automatically drained away. The depth of the "seal" B should be sufficient to ensure that the highest steam pressure obtained under working conditions is not sufficient to blow steam direct through the water in the seal pot. Since the

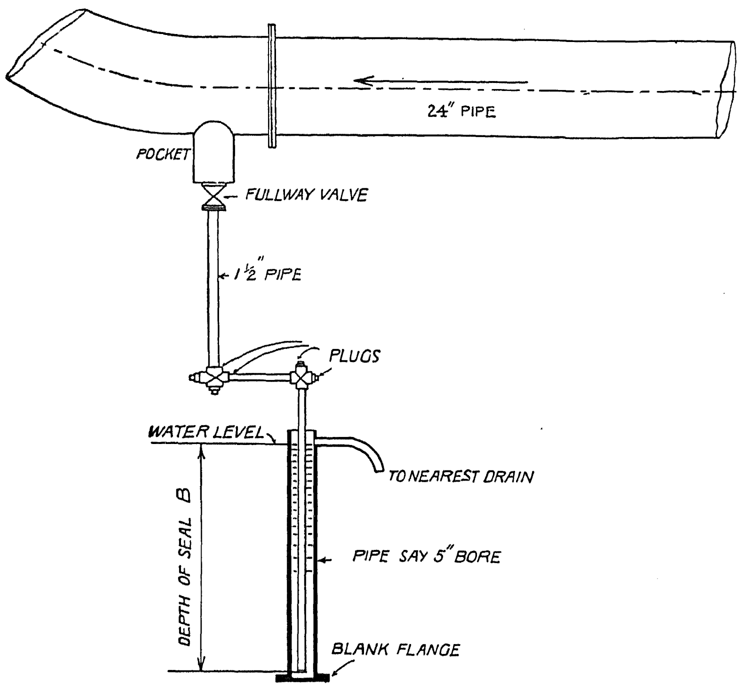


Fig. 44.—Seal Pot for Draining Low-pressure Steam-pipes

head of water equivalent to 1 lb. pressure is 2·31 ft., it follows that theoretically this head of water is required for every 1 lb. of pressure. In practice, however, it is advisable to allow at least 3·0 ft. head per 1 lb. pressure, as any pulsation in the steam main may cause a momentary oscillation of the water beyond the theoretical amount, and allow the steam to blow.

This form of automatic drain is not usually used for pressures above 2½ lb. pressure (gauge), as obviously the seal will then be very long. For pressures above 2½ or 3 lb., an ordinary steam trap as used on high-pressure mains may be used. Referring to fig. 44, it will be seen that cross pieces or tees with plugs are used rather than bends. This has been found advisable in practice, as drains from exhaust-steam mains are liable to become blocked up with dirt, and, owing to the small pressure available, cannot be satisfactorily

blown through to clear them. The use of tees or cross pieces with removable plugs facilitates cleaning without dismantling. For the same reason it is advisable to use full-way drain valves rather than globe or mushroom valves.

Gas Mains.—The remarks concerning the drainage of steam-pipes as regards direction of fall to suitable draining points also apply in the case of gas mains.

As a considerable amount of liquor is often found in gas mains from coke ovens, &c., it is advisable to provide drains of the type shown in fig. 44.

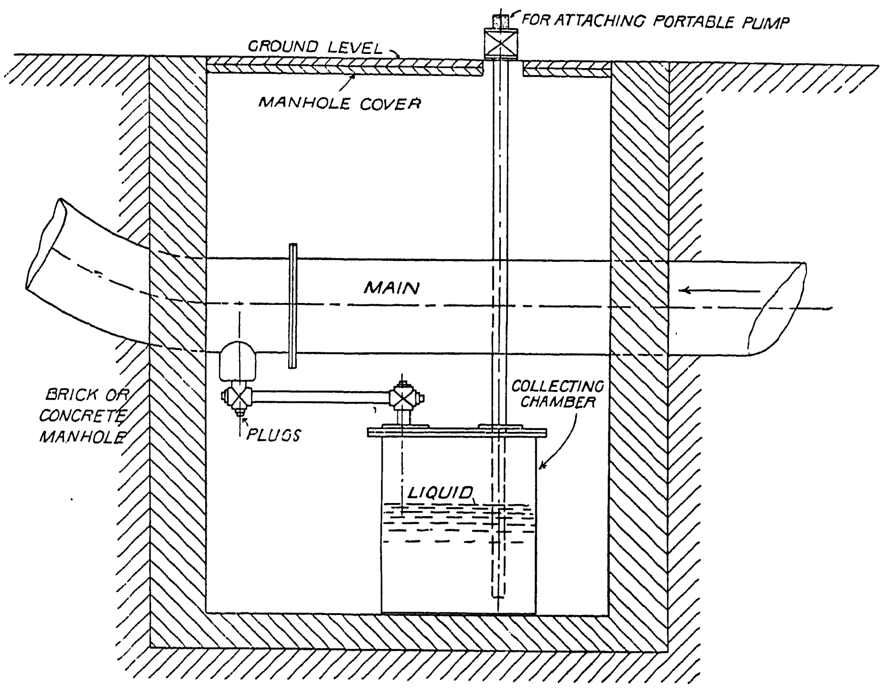


Fig. 45.—Closed Seal Pot

In the case of gas mains, however, the drain-pipes should not be less than 2-in. bore, as the effluent liquor is often of a tarry nature and will not flow freely through a pipe less than 2-in. bore.

Brass drain valves or cocks should not be used for gas mains as the gas attacks brass. Cast-iron plug cocks are generally used for drain connections.

For collecting the liquor from underground gas-pipes a closed seal pot or chamber, as shown in fig. 45, should be used. The vertical pipe projecting above ground-level should be provided at its upper end with a suitable cock and union for the periodic attachment of a hand-pump in order to pump out the accumulated liquor.

CHAPTER VI

Pipe Supports and Valves

Pipe Supports.—Pipes may be supported in various ways as follows:

1. By columns provided with suitable tops for carrying the pipes.
2. By hangers from adjacent walls or girders.
3. By brackets or saddles on which the pipes may rest.

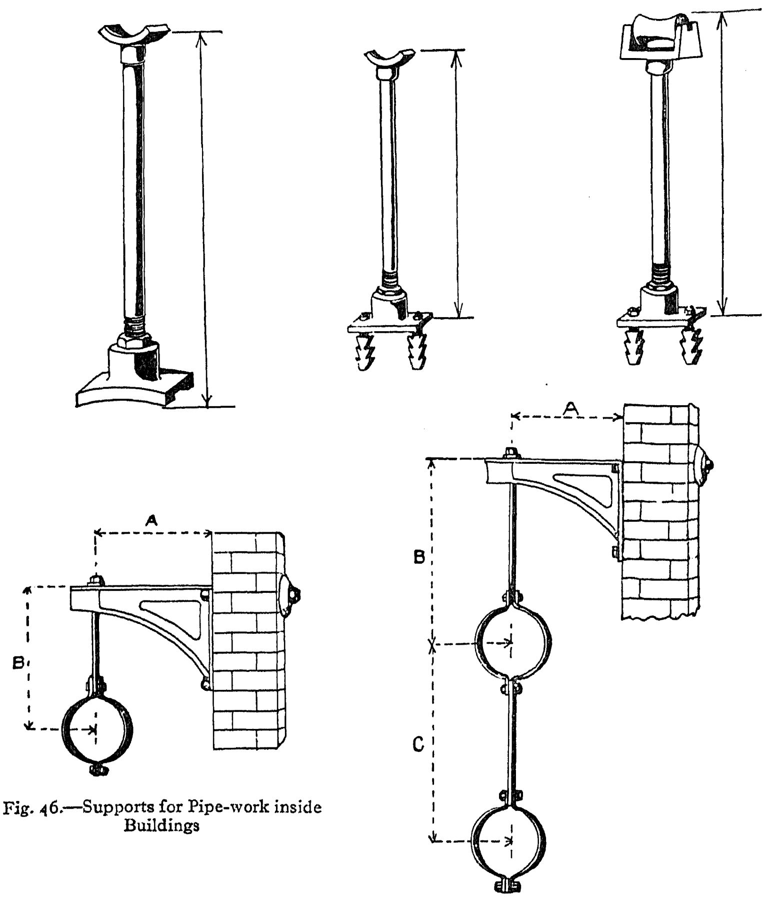


Fig. 46.—Supports for Pipe-work inside Buildings

Suitable supports for pipe-work inside buildings can usually be selected from more or less standard supports, typical examples of which are shown in fig. 46, as made by Babcock & Wilcox. An essential feature of a pipe support is that it should not restrict the movement of the pipe sideways when required to relieve the expansion stresses, and should allow of free movement along the line of the pipe for expansion.

It will be seen that all the above brackets and supports allow for a fair

amount of movement in both directions. In the case of long lengths of pipe subjected to large changes of temperature, it is, however, generally necessary to resort to some system of rollers to allow the pipes to move freely.

An efficient form of roller support, as made by Messrs. Babcock & Wilcox, is shown in fig. 47, and illustrates a supporting column for carrying a large steam-pipe above ground. The pipe is supported on a roller, which rests in a cast-iron tray secured to the column top, while side movement of the pipe is prevented, within the limits required to allow of free expansion, by means of rollers revolving on vertical pins attached to cast-iron side-brackets.

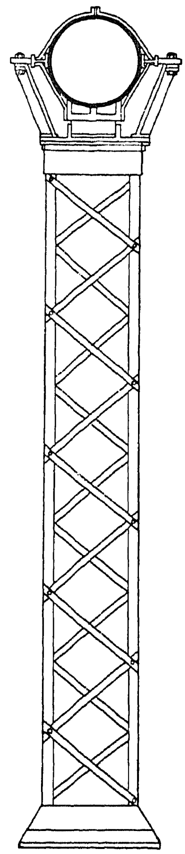


Fig. 47.—Supporting Column for carrying Large Pipes

Surfaces on the pipes for the three rollers to run on are clamped on by wrought-iron straps, arranged to grip the pipe without the necessity of drilling holes in the pipe itself.

In cases where the pipe is only a short distance above the ground, the column is dispensed with, and the cast-iron tray and side brackets are directly grouted-in on a concrete foundation. It is not necessary to use steel columns unless the pipe centre line is more than, say, 6 ft. above ground-level, as concrete pillars can be used up to this height with advantage.

Another type of support for large steam-pipes consists of a cast-iron cradle clamped to the pipe as shown in fig. 48. The cast-iron cradle A is provided with extended arms which carry small wheels B. Side motion is controlled by the projections C, which, together with the wheels, run on channels D, forming the top members of the column support. In cases where a column is not necessary, owing

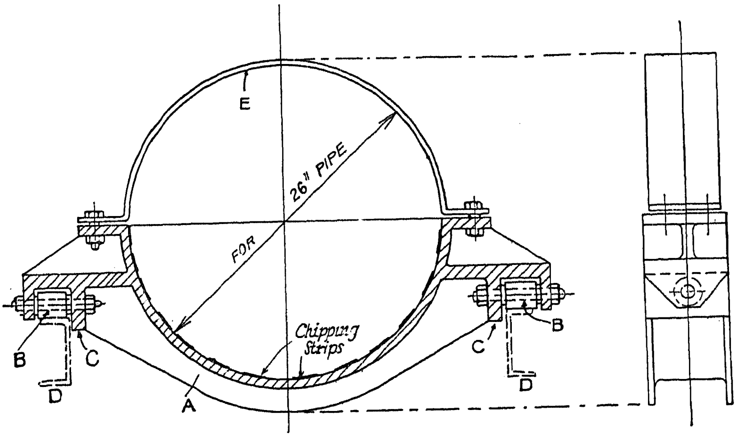


Fig. 48.—Cast-iron Cradle Support

to the pipe being near the ground, the channels are simply grouted-in on concrete foundations. It will be seen that no drilling of the pipe is necessary to attach the cradle, as the clamp E secures it in position.

It is sometimes required to carry two pipes on one set of supports, and fig. 49 shows a combination of the cradle type of support for a large pipe, and a side roller support for a smaller pipe carried over the large one.

Supports for long gas or air mains when placed above ground need not be provided with rollers, as the movement due to expansion is very much less than in the case of steam-pipes.

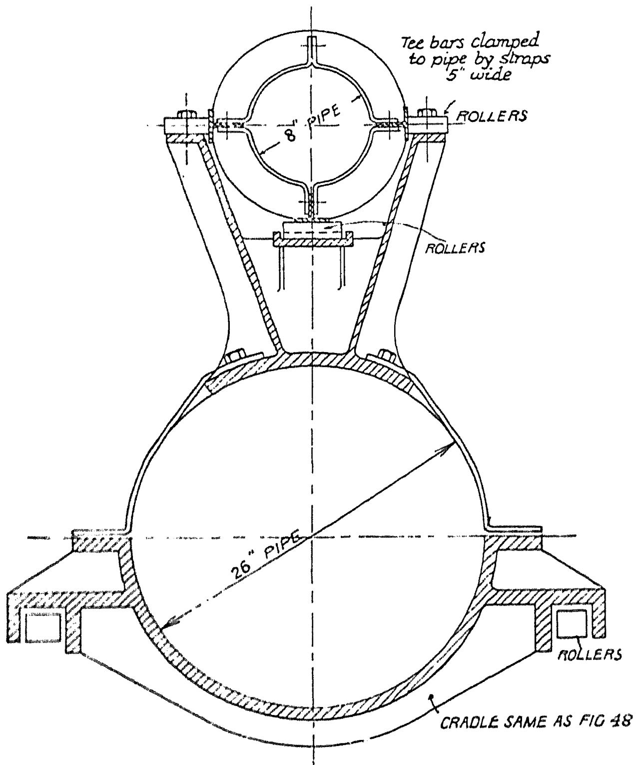


Fig. 49.—Combination Support for Large and Small Pipes

Figs. 50 and 51 show types of supports for a gas main laid above ground.

The support shown in fig. 51 has a side support for the pipe on one side, and is used where required to prevent the pipe travelling sideways under expansion stress.

Figs. 52 and 52A show an arrangement of supports in which two mains can be carried one above the other without the use of steel framework for the upper pipe. The lower pipe is provided with brackets cast on, which rest on the ends of large cast-iron rollers. The rollers which bear on the brackets give stability to the pipes, and it is generally found to be sufficient to provide brackets on alternate pipes only. The intermediate rollers simply carry the weight of the pipes and do not give stability.

Valves.—It is of course necessary to provide valves other than drain valves on a range of high-pressure steam-piping for isolating sections of the range, but the number of valves should be as small as is consistent with the requirements of operation, due to different parts of the machinery and boilers being periodically out of commission.

For high-pressure steam services, cast iron should never be used in the construction of valves.

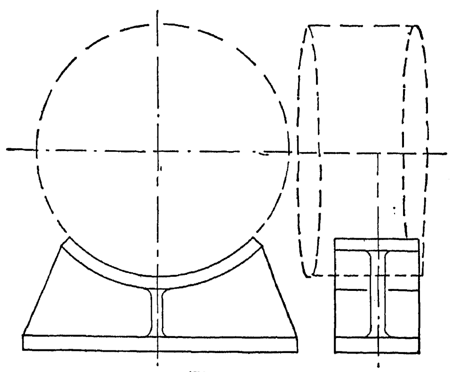


Fig. 50

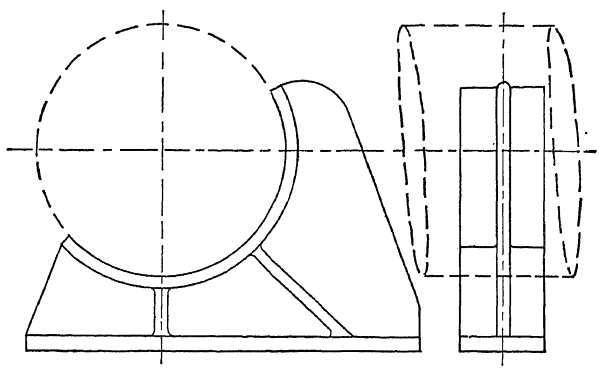


Fig. 51

Supports for Gas Main above Ground

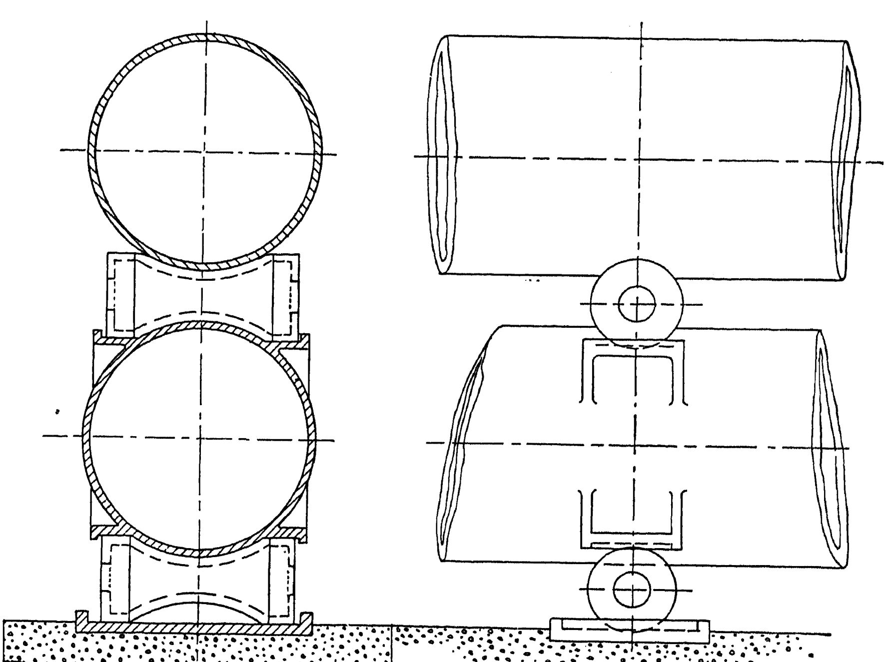


Fig. 52.—Arrangement of Supports for Two Mains

Fig. 52A.—General View of Circulating Water Mains

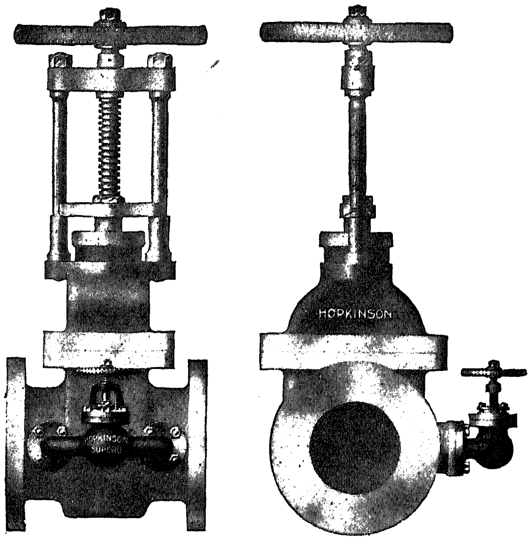


Fig. 53.—Arrangement of Bypass Valve

The smaller sizes of valves, say up to 2-in. bore, may be made of gun-metal in cases when a high degree of superheat is not used. Where high superheat is used, and temperatures of 700° F. and over are met with, cast steel should be used for the construction of all valve bodies.

When ordering valves, care should be taken to ensure that the arrangement of opening and closing gear is such that all valves are closed by rotating the hand-wheels in the same direction (preferably clockwise). It will be appreciated that, unless this is done, there is a danger of operators making a mistake by rotating the wheel in the wrong direction, especially in cases of emergency, which may lead to disastrous results.

For the same reason, and as an additional safeguard, all valve hand-wheels should be clearly marked, showing the direction of rotation required to close the valve, by casting on the words "to close" and an arrow indicating the direction of rotation.

Indicators should also be fitted on all important valves to show clearly at a glance whether the valve is open or closed.

Large steam valves should be fitted with bypasses to allow of a small amount of steam being passed at the commencement of warming up a steam range or prior to opening a large valve. A typical arrangement of bypass is shown in fig. 53, which illustrates a small valve on the side

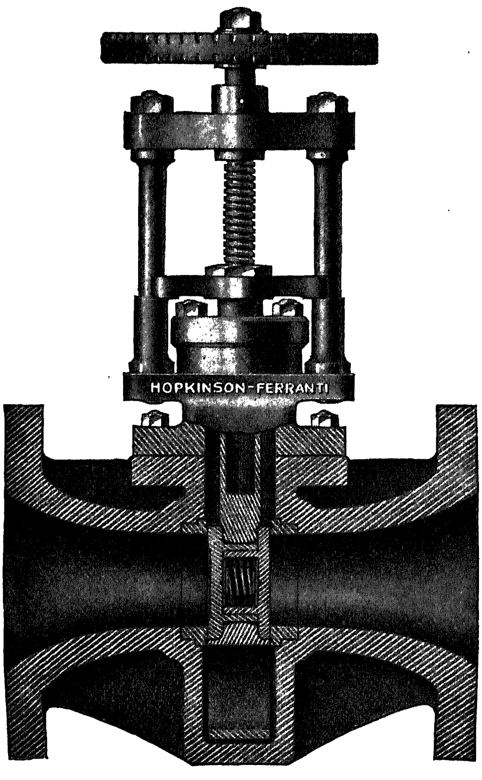


Fig. 54.—Hopkinson-Ferranti Valve

of a large parallel slide valve. Parallel slide valves above 5 in. diameter should be fitted with bypasses

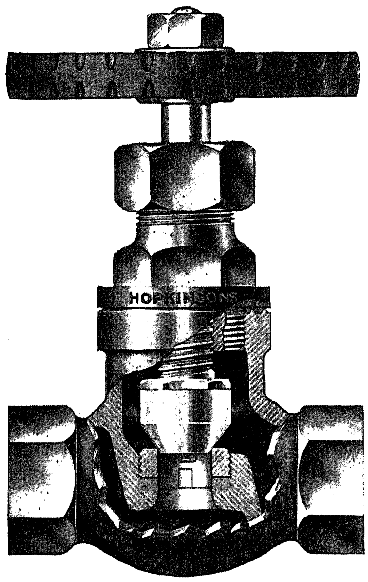


Fig. 55.—Small High-pressure Steam Valve

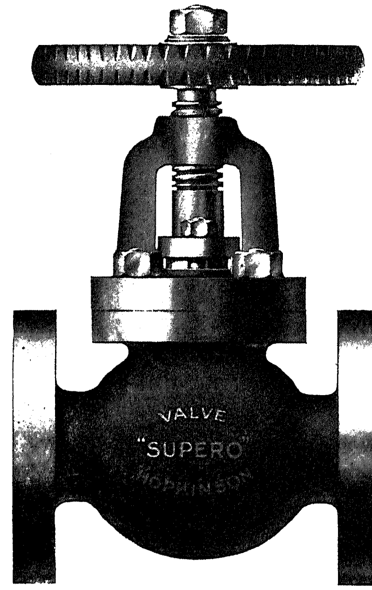


Fig. 56.—Valve for extra High Pressures

For high pressures, the valve shown in fig. 54, and known as the Hopkinson-Ferranti valve, offers many advantages.

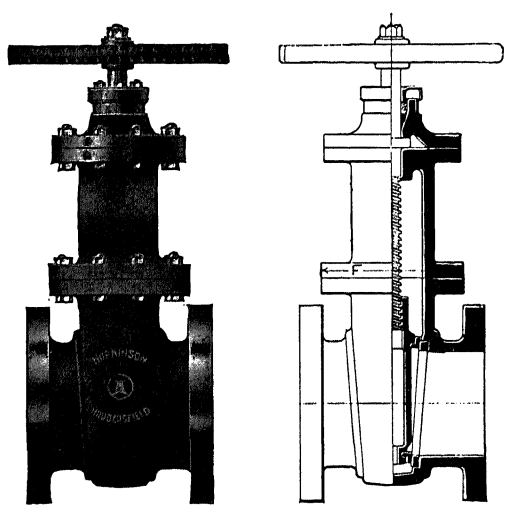


Fig. 57.—Valves for Exhaust-steam Mains

It will be noted that the body of the valve consists of a converging-diverging nozzle, and the slide itself and all moving parts are situated in the centre or smallest part. The actual slide is approximately half the diameter of the pipe to which it is coupled, and therefore there is less chance of leakage owing to the small size of the slide itself. This is of considerable importance in the larger sizes of pipes, say 6-in. bore and upwards. No bypasses are required in this type of valve for warming-up purposes on sizes below 10-in.-bore pipe.

This type of valve is made in sizes suitable for 4-in. pipes up to 24-in. pipes, and with cast-steel bodies and special metal seats and

slides to withstand the highest pressures (475 lb.) and high superheat.

For drain pockets and small high-pressure steam services, the type of valve shown in fig. 55 should be used. The illustration shows screwed end connections, but these valves are obtainable with flanged connections, and for high-pressure steam these are to be preferred.

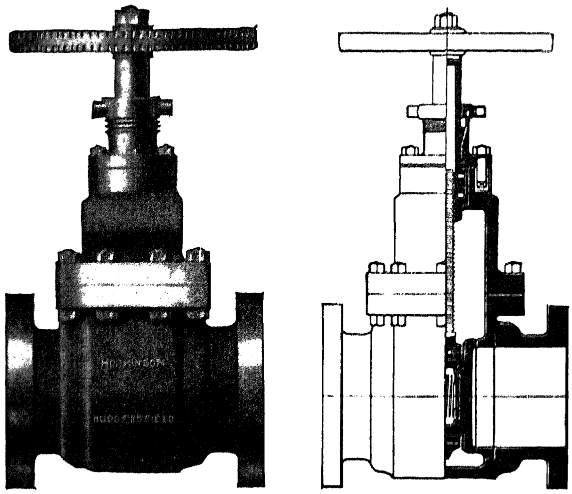


Fig. 58.—Valve for High-pressure Water or Oil Service

For extra high pressures and the highest superheats, the valve shown in fig. 56 with a cast-steel body should be used. The screws on the spindles of the high-pressure superheated steam valves should be outside the body, so that they may be lubricated. It is not necessary to use such expensive valves as illustrated above for exhaust-steam mains. In these cases the type of valve shown in fig. 57 may be used. The valve seat is in the form of a wedge provided with bronze facings. The body is of cast iron, and in cases when the steam is not superheated the screw on the spindle may be inside the body. In cases where exhaust steam is superheated, external screws should be used so that they may be lubricated.

The above valves may be used for water services where the pressure does not exceed 100 lb. per square inch.

For use with high-pressure water or oil services the valve shown in fig. 58 may be used.

This valve is provided with a parallel slide and bronze faces.

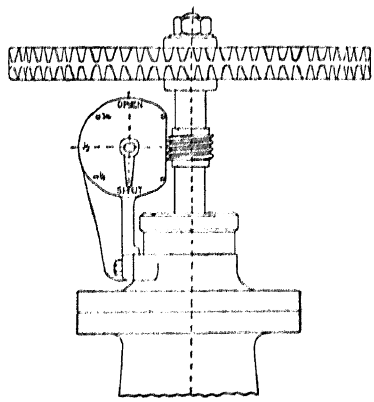


Fig. 59.—Indicator used for Valves fitted with Internal Screws

Fig. 59 shows a type of indicator used for valves fitted with internal screws. Valves fitted with external screws do not usually require a special indicator, as the bridge which spans the vertical pillars serves as an indicator to show the position of the slide (see fig. 54).

Atmospheric Valves.—An installation of exhaust steam-pipes carrying steam from engines to exhaust-steam turbines, or other exhaust-steam

utilization plant, requires an automatic atmospheric relief-valve to allow steam to escape to the atmosphere, in the event of the pressure in the exhaust-

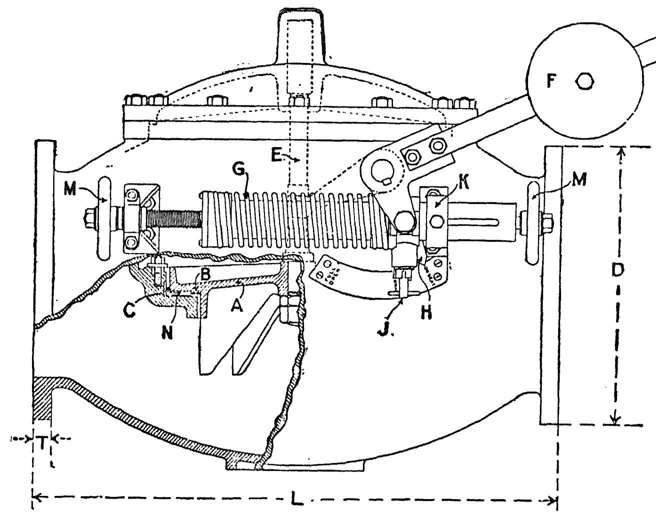


Fig. 60.—Spencer Bonecourt Valve

steam mains rising above a predetermined limit, due to the supply of steam exceeding the demand. If the steam is taken from reciprocating engines, as is often the case with an exhaust-steam turbine installation, there is a

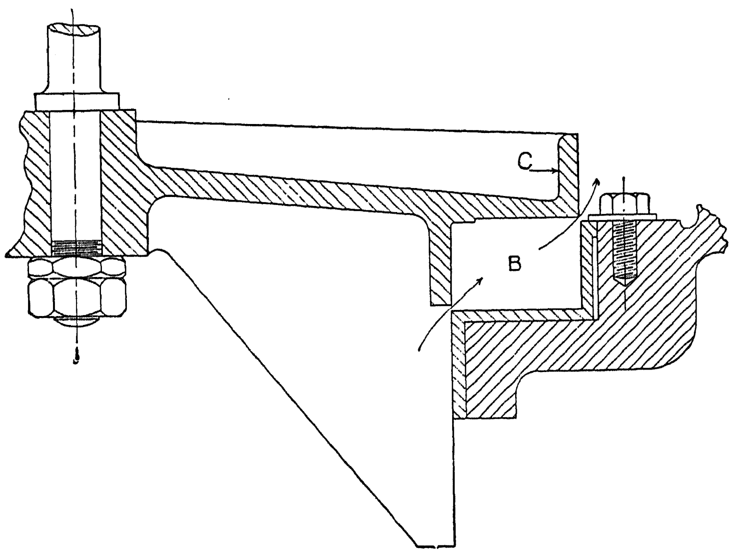


Fig. 61.—Diagram of Spencer Bonecourt Valve

periodic variation of pressure in the pipes, caused by the sudden exhaust of the reciprocating engine, which is very destructive to automatic atmospheric relief-valves. The pressure variation causes the valve to be continually

opening and closing, and unless the valve is fitted with an efficient dash-pot arrangement, the valve hammers on the seat and becomes damaged.

A type of valve made by Messrs. Spencer Bonecourt, which has been used successfully for the purpose of relieving when required the pressure in exhaust-steam installations, is shown in fig. 60. The valve A is provided with a seat at N, and an enlarged portion at C, which is an easy fit in the body of the valve. The valve is provided with a central guide at E, and the blow-off pressure on the valve is controlled by means of the spring G and the hand-wheel M, which rotates a screw carrying a nut for the purpose of altering the spring tension. The weight of the valve is counterbalanced by the balance weight F, and the stud J can be used to lock the valve in the full open position when desired. H is an indicator showing the position of the valve relative to its seat. In operation, the steam commences to blow off when the valve is lifted so that the enlarged portion C is above the central web of the body, as shown diagrammatically in fig. 61. The valve is prevented from hammering on its seat by reason of the steam trapped in the space B, which automatically acts as a dash-pot.

CHAPTER VII

Insulation of Steam Plant

A treatise such as the present work, dealing with the complex problems arising out of the economic raising of steam, cannot be considered complete without reference to insulation of heated surfaces.

Engineers in the past have been somewhat unwilling to realize the losses which occur through insufficient lagging of boilers, pipes, turbines, &c., and the following deals in a general manner with this portion of the problem.

A great number of insulating materials are on the market to-day, and are offered to engineers in a variety of forms; some of these consist of purely mechanical mixtures, while others are produced by scientific chemical and engineering plant. It is with the latter type of material that this section proposes to deal.

There are several base materials which can suitably be used in the manufacture of insulating material, but possibly none is so pre-eminently suitable as carbonate of magnesia produced from magnesium limestone or dolomite, with the addition of a suitable quantity of high-grade asbestos fibre. Such a material as this is the well-known Newall's Magnesia Covering. The works in which this lagging is made are situated in the north of England near an extensive dolomite bed, and have been in operation for 70 or 80 years. A great deal of research has been carried out by this firm on the complex problem arising out of scientific insulation, and, as a result of investigations made, materials have been evolved which cover practically all the problems

in connection with insulation. A variety of figures and tests is given below, showing the reduction which can be effected in the loss of heat, and also showing how boilers can, if necessary, be placed at considerable distances from the engine-room without undue loss of heat or condensation of steam. There are to-day in operation many plants where the boilers are separated from the turbines or other steam-utilizing machinery by over three-quarters of a mile, the pipes running across open and exposed country.

The following example from actual tests shows the benefit derived from covering a steam-pipe 100 ft. long, and conveying saturated steam at 100 lb. pressure. The diameter of the pipe was 4 in., and the condensation amounted to 108 lb. per hour with a bare pipe, whereas when covered with 1 in. of

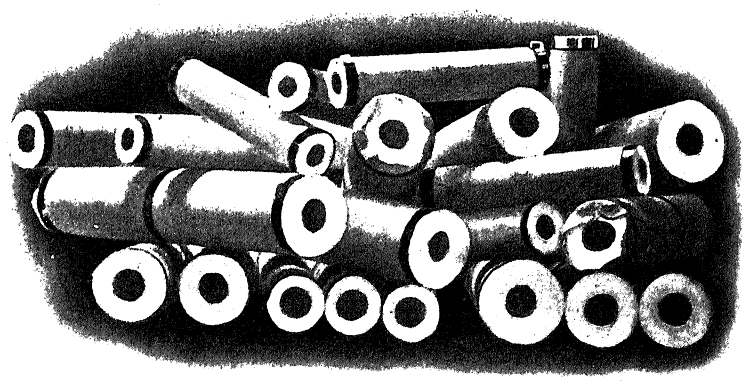


Fig. 62.—Lagging Sections

scientifically prepared non-conducting material, such as magnesia covering, the condensation amounted to only 17 lb. per hour.

Magnesia covering consisting of 85 per cent basic magnesium carbonate and 15 per cent asbestos fibre is probably the most efficient lagging obtainable, and when applied to heated surfaces retains from 87 per cent to 90 per cent of the heat which would be lost from uncovered surfaces under similar conditions. The remarkably low conductivity of this material is largely due to securing the best ratio of air space to solid matter in the process of manufacture, i.e. the air spaces are so small as to be practically free from convection currents; further, owing to its high porosity the material is extremely light. Magnesia covering is not affected by the action of water, excepting that it temporarily absorbs moisture which is afterwards given off without deterioration of insulating value. Magnesia either in the wet or dry condition has no corrosive action on any metal. It is manufactured in a variety of forms, and it can be applied equally successfully to hot and cold surfaces, and taken with the component materials which are placed on the market by Newall's Insulation Company, any condition for either high temperature work or for exposed work can be successfully met.

Magnesia covering is supplied in many forms to suit all purposes. It

is available in sectional form of various thicknesses to fit pipes from ½ in. to 15 in. diameter, the sections being secured to the pipe by bands, canvas, or wire. Fig. 62 shows an assortment of sections ready for application.

The material is also supplied in flat or curved blocks and as plastic magnesia covering; the latter is especially suitable for covering irregular surfaces. The plastic is mixed with water to form a soft mortar, and is applied by hand in two or more coats to the required thickness.

The loss of heat from flanges on steam-pipes is very considerable, and

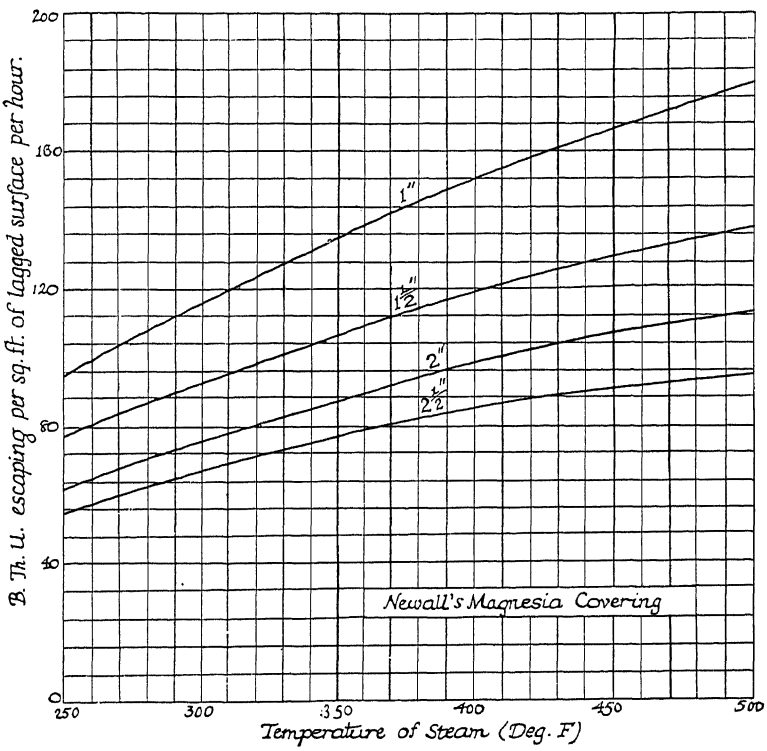


Fig. 63.—Curves showing Efficiency Tests of Magnesia

these should always be covered. The most efficient method of doing this is by means of detachable flange boxes consisting of a metal case lined with magnesia sectional covering.

Many tests of the efficiency of magnesia have been made, and these have shown the high efficiency obtainable, and also enabled the "economical thickness" (i.e. the maximum thickness which would repay its cost by heat saved) to be ascertained. The curves shown in fig. 63 give examples of the results obtained with various thicknesses and different steam temperatures.

As the use of highly-superheated steam became more general and the temperature of boilers, steam-pipes, and turbine casings and other heated

surfaces rose in proportion, it became evident that on account of the chemical constituents in magnesia covering, it was desirable to combine with it a material which would withstand the very high temperatures now coming into use.

As the result of considerable research, a material known as Newtempheit was evolved, and this material is now increasingly used in conjunction with magnesia covering. While Newtempheit is a good non-conductor, it is used only as a protective covering next the pipe, to reduce the temperature at the point of union with the magnesia to 600° F. The magnesia still forms the main insulation of the boiler, pipe, or other surface to be lagged. Newtempheit will stand the highest temperatures at present contemplated in steam-raising plant, and there is no doubt that after continued research improvements can be made in its manufacture, which will enable less thickness to be used even if the temperatures rise above what is contemplated.

The following table shows the temperature gradients and heat losses for a combination of Newtempheit and magnesia covering, and also the minimum thickness of Newtempheit required with various temperatures to reduce the temperatures to that at which magnesia covering can be safely employed.

Internal Temperature of Pipe.	Lagging.		Surface Temperature of Lagging.	Loss in B.T.U.'s per Sq. Foot per Hour.	Bare Pipe Loss in B.T.U.'s per Sq. Foot per Hour.	Saving of Heat per Cent.	Coal at 13,000 B.T.U. per Pound and Boiler Efficiency 75 per Cent.		
	Newtempheit.	Magnesia.					Loss from Bare Pipe per 100 sq. ft. per Hour.	Loss from Lagged Pipe per 100 sq. ft. per Hour.	Saving Effected by Lagging.
Deg. F.	Inches.	Inches.	Deg. F.				Pounds.	Pounds.	Pounds.
600	½	1½	120	199	1325	85·0	13·6	2·0	11·6
650	½	2	115	196	1450	86·5	14·9	2·0	12·9
700	½	2	125	210	1575	86·7	16·1	2·1	14·0
750	½	2½	105	218	1700	87·2	17·5	2·2	15·3
800	¾	2½	110	236	1825	87·1	18·8	2·4	16·4
850	1¼	3	108	263	1950	86·5	20·0	2·7	17·8
900	1½	2½	106	330	2075	84·1	21·6	3·4	18·2
950	2	3	108	332	2200	84·9	22·7	3·4	19·3
1000	2½	2½	112	394	2325	83·1	23·9	4·0	19·9

Fig. 64 indicates the temperature gradient through combined covering of Newtempheit and magnesia lagging.

The magnesia covering thus continues to form the insulation, and it is protected next the pipe by a small layer of Newtempheit, and may also be protected on the outer surface against rough usage or weather by a hard setting composition, asbestos cloth, or other similar protective wrapping.

The need for employing Newtempheit with temperatures over, say, 600° F. is on account of the calcination which commences at about this temperature, and causes a gradual conversion of the magnesium carbonate into magnesium oxide, carbon dioxide, and water vapour. This causes minute spaces to be formed in the lagging against the heated surface, and magnesia covering alone would tend to separate slightly from the pipe.

This mechanical difficulty, entirely due to a chemical change, is overcome by the use of a layer of Newtempheit between the magnesia covering and the heated surface.

Referring to the table (p. 80), the figures given are for a 6-in. pipe, and the heat lost is calculated on the area of the pipe surface, not the area of the surface of the lagging. The various thicknesses have been so adjusted as to maintain an efficiency of about 85 per cent. The method used in determining heat losses is as follows:

A piece of 6-in. pipe, 2 ft. long, with a close-fitting lid at one end and

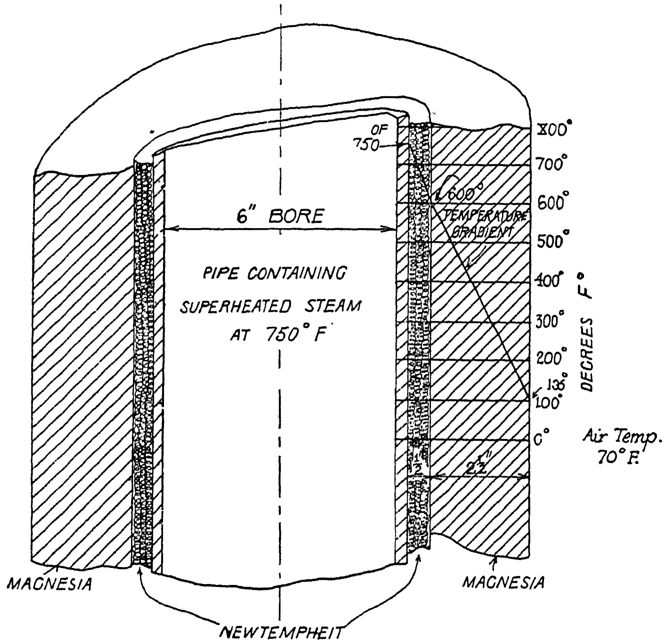


Fig. 64.—Temperature Gradient through Combined Covering of Newtempheit and Magnesia

closed at the other, is fitted with a grooved porcelain pot, round which is wound nichrome wire to serve as a heater. The current passing through this wire, and hence the B.T.U.'s generated, are measured by means of a watt meter, the current being adjusted by means of a variable resistance.

When the internal temperature measured by a standard thermometer or pyrometer is constant, as many B.T.U.'s are being developed by the heater as are escaping from the pipe surface. The pipe is then completely covered with the lagging to be tested, and the same internal temperatures obtained, and a second set of readings taken. The efficiency of the lagging is thus easily calculated.

Fig. 65 shows diagrammatically the arrangement of the testing apparatus. From the tables given (p. 80) it is a simple matter to ascertain the financial saving effected by lagging under any given set of conditions, and the following will serve as an illustration.

If it is assumed that the cost of coal is 20*s*. per ton, and the calorific value of the coal is 13,000 B.T.U. per pound with an average boiler efficiency of 75 per cent, the heat units passed into the steam-pipes for one penny would be approximately 91,000 B.T.U. From the table, the loss per square foot per hour from a bare steam-pipe carrying steam at 750° F. is 1700 B.T.U.,

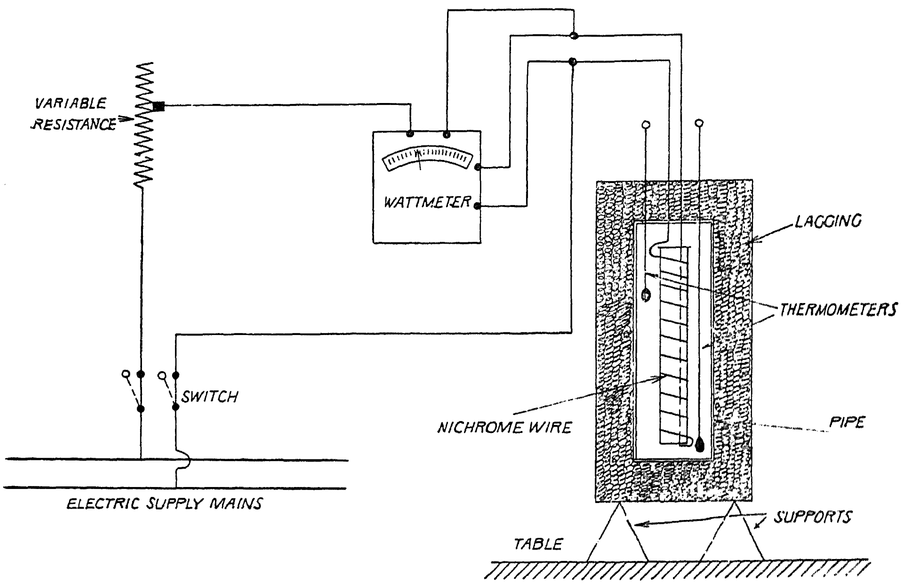


Fig. 65.—Diagrammatic Arrangement of Testing Apparatus

but when lagged with ½ in. of Newtempheit and 2½ in. of magnesia, the loss is only 218 B.T.U. The lagging therefore saves 1482 B.T.U. per square foot per hour, or in a working year of 300 days of 24 hours (equal to 7200 hours), 10,670,000 B.T.U. per square foot per annum. Dividing this figure by 91,000 shows that the financial saving resulting from the composite lagging is 117*d*., a sum far in excess of the cost of lagging one square foot of pipe. When considering the question of pipe covering, each case should be dealt with on the lines indicated above.

APPLIED MECHANICS

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Applied Mechanics

Co-ordinates.—Although all structures and machines exist in three-dimensional space, and in considering them this fact is of prime importance, the vast majority of them are such that much of their analysis can be carried out by the consideration of actions in a single plane and by the combination of these considerations, thus greatly simplifying the discussion of the problems involved.

The co-ordinates usually adopted in engineering work are either Car-

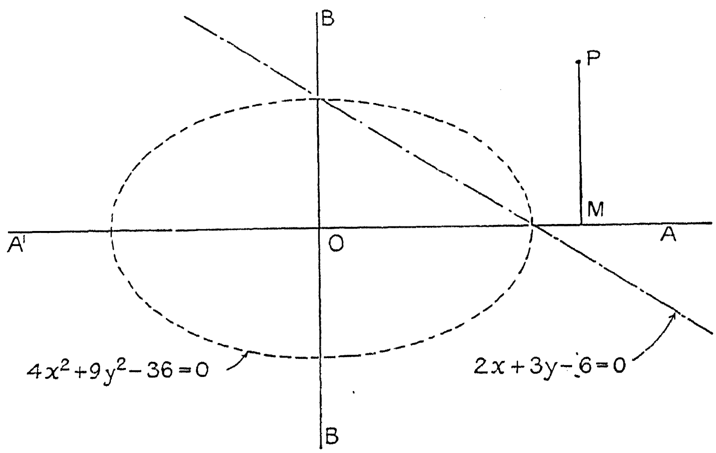


Fig. 1

tesian or polar In the former (fig. 1) a pair of reference lines OA, OB—here drawn and considered to be at right angles, as is customary, though lines at an angle (oblique) are sometimes employed—are taken, the co-ordinates of any point P being OM and PM (where PM is drawn parallel to OB to meet OA). The lines OA, OB are termed the axes, the point O the origin, and the point P is referred to as the point x, y, where OM $= x$ and PM $= y$. When OM is measured in the direction OA it is regarded as positive, when in direction OA′, as negative. The same convention applies to PM, which is considered as positive if it lies above OA and negative if below. In polar co-ordinates (fig. 2) the position of a point P is fixed by the

length of the radius OP (r) and the value of the angle POA (θ). If r be taken in the opposite direction, i.e. as OP′, it is regarded as negative, and if the angle θ be described in the opposite direction it is regarded as negative. In mechanical work it is customary to consider the counter-clockwise direction as positive. From the figure it is clear that the co-ordinates are connected by the relations

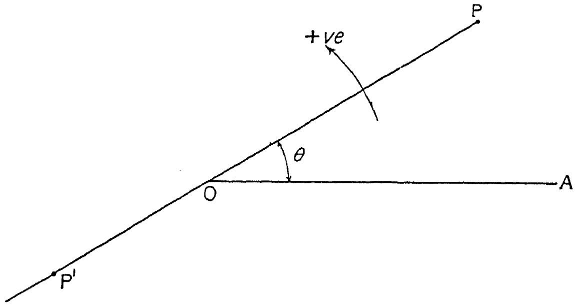


Fig. 2

$$r^2 = x^2 + y^2 \text{ and } \tan\theta = \frac{y}{x}.$$

The Expression of Relationships.—If the values of the co-ordinates (either x, y, or r, θ) be given, the position of the point is fixed, but if a relationship (such as $2x + 3y - 6 = 0$ or $y^2 = 4x - 3$) between the co-ordinates is given, the point is not definitely fixed but is confined to a curve, for to any selected value of x corresponds a certain number (real or imaginary) of values of y. Setting these out for a series of values of x, a curve is obtained upon which the points lie. The relationship, expressed generally by stating y as a function of x or by the typical equation $f(x, y) = 0$, is termed the equation of the curve. Thus $x^2 + y^2 = a^2$ is the equation to a circle having its centre at the origin, and of radius a, and $4x^2 + 9y^2 - 36 = 0$ represents an ellipse having its major axis along the axis of x and of length 6, while its minor axis is of length 4 (fig. 1).

If a pair of relationships $f(x, y) = 0$ and $F(x, y) = 0$ be given, the points whose co-ordinates satisfy them must lie on each curve and therefore are the points in which the two curves intersect.

Degrees of Freedom in a Plane.—A point in a plane may be thus considered to have " two degrees of freedom " (corresponding to its two co-ordinates), and any single condition to be fulfilled by the position of the point destroys one of these " freedoms ". To specify that the point lies on a particular curve leaves it with one degree of freedom only. Thus a locomotive on rails or the crosshead of an engine has one degree of freedom only, as the position of each can be represented by a point confined to a curve.

To specify the position of a body in a plane (i.e. a body which can only move so that the particles of which it is composed move in parallel planes) requires three co-ordinates, which are usually those of a particular point in the body and the angle which some definite line in the body makes with the the axis of x, and the body has three degrees of freedom. Thus the position of a connecting link (see fig. 3) in a machine is completely stated if the co-ordinates hk of one end and the angle θ which a line on the link makes with a given reference line be known.

If a number of bodies, moving in one plane, be under consideration, each of them, in the absence of restraints and connections, possesses three degrees of freedom, so that n bodies would possess $3n$ degrees. All position is " relative ", so that these degrees of freedom may be viewed from any desired standpoint: that is, in considering the geometrical relations between the bodies, any one may be taken as the standard of reference and considered as fixed. In a machine the main frame is usually regarded as fixed, and thus in a machine having n other parts there would be $3n$ degrees of relative freedom. Any connections between the parts would destroy one or more of these degrees of freedom. Thus if the mechanism of an engine consists of a frame containing a cylinder, a piston, connecting-rod, and crank with crank-shaft, the frame being taken as fixed, the three parts have nine degrees of freedom (in a plane); but of these the fit of the piston in the cylinder (confining the motion of its centre to the axis and preventing angular movement) destroys two degrees, the contact of the connecting-rod with the gudgeon-pin destroys

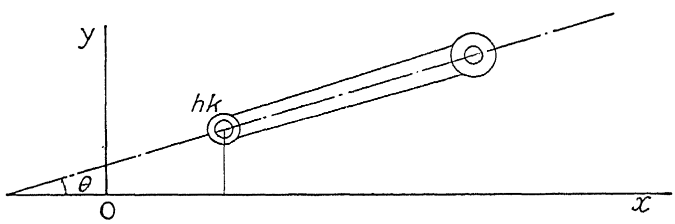


Fig. 3

two more, and its contact with the crank-pin yet another two, while the contact of the crank-pin with its bearings accounts for two more. Thus eight of the nine degrees are destroyed, leaving only one, which permits the constrained motion of the engine parts to take place.

Degrees of Freedom in Space.—If a point be not confined to a plane, but is in space of three dimensions, it requires three co-ordinates to specify its position and has three degrees of freedom. In fig. 4 the two Cartesian axes Ox and Oz are supposed to be drawn in the plane of the paper, and Oy perpendicular to it. From the point P, PM is drawn perpendicularly to the plane xoy, and then MN perpendicular to Ox, the Cartesian co-ordinates (x, y, z) of P being thus ON, NM, and MP. Polar co-ordinates, in which the position of a point is defined by its distance from the origin and two angles corresponding to latitude and longitude, are not much used in mechanical work. In any such co-ordinates any relationship, as $f(x, y, z) = 0$, between the co-ordinates of a point implies that it lies upon a surface; thus if $\frac{x}{2 \cdot 6} + \frac{y}{4} + \frac{z}{5} = 1$ the point x, y, z must lie upon the plane DEF, cutting the axes at distances 2·6, 4, and 5 from the origin as is indicated in fig. 4.

A body in space has three angular degrees of freedom in addition to the three translational degrees which any particular one of its points has: these correspond to the two polar co-ordinates (latitude and longitude) of a line or

axis fixed in the body, and to the amount which the body has turned through about this axis. Thus any set of n bodies in space has $6n$ degrees of freedom. Any restraint, which usually consists of the enforced contact of the surface of one body with that of another, destroys one of these degrees of freedom, but the nature of the contact is to be regarded carefully.

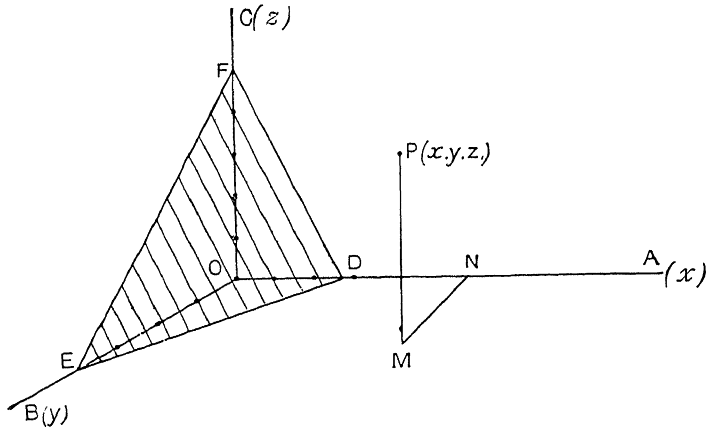


Fig. 4

Pin-jointed Perfect Frames.—Plane frames, such as roof and bridge trusses, consist of a number of parts, supposed to lie in one plane, and to be connected by pin-joints. A pin-joint connecting two parts destroys two degrees of freedom, as it enforces the coincidence of a point of one member with a particular point of another. Thus in the Warren girder of fig. 5 there are nineteen members (each part AB, BC, CD, &c., of a boom being considered as distinct and pin-jointed at its ends) and twenty-seven pin-joints (the pin-joints at A and F are single, those at G and L double and equivalent

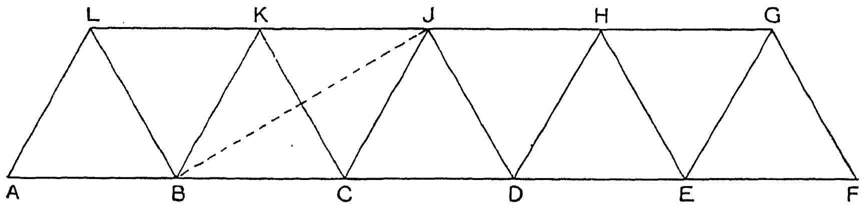


Fig. 5

to two separate joints, and the remainder treble) which gives $3 \times 19 - 2 \times 27 = 3$ net degrees of freedom, so that the whole set of connected parts has just the freedom of a single body. Reckoned relatively to any part, e.g. AB, there would be eighteen other members, giving no degrees of freedom. This implies that the parts would just go together, without any forcing, into one firm frame: thus the bars AB, BL, LA will form a triangle when the pins are slipped through the holes, and to this can be added the bars LK, BK and a pin slipped through the K holes in the position of coincidence,

and so the whole frame can be built up. Such a frame is termed "just rigid" or "perfect", and has the important property that no initial stress will exist in its members owing to inaccuracies in their lengths in construction or to variations caused by temperature.

Redundant Frames.—To place an additional bar in the frame (e.g. across BJ) will necessitate the length of the bar between the pin-holes to be the same as the length BJ in the frame, i.e. the bar must be accurately fitted, otherwise stresses will be produced in the frame in the erection, and these will be superposed upon those caused by the loading. Such frames are termed "redundant" or "over-stiff". The replacement of pin-joints by riveted connections somewhat alters the conditions, as the angles at a joint between the members cannot now alter to meet the new shape of frame required by the strain of the members. The main forces and stresses are the same, but are altered in the riveted construction by amounts termed secondary stresses, which are calculated in important struc-

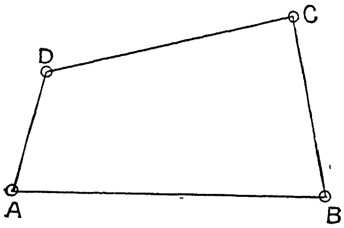


Fig. 6

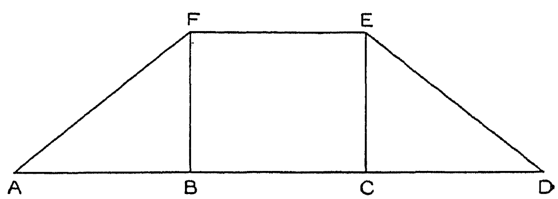


Fig. 7

tures only, being met by an allowance in ordinary work. The modern tendency is to employ perfect frames only in the main members of important structures. In a girder such as is sketched in fig. 5, the secondary stresses amount to from twenty to thirty-five per cent of the main stresses.

Linkages.—The systems sketched in figs. 6 and 7 have, relatively to one bar, each one degree of freedom remaining, e.g. fig. 6 shows three bars in addition to AB and four single joints, giving $3 \times 3 - 2 \times 4 = 1$ degree of freedom. Such frames are termed "imperfect", and viewed in this manner are collapsible; but fig. 7 is a common (queenpost) roof truss, and the rigidity of the truss depends partly upon the bar ABCD being continuous through the joints B and C, and partly upon BF and CE being morticed into ABCD. Actually the truss is then redundant.

The arrangement of bars shown in fig. 6 is also regarded as a "linkage", and is typical of certain types of mechanism. It has in itself one degree of freedom and no more, which permits the desired motion to take place; but when, besides the frame AB, a second link BC has a definite position, the position of every point in the system is determined. The simplest connection between two parts of a machine is the enforced contact of their surfaces at a single point, this type of contact being maintained during the motion, although the specific points of the parts of the bodies continually change.

The action of a cam, or of the teeth of spur gearing, is of this nature. Such contact destroys one degree of freedom only. Thus a mechanism (fig. 8) consisting of a frame in which two gears are mounted on shafts, and rotate so that their tooth surfaces are always in contact, has in itself one degree of freedom only—two parts, the gears, give six degrees of freedom, of which two are destroyed at each bearing and one at the tooth contact.

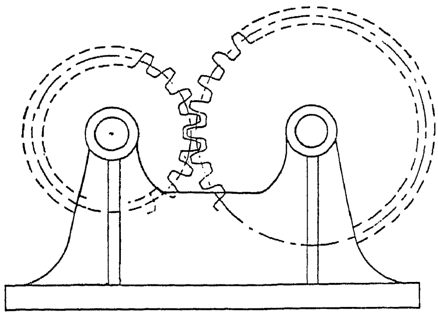


Fig. 8

Lower and Higher Pairs.— In the sketch the teeth touch at a point, in actual space along a line, but in three types of contact only one degree of relative motion of the parts is possible, and contact is over a surface and so maintained during the motion. These types — journals and bearings, slides, and screws and nuts — are termed lower pairs, while those in which contact is at a point or along a line are termed higher pairs. Actually there is always surface contact, as under the transmitted load, sur-

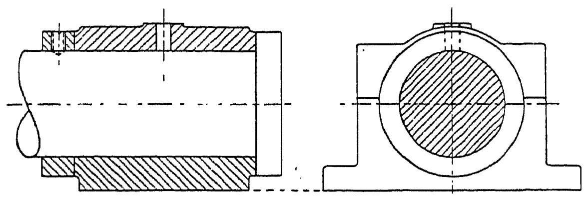


Fig. 9.

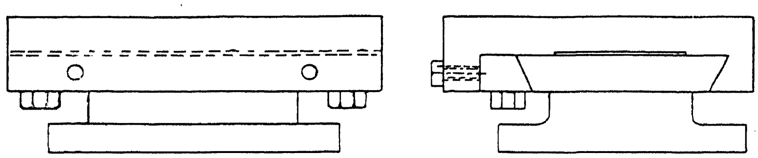


Fig. 10

faces which would touch along a line if undistorted, yield elastically so that they touch over small surfaces. The amount of "surface of contact" in the lower pairs enables large loads to be carried, but the motion always involves simple sliding, while rolling of the surfaces may take place with the higher pairs. A lower pair permits of one degree of freedom between the parts, so that a turning pair, or journal and bearing, must be considered with regard to end as well as lateral movement. Such a complete pair is shown in fig. 9, a sliding pair in fig. 10, and a screw pair in fig. 11. In all

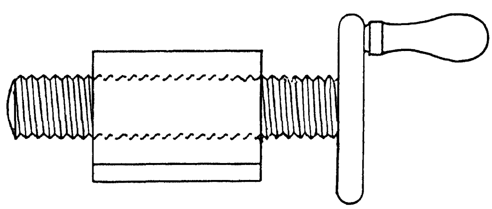


Fig. 11

these the "constraint" is complete; but this is not always the case, as is illustrated in fig. 12, which shows a sliding pair in which the contact is enforced by the weight of the upper part, this being termed force-closure. It has advantages where easy motion is desired.

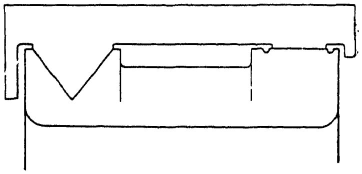


Fig. 12

Point Paths.—Usually in a machine, considering the frame to be fixed, if the position of one part be known, the position of every point in the machine is determined, as the movement is positive. In the motion, however, there occasionally occur positions in which the resulting motion is not definite; the case of an engine mechanism with the crank on a dead centre is illustrative. In these, corresponding to a near position of one part, there are alternative configurations possible for the other parts: thus for a piston position close to the dead centre, there are two possible connecting-rod and crank positions corresponding to the two directions of rotation. Such positions of a mechanism are termed "change points". In other machines, for instance those containing a belt drive or a fluid power transmission, slip may occur, so that the motion is not definite or "positive". As illustrating a positive movement a drawing is made in fig. 13 of an indicator, in which some essential points of design are that the parts should be light (and stiff) to avoid inertia effects, that the pencil point P should move (within

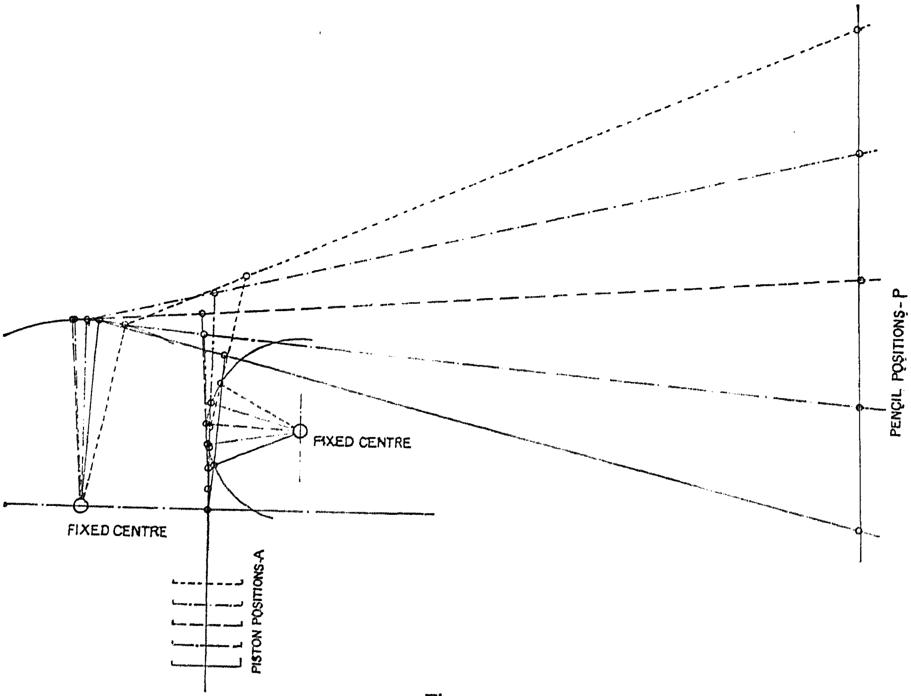


Fig. 13

its range) in a straight line, and that its movement should be proportional to that of the piston A. A few positions of the mechanism are indicated, taking equal piston movements. For each position of the piston the position of every point of the instrument is definitely fixed—apart from effects due to elastic bending of the parts or to looseness of the joints, which effects should be negligible. The path traced out by any point in its motion can be determined by drawing a series of positions of the mechanism and marking the sequence of positions occupied by the point.

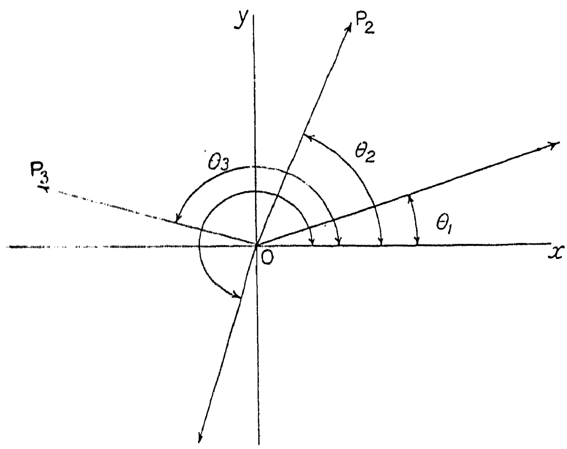


Fig. 14

Forces acting at a Point.—The parts of a structure or machine are subjected to forces, communicated through the connections or due to the masses of the parts. In structures and approximately in slow-running machinery the forces on any part or portion of a part are in equilibrium, and are subject to the laws of statics.

The conditions for equilibrium may be expressed, and the cases treated, either algebraically or graphically as is the more convenient. The simplest cases are those in which the forces act at a single point, either exactly or with sufficient closeness to be regarded as doing so, as is the case with a pin-joint in a structure. If a number of forces P_1, P_2, P_3... in directions given by the angles θ_1, θ_2, θ_3... which their lines of action make with a conveniently chosen axis Ox (fig. 14), act at a point (taken as the origin), their resultant is given by its components X along Ox and Y along Oy, where

$$X = P_1 \cos\theta_1 + P_2 \cos\theta_2 + P_3 \cos\theta_3 \dots = \Sigma . P \cos\theta,$$
$$Y = P_1 \sin\theta_1 + P_2 \sin\theta_2 + P_3 \sin\theta_3 \dots = \Sigma . P \sin\theta,$$

the total magnitude of the resultant R being given by $R^2 = X^2 + Y^2$ and its direction by the angle Θ which it makes with Ox, when $\tan\Theta = \frac{Y}{X}$. If the forces are in equilibrium both X = o and Y = o.

Thus if P be the steam force (= pA when p is the pressure and A the area of the piston) acting upon the piston and so upon the crosshead of the engine sketched in fig. 15, and R be the thrust along the connecting-rod in a position which makes the angle θ with the piston-rod, and Q the reaction of the crosshead guide, then, resolving along and perpendicular to the line of the stroke,

$$\left.\begin{aligned} P - R\cos\theta &= o \\ R\sin\theta - Q &= o \end{aligned}\right\},$$

and Q and R are known.

The force R acts upon the crank-pin D and may be resolved into R $\cos\psi$ along the line DC (which causes a thrust along the crank, and is balanced by

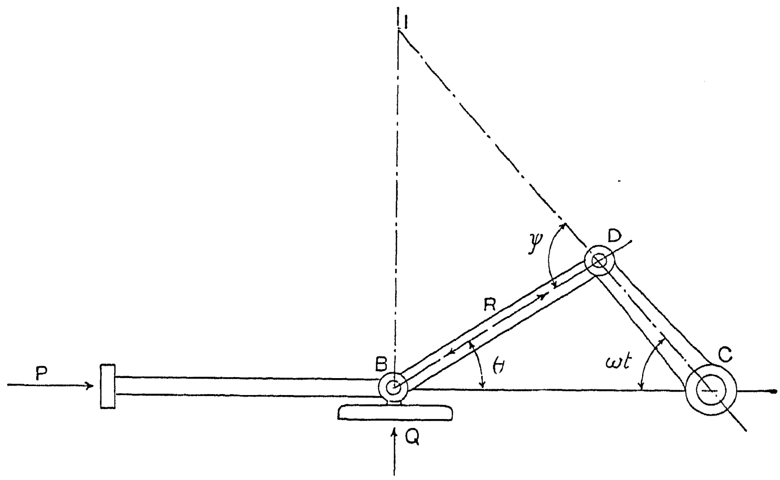


Fig. 15

reaction at the crank-shaft bearings), and R $\sin\psi$, perpendicular to CD (which causes a bending action along the crank, a turning moment on the crank-shaft, and is balanced by the resistance of the work) where ψ is the angle between the crank and connecting-rod.

These two conditions, X = o and Y = o, for equilibrium correspond to the two degrees of freedom which a point has in space of two dimensions.

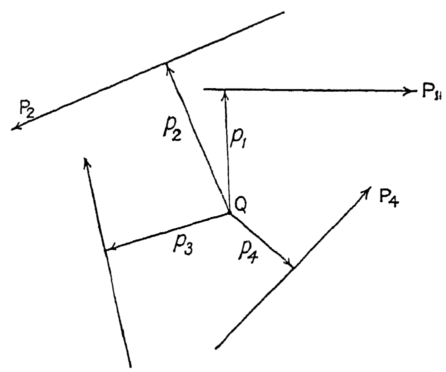


Fig. 16

Forces acting on a Body.—When the forces act upon a body or a part of a body they need not pass through a single point, and corresponding to the extra (angular) degree of freedom which a body possesses over a point is the moment equation

$$M = P_1p_1 + P_2p_2 + P_3p_3 + \ldots . = \Sigma Pp,$$

where $p_1, p_2, p_3 \ldots .$ are the perpendiculars (see fig. 16) from the point Q about which the moments are taken, on the lines of action of the forces, due regard being had to the sign or angular direction of each moment. For equilibrium the condition is that the moment M or ΣPp should be zero. This form of the conditions of equilibrium of a body, viz.

$$\Sigma P \cos\theta = o, \ \Sigma P \sin\theta = o, \text{ and } \Sigma Pp = o,$$

and that obtained by equating to zero the moments of the forces about three points Q_1, Q_2, Q_3 not lying in one straight line, are the most generally serviceable. In fig. 17 is shown a rafter or inclined beam AB, supported as sketched, and carrying known loads W_1, W_2, W_3 at definite positions. These are given by the horizontal distances a, b, c of the loads from the end A of the beam, and h is the known height of B above A. The horizontal (P) and vertical reactions at A, and the horizontal (the friction at the wall-face not being

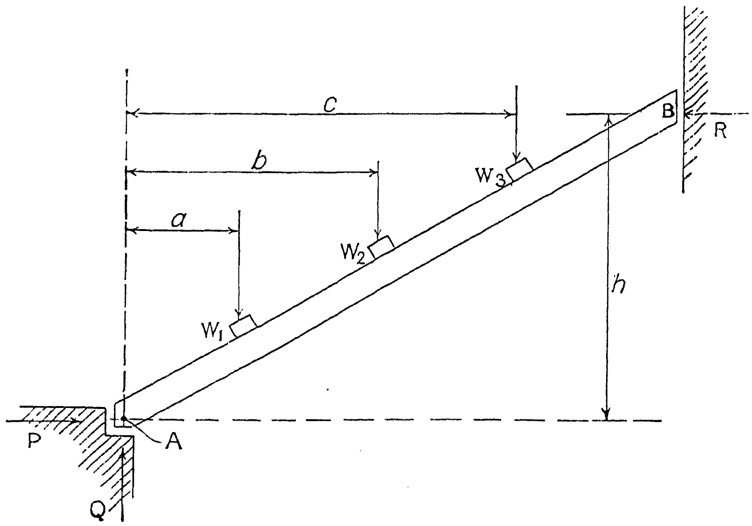


Fig. 17

taken into account) (R) action at B are given by the conditions of equilibrium.

Resolving vertically .... $Q - (W_1 + W_2 + W_3) = 0$;
Resolving horizontally .... $P - R = 0$;
By moments about A .... $Rh - W_1a - W_2b - W_3c = 0$.

Systems of Forces.—From the conditions of equilibrium it is seen that any system of forces reduces to a single force at any selected point, and a couple or moment about that point.

If the force is zero the system reduces to a couple or moment only, but if the force exists the system reduces to a single force acting along a defined straight line, for the couple M may be regarded as equivalent to a force R, equal to and opposing the force at the chosen point O, and a parallel force at a distance $\frac{M}{R}$ from O.

If two forces only act upon a body which is in equilibrium they must be equal and opposite. Thus when the weight of a part connected by pin-joints to other parts of the structure or machine is small enough to be disregarded in comparison with the force transmitted, this force is considered as acting directly from pin-joint to pin-joint. The usual practice in struc-

tural work is to disregard the weight of the members initially and allow for them when refining the design.

The Case of Three Forces Only.—If three forces only act upon a body in equilibrium they meet in a point; otherwise there would be an unbalanced moment due to one force about the point in which the others intersect. Thus the actions at the hinged support A and at the sliding support B of a roof truss ABC, acted upon by a wind pressure causing a force P

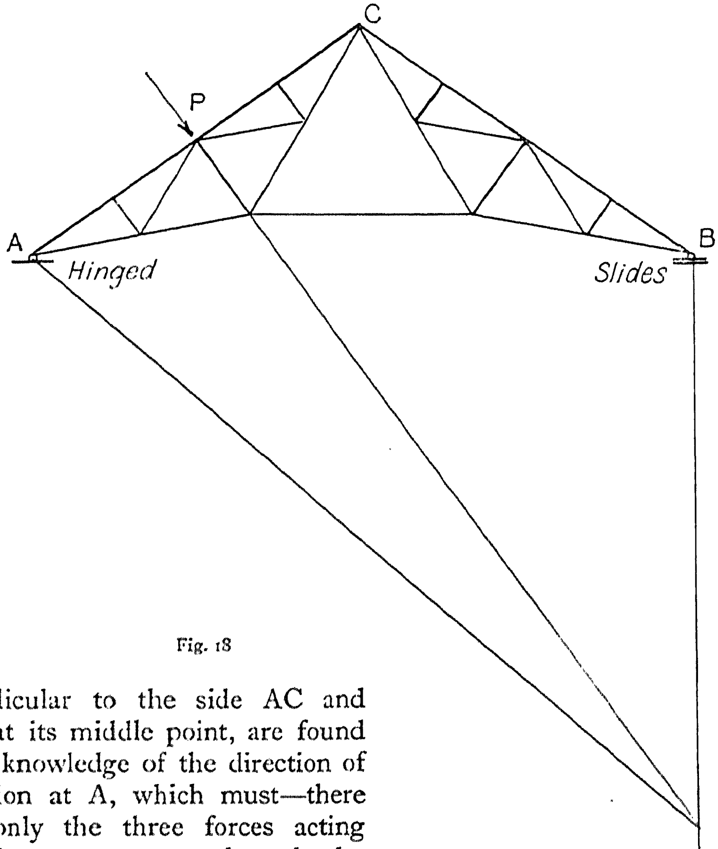


Fig. 18

perpendicular to the side AC and acting at its middle point, are found from a knowledge of the direction of the action at A, which must—there being only the three forces acting upon the truss—pass through the point in which P intersects the reaction at B, which must be vertical owing to the slide at B, as is shown in fig. 18.

The Method of Sections.—The force action at any point of contact of two parts of a body is found by supposing the parts separated at the point of contact and the force in question to act there. If the force action transmitted by a part across any particular imaginary section be required, the part is supposed divided at the section, and the whole stress action exerted by one part on the other replaced by any simple actions to which the system of stress forces is equivalent. By the consideration of the equilibrium of either part into which the structure or machine is supposed to be divided, the action across the section can be found. This process is known as the

"Method of Sections". Thus the action can be reduced to a force through the centre of gravity of the area of the section and a couple or bending moment; and the force action in turn resolved into one perpendicular to the section, or an axial force, and one in the plane of the section or shear force. Or the whole stress action may be reduced (if not equivalent to a pure couple) to a single force of definite direction cutting the section, or the plane of it produced, in a particular point. Thus the action across each section of a steel arch may be expressed, the resulting curved line being known as the line of thrust.

The Super-position of Force Systems.—If one system of forces is in equilibrium, and a second system, also in equilibrium, be supposed to coexist with the first system, the total system of forces will be in equilibrium. And conversely a system of forces in equilibrium may be supposed divided into two or more systems of forces in equilibrium. In discussing a structure or machine each system of forces which may act independently must be considered independently, and then the cases of possible coexistence considered.

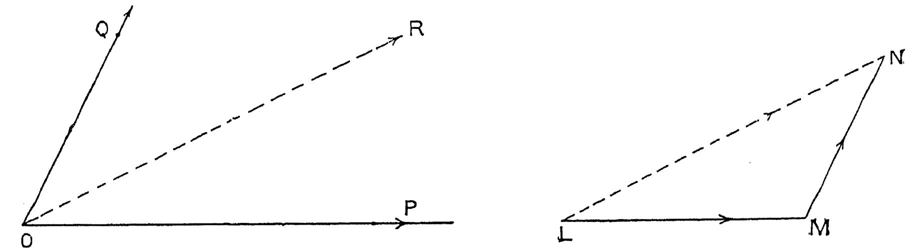


Fig. 19

Thus members of a roof truss have always to sustain the forces due to the deadload of purlins, roof covering, &c., and may at any time have to simultaneously sustain wind forces which may act from either side, but not from both at once. It may also be called upon to sustain a snow-load, but the snow-load cannot well occur with a high wind-load. Also the roof may be used to support runways upon which certain loads may occur. In such a case then the forces caused in the various members by these separate actions are to be determined separately, and finally the forces for which the members are to be designed determined by a consideration of the possible combinations.

Force Diagrams and Bow's Notation.—The problems of statical equilibrium are treated graphically by the vectorial representation of the forces, any force P (fig. 19) being represented by a line LM drawn parallel to P and in the same direction sense, as is indicated by the arrowheads, and of a length to represent the magnitude of P. The resultant R of P and a second force Q acting at O is obtained by drawing from M the vector MN to represent Q, when the vector LN (in sense marked) will represent R. If R be reversed and act at O the system will be in equilibrium, and the vectors will form a triangle LMN, the senses of the vectors being continuous round the figure. So if a number of forces act at a point and are in equilibrium

their vectors will form a closed polygon. Practical work is usually much simplified by the use of Bow's notation, in which a letter is assigned to each of the spaces on either side of the forces, and a force specified by the letters of the spaces on either side of it, the direction of the force being associated with the order of the letters. The forces in the vector or force diagram are indicated by the same letters, a distinction being usually made between the

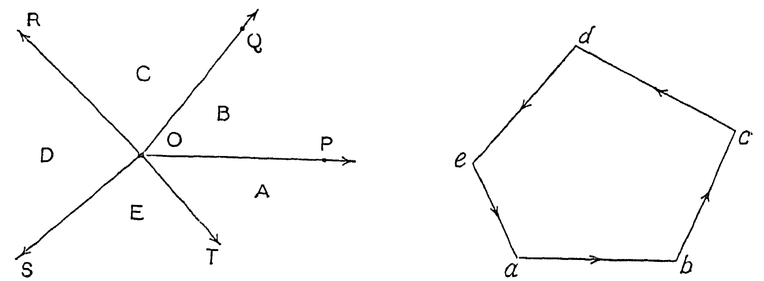


Fig. 20

drawing and the diagram by using capital letters in the former and small in the latter. Thus in fig. 20 the forces P, Q, R, S, T, acting at the point O, are denoted by AB, BC, CD, DE, and EA, where the letters are placed in the spaces separating the forces, and the vector diagram takes the form *abcdea*, the letters *a* and *b* being the ends of the vector representing the force AB or P. When the forces are in equilibrium the force polygon is closed as in the figure.

In fig. 21 is sketched the jib of a crane, the load W being supported by

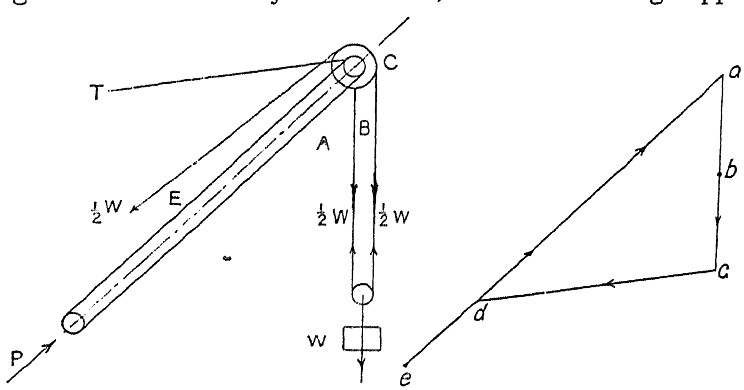


Fig. 21

a block in the bend of a rope fastened to the end of the jib and carried back round a pulley at that end parallel to the jib. The jib is hinged at its lower end and supported by a chain, the tension in which, indicated by T, is to be found, and also the thrust along the jib. The various forces acting at the upper end of the jib are denoted by the lettering of the spaces separating them. The tension in the rope is known to be $\frac{1}{2}$W. The force diagram is drawn by drawing *ab* vertically downwards to represent AB or $\frac{1}{2}$W on any convenient

scale; then *bc* also vertically downwards to represent BC or $\frac{1}{2}$W; and by drawing *cd* parallel to T and *ad* parallel to the jib to meet at *d*. Along *da* is set off *de* equal to the rope tension $\frac{1}{2}$W. Then the length *cd* represents the tension CD or T, and *ae* represents EA or the thrust along the jib. In drawing the force diagram it is supposed that the forces acting at the point are crossed in the direction of the arrow, and the vectors drawn in the known direction of the known forces: the determined vectors will then give the direction of the forces according to the same convention. Thus the force T is CD (not DC) and its direction sense is from *c* to *d*.

The Funicular Polygon.—When the forces of a system acting upon a body do not meet in a point, their resultant force or couple may be found by combining two of them considered as acting at the point in which their lines meet; then combining their resultant with a third force, considering both to act at the point in which this resultant cuts the line of action

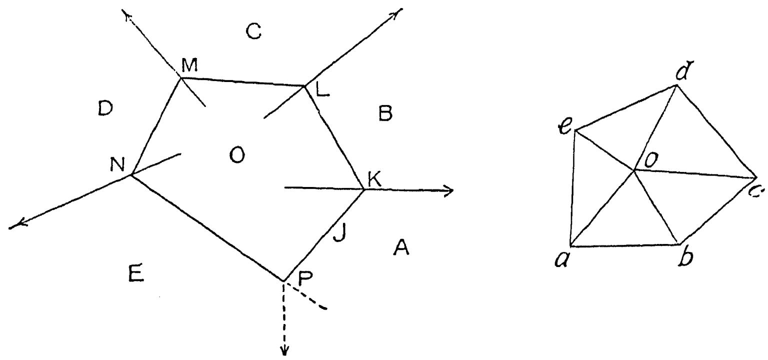


Fig. 22

of the third force, and continuing the process. The final result is either a single resultant, two parallel forces, or equilibrium. By first drawing the polygon of forces for the system, the direction and magnitude of the various intermediate resultants are at once determined and the work simplified.

The lines in this construction often run inconveniently, and the following method—that of the "funicular polygon"—is usually preferable. Suppose that the system of forces AB, BC, CD, DE are under discussion. The force polygon *abcde* is first drawn, and in this particular example it shows that there is a resultant represented by *ae* in magnitude and direction, and what is still required is to ascertain its position upon the first diagram. Any convenient point O is taken as a "pole" in the force diagram and joined to the points *a*, *b*, *c*, *d*, *e*. Then any convenient point K is selected upon the line of the force AB, and the force AB resolved into the pair of forces AO, OB, along lines JK, KL parallel to *ao* and *ob*. The magnitude of the forces AO, OB will be proportional to the lengths *oa*, *ob*. The force BC is now supposed to act at L, where KL intersects its line of action and is resolved into components along KL and LM, the latter being drawn parallel to *oc*. The magnitudes of these forces being proportional to the lines *ob* and *oc*,

the resolved parts of AB and of BC along LK are equal and opposite and therefore cancel. Proceeding thus, the system reduces to forces *oa* along JK, and *oe* along the final line NP parallel to *eo*. Let these meet at P; the resultant of all the forces then passes through P, and in magnitude and direction is represented by *ae*.

If the force diagram in fig. 22 is closed, the system of forces may either be in equilibrium or be equivalent to a couple; in the latter case the final lines JK and NP are parallel, and the couple is measured by the product of the force *oa* (if *a* be the closing point) into the distance between JK and NP.

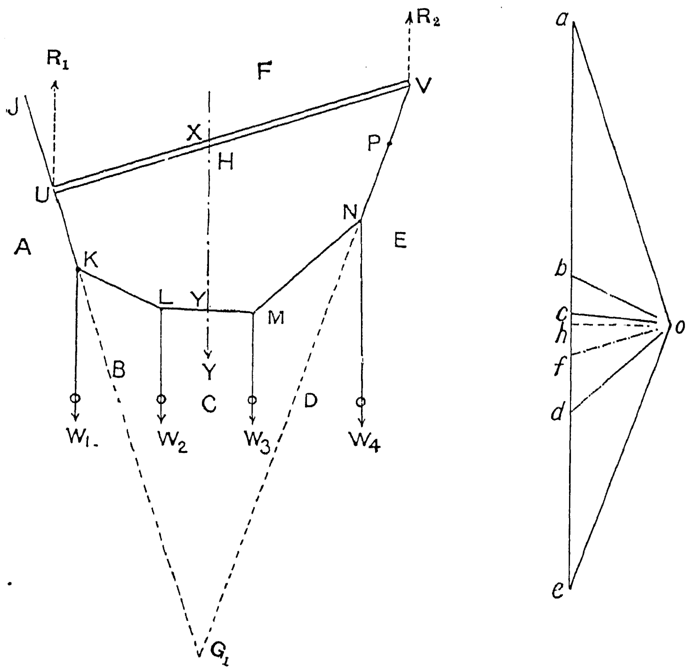


Fig. 23

The figure JKLMNP is known as the funicular polygon. It is to be noted that since any point in the force diagram may be taken as the pole, and since any point in a force line in the original figure can be used as the starting-point, an indefinitely large number of funicular polygons can de drawn.

Bending-moment Curves.—The case of most importance is that in which the forces are due to gravity, in which case the force polygon closes up to a vertical straight line. The following discussion illustrates the uses of the method. Consider the loads W_1, W_2, W_3, W_4 (fig. 23), acting at any points in the fixed vertical lines AB, BC, CD, DE. Draw a vertical load line and set off *ab* to represent W_1, *bc* to represent W_2, &c. Take any pole *o* and join it to the points *a*, *b*, *c*, *d*, *e*. Take any point K on AB, and draw the lines JK, KL, LM, MN, and NP of the funicular polygon, parallel to *oa*, *ob*, *oc*, *od*, and *oe* respectively. Then, as before, any triangle as *ocd* in

the force diagram corresponds to forces acting at the point M: *oc* along ML, *cd* the weight W_3, and *do* along MN. The whole system of loads W_1, W_2, W_3, and W_4 is equivalent to forces *ao* along JK and *oe* along PN, so that if JK and PN produced meet in G, this point lies on the resultant of the weight systems, i.e. the centre of gravity of the system of weights lies in the vertical line through G.

When the positions of the weights are definitely given, by turning the system through an angle (or by imagining the direction of gravity altered in the drawing), another line passing through the centre of gravity can be found and thus the centre of gravity of the whole fully determined.

The loads W_1, W_2, &c., may be supposed supported in equilibrium by a weightless cord along JKLMNP, from which arises the term funicular polygon. The tensions in the various parts of the cord will be *oa*, *ob*, *oc*, &c. The force along any part of the cord, e.g. LM, can be resolved into its horizontal and vertical components by drawing *oh* perpendicular to the load line; the vertical component in LM is then *ch* and the horizontal *oh*. This horizontal component is constant throughout the funicular polygon, and is determined solely by the perpendicular distance of the pole *o* from the load line *ae*.

Suppose that the terminal cords of the polygon be attached at any points U and V to a weightless rod. The whole system can then be carried in equilibrium by vertical forces (either in vertical cords as suggested in the sketch or by reactions at bearing plates) at U and V. Considering the equilibrium of the point U under the tension *ao* in UK, the vertical reaction or force R_1 and the thrust along the weightless rod UV; and lettering the space between R_1 and the rod as F, the triangle of forces is obtained by drawing *of* parallel to UV to meet the load line. Thus the vertical force at U is *fa* and similarly that at V is *ef*. In this manner the vertical actions at the supports of a structure under vertical loads are readily determined. It is to be noted that the position of the point *f* is independent of the position of the point selected as pole in any particular case.

Let the system of rod and cord polygon be cut across by any vertical line, as at XY, and suppose one part, let it be the right-hand side, removed. The action exerted by it upon the remaining portion consists of the force *of* along VX, and the force *co* along YM, which may be replaced by the forces *oh* (or H) horizontally and *hf* vertically at X, and *ho* horizontally and *ch* vertically at Y. Taking moments for the equilibrium of the left-hand side, the moment H . XY is equivalent to the moment of the reaction R_1 and all the vertical loads to the left of the section, and is the moment which must be transmitted across the section of any structure so loaded and supported. The shear force transmitted across the section is, by resolving vertically, equal to $ch + hf$ or *cf*.

This funicular polygon gives a curve showing the bending moment which is transmitted across any section of a loaded structure.

The value of the bending moment being given by the product of the vertical depth of the diagram XY measured to the scale of the drawing,

and the horizontal force *oh* measured to the scale of the force diagram. This distance *oh* should therefore be chosen so that the funicular polygon gives the bending moment on some convenient scale.

Usually it is desirable that the base of the diagram UV should be horizontal, which is secured by first determining the point *f* and then selecting a new pole in a horizontal line through *f*.

Centres of Gravity and Moments of Inertia.—Although the term "centre of gravity" is not strictly applicable to an area, the expression is generally accepted, as is the similar term "moment of inertia". The position of the centre of gravity and the moment of inertia of a figure about any line are most easily found by mathematical methods if the figure is a simple one. Taking any axes Ox, Oy in the plane, the values of $\bar{x}$, $\bar{y}$, the co-ordinates of the centre of gravity, are given by

$$A\bar{x} = \Sigma\delta A \,.\, x = \iint x\,dx\,dy,$$

$$A\bar{y} = \Sigma\delta A \,.\, y = \iint y\,dx\,dy,$$

$$\text{or by } A\bar{x} = A_1x_1 + A_2x_2 + A_3x_3 \,.\,.\,,$$

$$A\bar{y} = A_1y_1 + A_2y_2 + A_3y_3 \,.\,.\,,$$

for areas A_1, A_2, A_3, whose centres of gravity are known, where A is the total area of the figure; δA an element of this area, and the summation is to be taken over the whole area. The expressions for the moment of inertia about the lines are:

$$A \,.\, I_x = \Sigma\delta A \,.\, y^2 = \iint y^2\,dx\,dy,$$

$$A \,.\, I_y = \Sigma\delta A \,.\, x^2 = \iint x^2\,dx\,dy;$$

and the condition that the axes should be principal axes is:

$$\iint xy\,dx\,dy = 0.$$

When the moment of inertia I_o of a figure about a line through its centre of gravity is known, its moment of inertia I about any parallel line is given by the expression

$$I = I_o + Ah^2,$$

where h is the distance between the lines. For if the first line be taken as the axis of x, the new co-ordinates of the point xy will be x, $y + h$, so that

$$I = \iint (y + h)^2\,dx\,dy = \iint y^2\,dx\,dy + 2\iint yh\,dx\,dy + \iint h^2\,dx\,dy = I_o + Ah^2.$$

Simple and Standard Sections.—The centres of gravity and moments

of inertia of some simple figures are given in Table I, and these are given for the sections of rolled-steel joists, angles, &c., in tables supplied by the makers of the products, and by the Standards Committee. By the aid of these, built-up sections can be dealt with. As illustrative, suppose that the position of the centre of gravity of the section shown in fig. 24 be required, and its

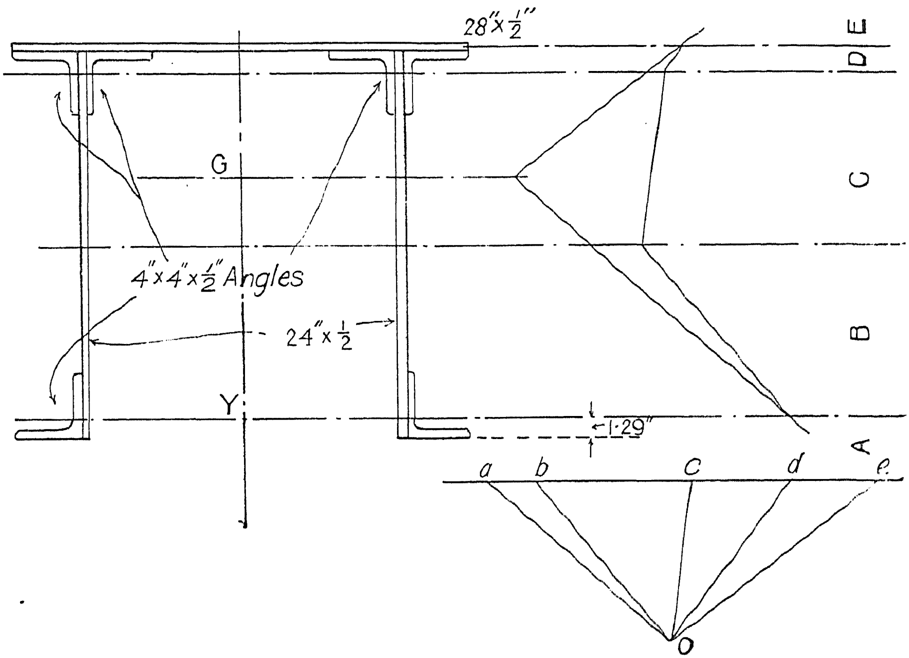


Fig. 24

moment of inertia about lines through the centre of gravity parallel to the edges. If $\bar{y}$ be the distance of the centre of gravity below the top,

$$\begin{aligned} A\bar{y} &= \Sigma\delta A \,.\, y = 28 \times \tfrac{1}{2} \times \tfrac{1}{4} + 2 \times 24 \times \tfrac{1}{2} \times (12 + \tfrac{1}{2}) + 4 \\ &\qquad \times 3\cdot75 \times (1\cdot29 + \cdot5) + 2 \times 3\cdot75\,(\cdot5 + 24 - 1\cdot29) \\ &= 504\cdot4, \\ \text{and } A &= 60\cdot5 \text{ sq. in.,} \\ \text{so that } \bar{y} &= 8\cdot34 \text{ in.} \end{aligned}$$

The section and properties of the angle, as taken from standard data, are given in the figure, and used in the calculation. Area $= 3\cdot75$ sq. in., and distance of centre of gravity from outside $= 1\cdot29$ in.

The determination by graphical means described above is also shown in the figure, the line GY being supposed to be placed horizontally. The "forces" AB, BC, CD, DE are proportional to the areas of the sections of two angles, the two side plates, four angles, and the top plate.

Since GY is a line of symmetry it is a "principal" axis; the moment of inertia about it is

$$\begin{aligned}I &= \tfrac{1}{12} \times \tfrac{1}{2} \times 28^3 + 2 \times \tfrac{1}{12} \times 24 \times (\tfrac{1}{2})^3 + 2 \times 24 \times \tfrac{1}{2}(14 - 4 - \tfrac{1}{4})^2 \\ &\quad + 6 \times 7{\cdot}92 + 4 \times 3{\cdot}75 \times (14 - 4 + 1{\cdot}29)^2 + 2 \times 3{\cdot}75 \\ &\quad \times (14 - 4 - \tfrac{1}{2} - 1{\cdot}29)^2 \\ &= 5584 \text{ in}^4,\end{aligned}$$

the value for the moment of inertia (7·92) of the angles being taken from the tables.

The Graphical Determination of the Centres of Gravity and Moments of Inertia of Sections.—Graphically the area whose centre of gravity or moment of inertia are to be found is divided into parts whose areas, &c., are known with sufficient exactitude, the division being usually into strips. The following method is frequently used. The distance of the centre of gravity of the area ABCDE and its moment of inertia about FG (fig. 25) are to be found. A point O in FG is selected as origin, and OG taken as axis of x with Oy perpendicular to it. The line Oy is here taken across the area, and the construction shown for the part ABCD on one side of Oy only. A line HK is taken parallel to FG and at any convenient distance OH $= h$ from it. Take a point C on the boundary, join OC and produce to K. Draw CL perpendicular to HK and join OL, cutting the perpendicular CN on OH in M. Then

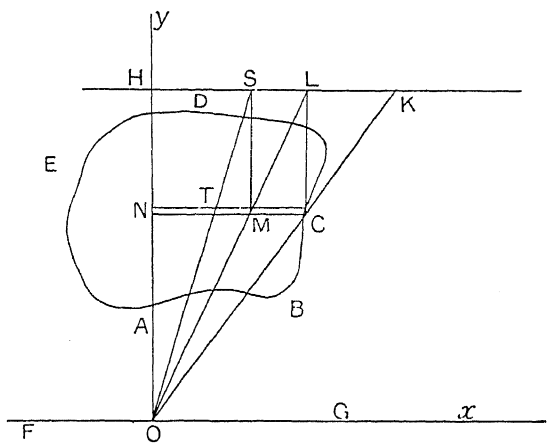


Fig. 25

$$\text{MN} = \frac{\text{ON}}{\text{OH}} . \text{HL} = \frac{y}{h} . x$$

where x and y are co-ordinates of C. Hence, taking a sufficient number of points corresponding to M to obtain a curve, the area of this curve is

$$A_1 = \int \text{MN} . dy = \int \frac{y}{h} . x dy = \frac{\bar{y}}{h} A_o,$$

so that by measuring the areas of the two curves $\bar{y}$ is found.

Again, drawing MS perpendicular to HS, and joining OS cutting CN in T,

$$\text{NT} = \frac{\text{ON}}{\text{OH}} . \text{HS} = \frac{y}{h} . \text{MN} = \frac{y}{h} . \frac{y}{h} x,$$

and if A_2 be the area of the locus of T,

$$A_2 = \int NT \, . \, dy = \int \frac{y^2}{h^2} x dy = \frac{I}{h^2},$$

so that measuring the area A_2 gives the value of I, the moment of inertia of the original figure about FG. The loci of M and T are called modulus figures.

Loaded Beams treated Graphically. As an example of the use of the funicular polygon method, consider the case of a beam carried upon supports at X and Y, 30 ft. apart and projecting 12 ft. beyond Y, loaded with 2 tons per foot run uniformly, single load of 24 tons placed 12 ft. from X, and a load of 20 tons at the projecting extremity, as in fig. 26.

In determining the reactions graphically, the whole uniform load of 84 tons may be supposed concentrated at the centre of the beam. The loads now are AB, BC, CD and the reactions DE and EA, and for this determination all the known loads are to be taken in a sequence, which enables the complete load line *abcd* to be drawn. The line parallel to O*a* cuts the vertical through X in *x*, and that parallel to O*d*, when produced backwards, cuts the vertical at Y in *y*, giving *xy* as the closing line. Drawing O*e* parallel to *xy*, the reactions *de*, *ea* at the supports are found to be 96·5 and 31·5 tons as in fig. 26.

For the purpose of drawing the curves of shear force and bending moment, the uniform load is divided into a number of parts, here seven, and the load on each, 12 tons, is supposed to act at the middle point of the portion. This gives the lower figure in which all the loads, including the support reactions, are known. The loads are taken in sequence as they come along the bar and relettered, and in consequence, after drawing the load line downwards from *b* to *h*, it goes upwards to *j* and then downwards to *a*, and finishes upwards to *b*. There being no shear force outside the ends, a horizontal line through *a* gives the zero shear force line. Just to the right of X the shear force is *ab*, and considering the loads as applied along the lines BC, CD, . . ., the shear-force diagram takes the form of the broken stepped line, but actually, as the load is applied uniformly except at the point loads, it takes the form of the double line curve. From this the shear force at any point can be read off upon the scale upon which the load line *bc* . . . *h* was set out—which in the figure is 1 in. = 32 tons.

By taking the pole *q* in the horizontal line through *a*, a funicular polygon started at one end of the beam must finish at the other. With the loads considered as point loads, and the polar distance selected, it is the rectilinear figure, having positive values, tending to make the beam concave upward on the left, and negative values on the right. Owing to the main load being uniform, the bending-moment curve will consist of curves (parabolic arcs) joined together at angular points at the concentrated load lines and at the support reaction at Y. Thus the bending-moment diagram may be drawn as accurately as is required, and from it the value of the bending-moment at any section can be measured and the maximum ascertained.

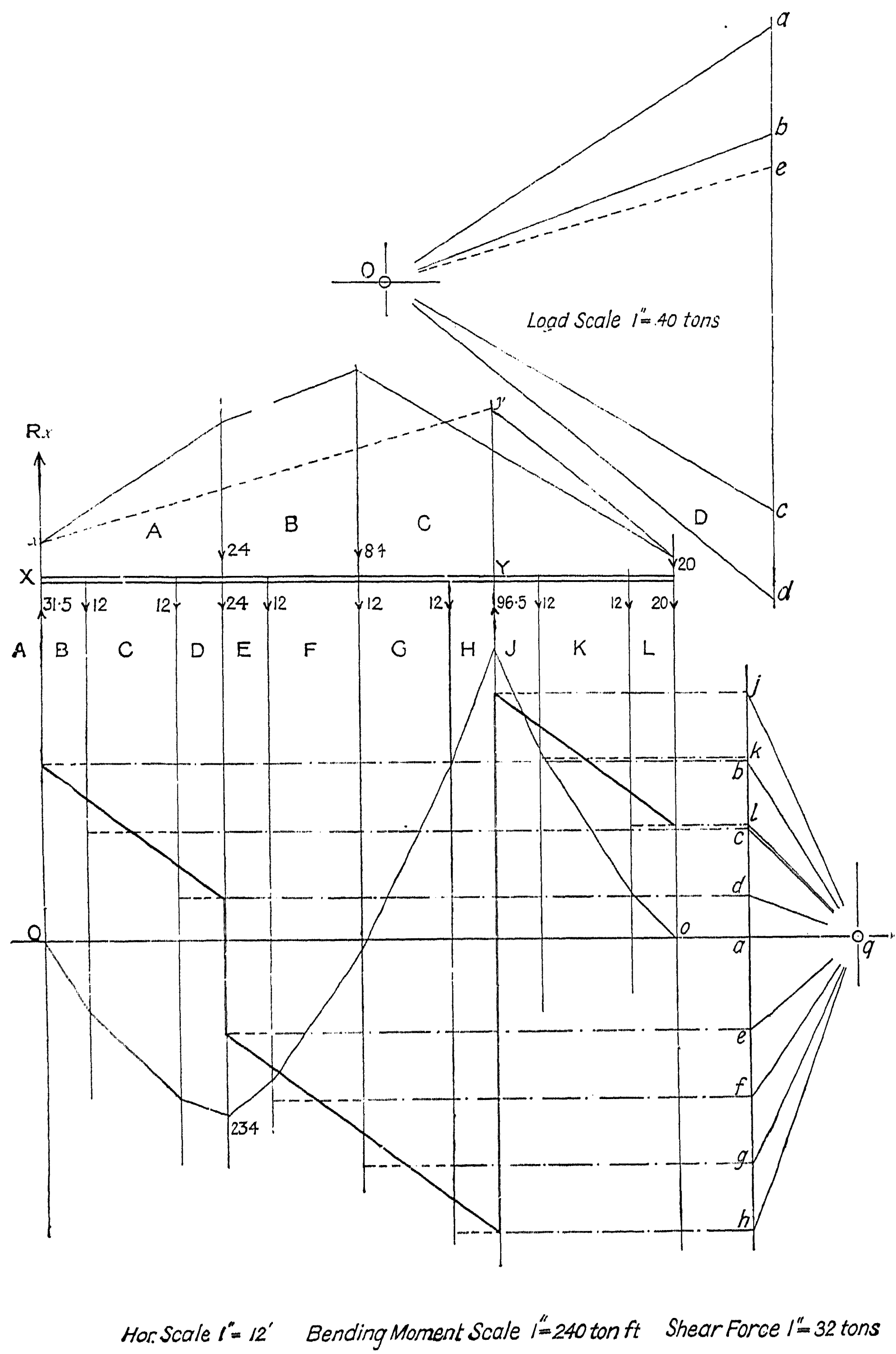

a
b
e
O
c
d
Load Scale 1" = 40 tons
Rx
A
B
C
D
24
84
20
X
Y
31·5
12
12
24
12
12
12
96·5
12
12
20
A
B
C
D
E
F
G
H
J
K
L
j
k
b
l
c
d
O
o
a
q
e
f
234
g
h
Hor. Scale 1" = 12'
Bending Moment Scale 1" = 240 ton ft
Shear Force 1" = 32 tons

Fig. 26.

The polar distance taken in the figure represents 20 tons, so that the horizontal scale being 1 in. = 12 ft., the bending moment scale is 1 in. = 20 × 12 or 240 tons-ft.

The Loaded Beam treated Analytically.—The reactions, shear forces, and bending moments can also be found by direct calculation. Thus the reaction at one support is to be found by taking moments about the other for the equilibrium of the whole beam and loads. Hence if R_x by the reaction at X,

$$R_x 30 - 24 \times 18 - 84 \times 9 + 20 \times 12 = 0,$$
$$\text{or } R_x = 31{\cdot}6 \text{ tons},$$
$$\text{and so } R_y = 96{\cdot}4.$$

The shear force at the centre of the 30-ft. span will be given by

$$S = 31{\cdot}6 - 2 \times 15 - 24 = -22{\cdot}4,$$

and the bending moment at the same place by

$$M = 31{\cdot}6 \times 15 - 2 \times 15 \times \frac{15}{2} - 24 \times 3$$
$$= 177 \text{ tons-ft.}$$

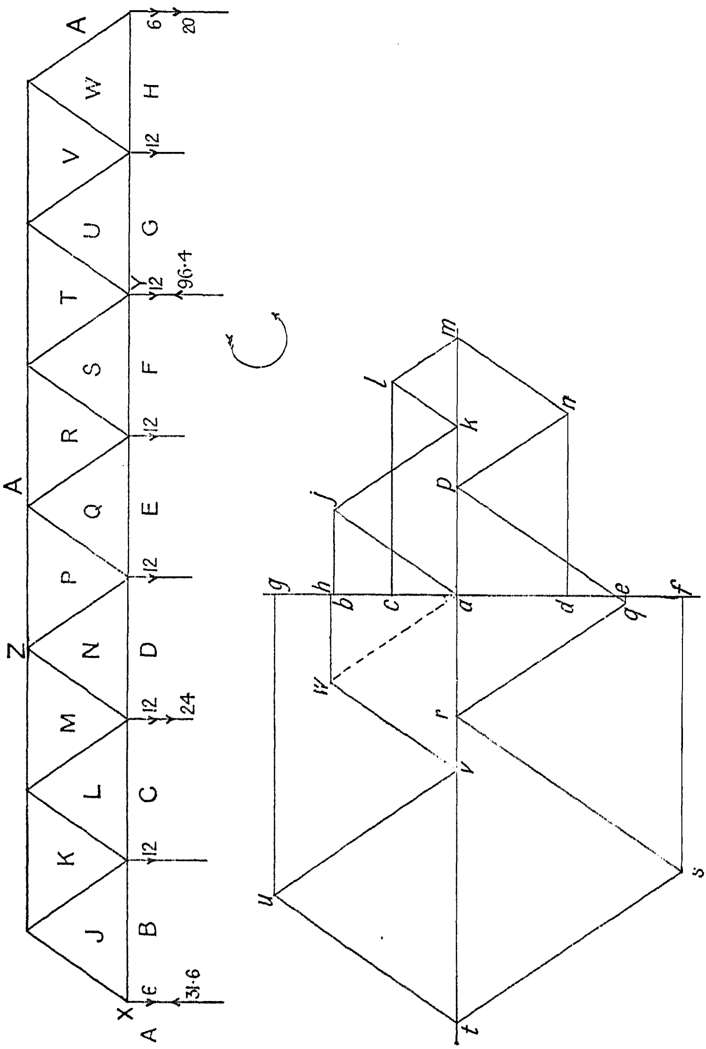
By calculating these quantities at a series of sections, taking in particular those at which point loads are applied, the shear-force and bending-moment curves can be constructed, the change of shear force across a point load being noticed.

The Effect of Width of the Supports.—If the supports are bed-plates of some width and the beam is carefully bedded so that the bearing pressure is uniform, these curves will hold to the edge of the plates and change gradually across them, the shear force by a straight line and the bending moment according to a parabolic law.

Force Diagrams for Frameworks.—In many cases, such as floors, the load is applied to the main girder at series of points, where the secondary girders rest upon it, and thus all the loads may be considered as point loads. Suppose that the case just considered be of this nature, and that the girder be a Warren girder 4 ft. 3 in. high with 5-ft. bays, as in fig. 27. The loading is, as shown there, applied to the lower joints of the girder. The reactions will be the same as before, 31·6 and 96·4 tons.

The force in any member can be found graphically by continued application of the force diagram. The lettering according to Bow's notation is as shown. The diagram for the applied forces A . . . H is first drawn and forms a vertical line, lying upon itself and closing. The triangle of forces *abj* for the end joint is then drawn. From it is drawn the triangle *ajk* for the top left-hand joint, followed by the lower joint *kjbcl* next the corner, and the method continued until the figures for all the joints are completed. The curved arrow indicates the direction round a joint in which the forces are taken. Towards the end of the construction a check is obtained upon the accuracy of the work. In this case when the force polygon for the last

lower joint but one, *vughw*, has been drawn the point *w* is determined, and proceeding to either of the remaining joints it is seen that the line *aw*, which should be parallel to AW, is already fixed. Thus drawing from *a* the line *aw*, shown broken, the nearness with which it passes *w* gives a measure of



Load Scale 1" = 40 tons

Fig. 27

the accuracy of the graphical work done. The lengths of the lines *aj*, *bj*, . . . on the force diagram are then measured and tabulated as being the forces in the corresponding members, AJ, BJ, . . .

The Employment of Sheer-force and Bending-moment Curves.—The forces in the various members can also be calculated directly, or from the shear-force and bending-moment curves, by imagining the frame cut across so as to divide the member in which the force is required. Thus

if the force in DN be required, the frame may be supposed to be cut across through DN, NP, and PA, when, since the forces along the members NP, AP intersect at the joint Z, by taking moments about this joint, the moment of all the applied forces on the beam to either side of the imaginary section is equal to the moment of the force in the section. Hence the force required is obtained by dividing the moment by the height of the girder, or 4.25 ft. To obtain the force in NP, it is to be noticed that the forces in DN and AP

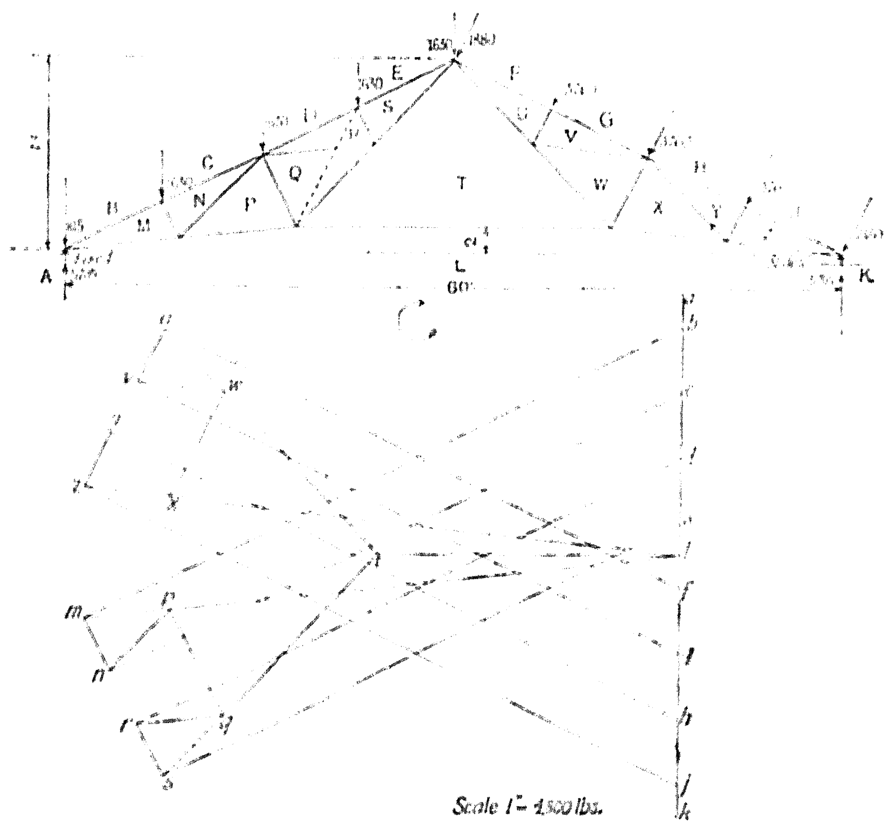


Fig. 28

are horizontal, so that the vertical force in NP is equal to the shear force. Hence the force required is equal to the shear force multiplied by the secant of the inclination of the member to the vertical.

A Difficulty in the Construction of Force Diagrams.—The graphical method, besides having the advantage of containing the check upon the accuracy of the work previously alluded to, has the advantage of rapidity in cases where the girder is not so simple in form as the Warren girder considered on p. 107. As illustrating a further point the case of a French type shop roof is considered below (see fig. 28), the span of the principal being 60 ft. and its rise the quarter of the span. The dead load of the roof covering, purlins, and principals is taken as giving an average load of 1630 lb. weight at the rafter

joints and half this amount at the ends, as is indicated in the sketch, where the dead loading is shown on one half only, and the wind loading on the other for clearness, although the problem is worked with the complete dead-load. Lettering the spaces according to Bow's notation, the diagram for the external forces becomes the vertical line *abc . . . k*, and the construction is started by drawing the force diagram *abm*, *cbmnc*, *lmnp*. At this stage a difficulty arises, as at each of the next joints there are two unknown forces—either *qr*, *rd*, or *qt*, *tl*. The difficulty may be overcome by imagining the member QR removed and replaced by one shown in broken lines across the same space, which will not affect the forces along members except those immediately concerned, and hence the construction can be proceeded with and later —say when force *lt* has been determined—the bar QR reinstated. Or it may be overcome by ascertaining one of the forces involved by imagining the frame suitably cut. Taking the latter method, and imagining the frame cut across at ESTL, taking moments about the apex for one half of the frame gives for the force *tl* the value 7530 lb. Using this value, the lower joint LPQT can be treated, and the force diagram then proceeds normally. Owing to the employment of a calculated value for the force *tl* a check occurs before the end of the diagram is approached, in that the line *sq* (shown broken) should be in line with *qt*. The final check is shown at *zx*, the whole diagram being drawn, although since the loading is symmetrical one half is actually sufficient. Here the central tie is 13 ft. below the apex.

Wind-load Diagrams.—The roof has also to sustain a wind load, which is usually taken as producing a pressure normal to the roof on the side from which the wind blows, although it is well known that the action is more complicated. Since the roof is fixed by a pin-joint at one end and free to slide at the other—otherwise the racking due to temperature effects would disintegrate the supporting walls—the forces in the members caused by wind pressure on the two sides are not similar. Knowing that the reaction at the sliding end is vertical (friction not being considered), the direction of the action at the hinged end is determined by the method of p. 95, and the final determination of the complete force diagrams carried out as above described.

The diagram for the wind blowing from the right is given in fig. 28*a*, and that for a wind from the left in fig. 28*b*. The various forces having been found for a member, the combination of independent loadings giving the maximum force is found and the member designed for it.

Wind loads were not carefully considered until the Tay Bridge disaster called attention to their effect, and since that time numerous experiments have been carried out on the forces produced.

In order to determine whether the force in a member is tension or compression, the sense of the force which it exerts at one of its joints is determined from the force diagram. Thus to determine whether XY is in tension or compression under a wind load from the right, consider the joint XYZL. The curved arrow indicates that at this joint the XY member is crossed from X to Y, and reference to the force diagram gives the direction indicated by

the arrow. Thus, transferring the arrow to the truss drawing, the member pulls at the joint and is therefore in tension.

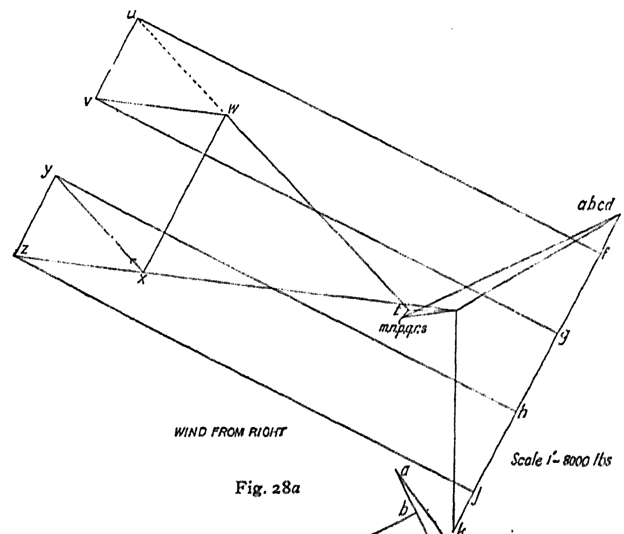


Fig. 28a

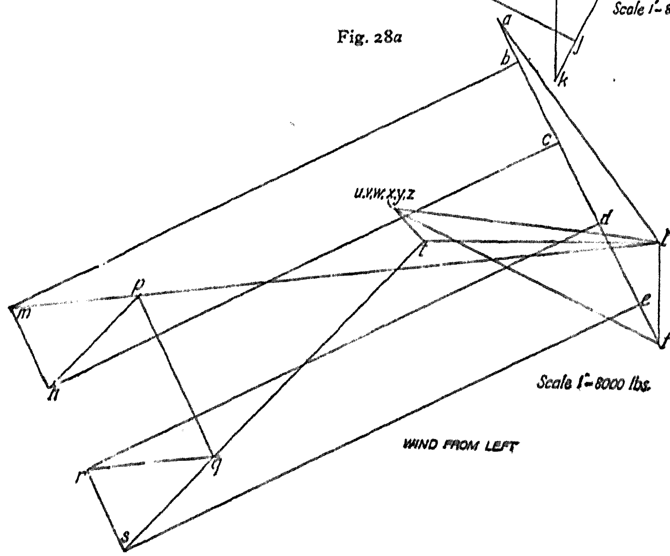


Fig. 28b

The Case of Inclined Beams.—If the beam be not horizontal, or if there be forces applied to it which are not perpendicular to its axis, there will be axial force in addition to the shear force and bending moment. Such variation of the axial force is again conveniently shown by a curve drawn on the beam as a base. In fig. 29 is shown a rafter, similar to that of fig. 17, pin-jointed at the lower end and resting against a smooth vertical surface at the upper, the loads and dimensions being as shown in the sketch. The reactions (p. 94) are found to be R = 7520 lb. = P and Q = 7520 lb. Resolving these along and perpendicular to the beam, R gives X_1 = 6730 lb. along the bar, and Y_1 = 3364 lb. perpendicular to the bar, with amounts

10,098 lb. and 3364 lb. for the lower end. The shear force along BE is therefore 3364 lb. less the component due to the 940 lb. vertical load (842 lb.), or 2523 lb. At section E this is reduced by the component of the vertical load of 1880 lb., and becomes 841 lb., while at the next section D it has an equal reduction, becoming −841 lb., and beyond the section C it becomes

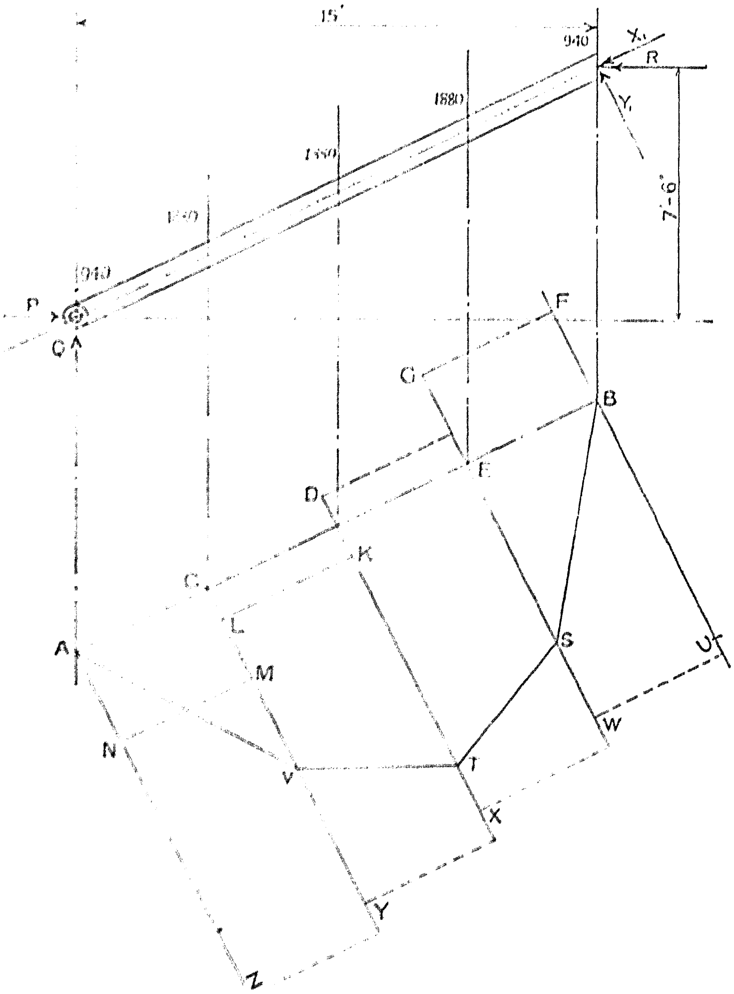


Fig. 29

− 2523 lb. The component of the end load of 940 lb. makes this equal to the normal force, 3364 lb., at the end of the bar. This variation of the shear force is indicated by the broken line FGDKLMN. So the bending-moment curve is given by the full line BSTVA. The axial force in part EB is X_1 together with the component (421 lb.) of the 940 lb. vertical load, or 7151 lb. At E it is increased by the component (842 lb.) of the 1880 lb. vertical load, becoming 7993 lb., and so at DK it becomes 8835 lb.; at C,

9677 lb., which with the component of the 940 lb. end load makes up the end thrust 10,098 lb. This curve is sketched at UWXYZ, the base line for all the curves being AB.

In fig. 30 is given a diagram representing a structure in which a single horizontal member ABC is connected to a single vertical member CDE by a pin-joint at C and by a member BD pin-jointed at B and D. The lengths of AB, BC, and CD are a, b, and c respectively. The frame is loaded

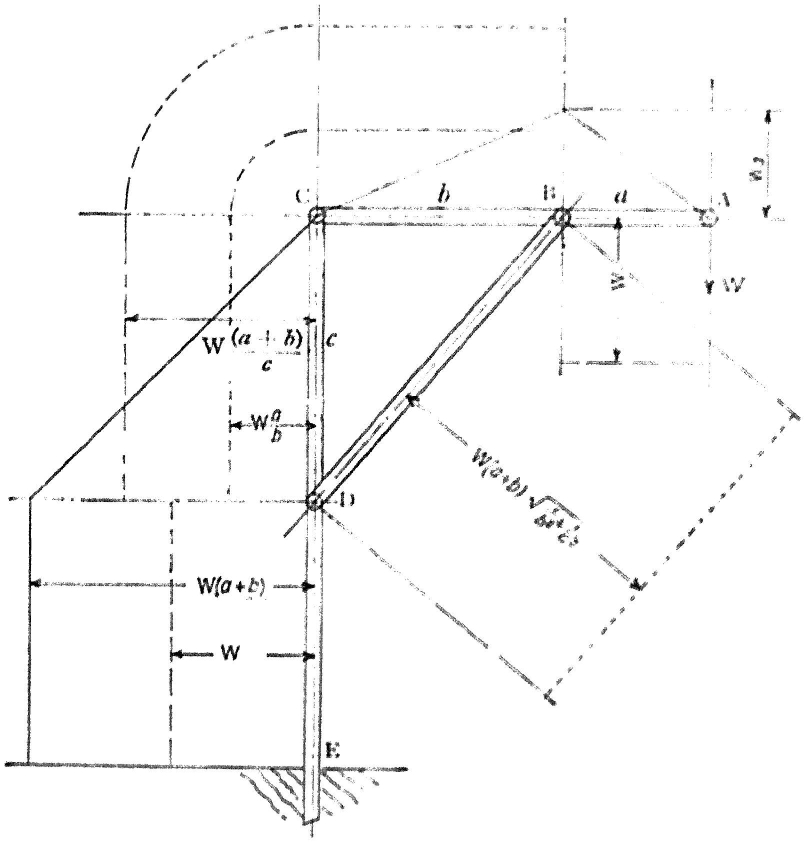


Fig. 30

by a weight W at the outer end of the horizontal member. The curves (straight lines) giving the actions along the members are drawn, the shear forces being indicated by the broken line with the longer parts, the axial forces by the broken line with the shorter parts, and the bending moments by the full lines. Thus along AB there is constant shear force W and a bending moment increasing from zero to Wa; and along BC the shear force is Wa/b, the axial force W$(a + b)/c$, and the bending moment decreases from Wa to zero. The shear force in CB becomes the axial force in CD, and the axial force in CB the shear force in CD, as is indicated by the connecting quadrantal arcs.

Forces in Space.—When forces P_1, P_2, P_3, ... act at a point and exist in three-dimensional space, the analytical expressions for the components of the resultant parallel to three rectangular co-ordinate axes Ox, Oy, Oz (fig. 31) are:

$$X = \Sigma P \sin\theta \cos\phi, \quad Y = \Sigma P \sin\theta \sin\phi, \quad Z = \Sigma P \cos\theta,$$

where θ is the angle which the direction of P makes with the axis Oz, and ϕ the angle which the plane through Oz and the line of action of P makes with the plane zOx; or, if l, m, n be the direction cosines of the line of action of P,

$$X = \Sigma Pl, Y = \Sigma Pm, Z = \Sigma Pn,$$

the resultant being R where $R^2 = X^2 + Y^2 + Z^2$. For equilibrium R = 0, or X = Y = Z = 0.

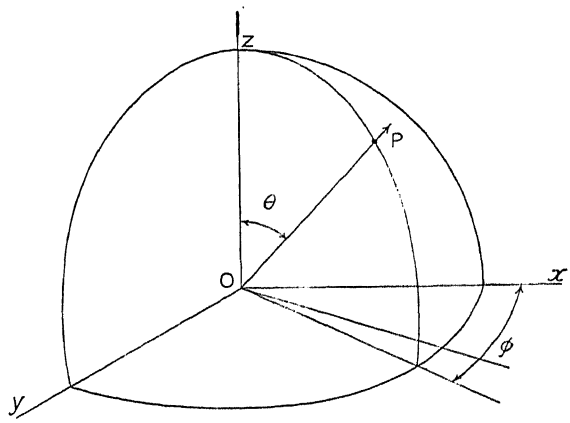


Fig. 31

Vectorially the force polygon again determines the resultant or closes in the case of equilibrium. As it is a polygon in space it is treated graphically by considering its projections on suitable planes. Thus in fig. 32 is given the diagram of a "shear legs", AB and AC being the struts pivoted along the axis BC, and AD the elevating rope in the vertical plane ATDE. The projection of the forces on this plane gives the force triangle whose sides are W the load, T the tension in AD, and R the resultant of the forces P in the struts, and the consideration of the triangle ABC gives P and P in the force diagram as the components of R.

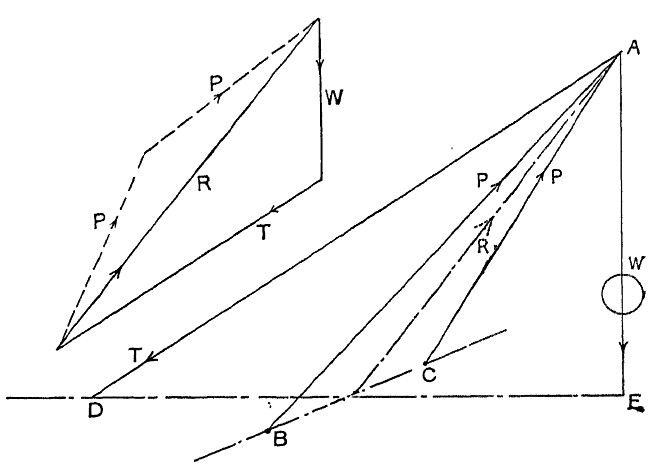


Fig. 32

When the system of forces act upon a body, each force may be re-

placed by an equal force at any convenient point, considered as origin, and a couple. The forces yield the same expression as above for the components of the resultants parallel to rectangular co-ordinates, and the couples give as components of the resultant couple G,

$$L = \Sigma(yZ - zY), \quad M = \Sigma(zX - xZ), \quad N = \Sigma(xY - yX),$$

where X, Y, Z are the components of any force, and x, y, z its point of application. For equilibrium all the quantities **X, Y, Z,** L, M, N are zero. These six equations correspond to the six degrees of freedom possessed by a body in space.

Any system of forces, therefore, reduces to a single resultant R acting at any selected point and a couple G, where $R^2 = X^2 + Y^2 + Z^2$ and $G^2 = L^2 + M^2 + N^2$. The couple can, however, be resolved into a component about the line of force and a component about one perpendicular to it, and the latter can be represented by a force R opposed to the primary force, and a force parallel to it at a distance $\frac{G}{R}$ from it, so that any system of forces is equivalent to a force acting about a particular line and a couple about this line. Such a combination is termed a wrench.

Forces in a Mechanism.—In fig. 33 is given a sketch of an engine mechanism. The force on the piston being P, it is required to find the torque transmitted along the crank shaft and the forces at the bearings, the engine speed being such that its effect is negligible. Taking the side view

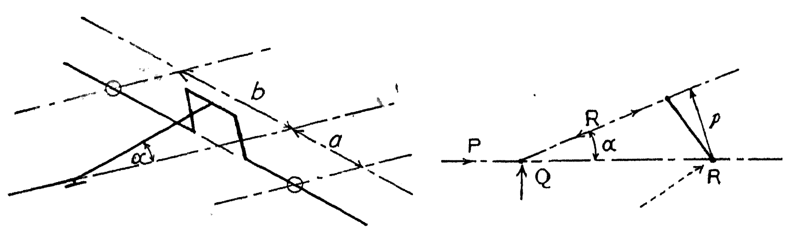


Fig. 33

given in the second figure, consideration of the equilibrium of the gudgeon-pin gives

$$R \cos\alpha = P,$$

and

$$Q = R \sin\alpha = P \tan\alpha.$$

where R is the force along the connecting-rod and Q the force on the guides. Consideration of the equilibrium of the crank then gives as the couple transmitted as drive by the crank-shaft the value Rp where p is the length of the perpendicular from the axis on the connecting-rod centre line, and there remains the force R supposed to act at the crank-shaft axis. This gives from the first figure as the forces at the bearings $\frac{a}{a+b}$ R and $\frac{b}{a+b}$ R acting

parallel to the connecting-rod. If there be more cylinders, or if the power be not taken from the engine by a pure torque, the effects of these at the bearings can be calculated similarly and then the forces acting at each bearing combined.

Static Friction.—Equilibrium is preserved and motion prevented by friction, referred to as static friction as opposed to the action which takes place between moving parts separated by a lubricating oil film. In statical cases the frictional force which is called into play is that which is necessary and only just sufficient to prevent motion. This limitation is usually expressed by the equation $F = \mu R$, where F is the frictional force acting in the tangent plane to the surfaces in contact, R the normal force between the surfaces, and μ is a coefficient which has a maximum value independent of the area of contact between the surfaces, and dependent only upon the nature and condition of the surfaces. The same relationship is regarded as holding in cases of motion at slow speeds, and also at higher speeds when there is no lubrication, e.g. between wheels and rails. The value of μ, however, varies in such cases and is liable to be reduced considerably by polishing of the surfaces by sliding. If the surfaces are oily there is a very considerable reduction of the coefficient of friction immediately motion occurs, although the phenomena of correct lubrication are not established.

The effects of friction may be discussed either by considering a force $F = \mu R$ to be applied tangentially at the point of contact, R being the normal force between the surfaces, or by considering the resultant force between the surfaces to be inclined at an angle ϕ, where $\tan\phi = \mu$, to the common normal.

When a body "rolls" upon another without sliding as a whole, there is a very slight amount of sliding of the elastically distorted surfaces, and this produces a slight resistance to the motion, which is termed rolling friction.

Friction Locks.—As illustrative of the former method, consider an adjustable stop A fitted in a slot, and having a tail B against which a part C comes into contact. The dimensions are as shown in the sketch fig. 34, and it

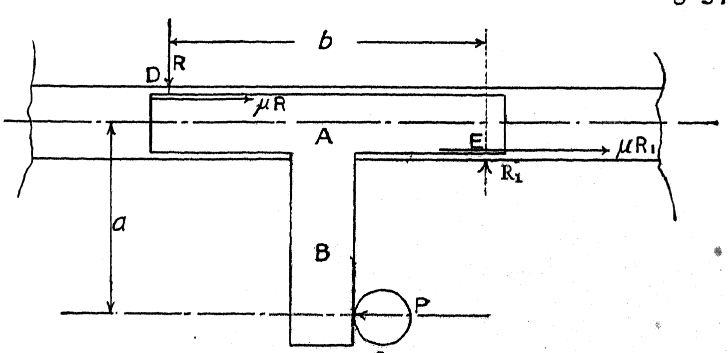


Fig. 34

is required to ascertain what they should be in order that the stop will be effective even if it be not properly clamped in the slot. If the part C exert a force

P on B which will force the stop to make contact at D and E with the slot, there will be normal forces R and R_1 acting at D and E, with frictional forces μR and μR_1 opposing the tendency to slide. As shown, P is normal to the tail B and parallel to the frictional forces μR, μR_1; hence resolving perpendicularly to the slot $R = R_1$, and parallel to it $P = 2\mu R$. Also, by moments about a point on the centre line of the slot, $Pa = Rb$. Hence $b = 2a\mu$, and hence if b is less than the maximum value of this quantity the stop will not slip, however large P may become.

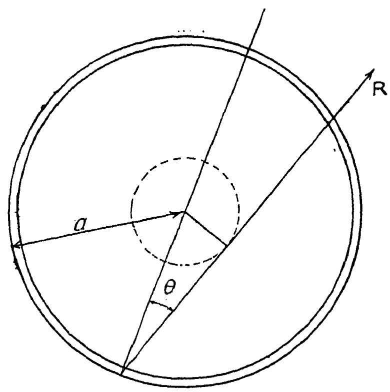


Fig. 35

If a journal in a bearing be in limiting equilibrium under a force R between the surfaces, the direction of R (fig. 35) makes the angle ϕ with the radius to the point of contact. Hence the length of the perpendicular from the centre in the direction of R is $a \sin\phi$, where a is the radius of the journal. Hence R touches a circle of radius $a \sin\phi$ about the journal centre, which circle (shown in broken lines) is termed the friction circle.

The Centre of Gravity and Moments of Inertia of a Mass.—The positions $\bar{x}$, $\bar{y}$, $\bar{z}$, in Cartesian co-ordinates, of the centres of gravity of bodies in three-dimensional space are given by the equations

$$M\bar{x} = \Sigma M_n \bar{x}_n = \Sigma dM \,.\, x = \iiint \rho x dx dy dz,$$

and similar equations for $\bar{y}$ and $\bar{z}$, M being the total mass, M_n any particular mass whose centre of gravity is x_n, and ρ the density of the body, the integral being taken over the whole mass considered. The positions for some simple shapes are given in Table I, together with the volumes and moments of inertia.

The moments of inertia of a body are given by the equations

$$I_x = \iiint \rho(y^2+z^2)dxdydz,\ I_y = \iiint \rho(z^2+x^2)dxdydz,\ I_z = \iiint \rho(x^2+y^2)dxdydz,$$

where I_x, I_y, and I_z are the moments about the axes of co-ordinates, and the conditions for these being principal axes are

$$\iiint \rho yz dxdydz = 0,\ \iiint \rho zx dxdydz = 0, \text{ and } \iiint \rho xy dxdydz = 0.$$

The radius of gyration k about an axis is given by $Mk^2 = I$, when I is the moment of inertia of the body about that axis.

Work.—The work done by a force R when it moves its point of application through a distance x, inclined to the line of action of R at an angle θ,

is $Rx\cos\theta$, or if the quantities are variable by $\int R\cos\theta dx$. If P be the resolved component of the force along the direction of motion, this is $\int Pdx$. If the values of P be set off as ordinates against abscissæ corresponding to x, this is the area between the resulting curve and the axis of x.

The simplest case is that where the force is constant, as is the case of hoisting a load W against gravity through a moderate distance h, the work then being measured by the product Wh, and usually being expressed in feet and pounds weight as units. Thus any raised weight, such as water pumped into an elevated reservoir, possesses by virtue of its position potential energy, or the power of doing work equal to that done upon it, and measured in foot-pounds.

Frequently the force is proportional to the distance through which its point of application has moved, as is the case in stretching a tie-bar or a spring. The force P to maintain any configuration is then given by $P = kx$, x being the extension from the unstressed state, so that the work done in extending a spring by a force P_0 through an amount x_0 is

$$\int Pdx = \int kxdx = k\int xdx = \frac{P_o}{x_o} \times \left[\tfrac{1}{2}x^2\right]_0^{x_o} = \frac{P_o}{x_o}\cdot\frac{x_o^2}{2} = \tfrac{1}{2}P_o x_o.$$

The work done by a couple or torque L in turning a body through an angle θ is $L\theta$, or if the torque be variable $\int Ld\theta$. Again in many (elastic) cases the torque is proportional to the angle θ in which case the work done up to a position θ_o in which the torque is I_o is $\int Ld\theta = \int R\theta d\theta = \tfrac{1}{2}L_o\theta_o$.

Indicator Diagrams.—The work done by the steam or gas upon the piston of an engine is ascertained by means of the indicator, which carries a pencil point moving through a distance proportional to the force upon the indicator piston and so to the pressure in the cylinder, and simultaneously moves a card under the pencil point through a distance proportional to the movement of the engine piston. Thus a curve is drawn having abscissæ proportional to the piston movement and ordinates proportional to the pressure acting upon the piston, so that the work done upon the piston is proportional to the area of the diagram. Such a diagram is shown in fig. 36. At A the piston is at the commencement of its movement, the pressure being represented by AL. As the piston moves to B the pressure is nearly constant, but after B it falls rapidly, cut-off having taken place. When the piston reaches C a rapid fall of pressure occurs, owing to the opening of the exhaust valve, and D is the end of the stroke. The work done on the piston during this period is measured by the area LABCDM, AL and DM being the extreme ordinates drawn to Ox, the line of no pressure. During the return stroke the piston moves against the pressure force, so that the work done on it is negative. The curve during this portion of the stroke is DEA, the exhaust being closed at E and compression taking place until the readmission of the pressure just before the end of the stroke. The area MDEAL

must hence be deducted from the area LABCDM to obtain the net work, and hence the area of the closed curve ABCDEA represents the net work done upon the piston. The actual position of the line Ox is not therefore of importance to this end: such a line is usually drawn giving atmospheric pressure. It is usual to find the mean pressure, by finding the area of the figure and dividing by the length LM, which gives the mean ordinate, the pressure corresponding to which is known from the spring used in the indicator. The work is then found by taking the product of this mean pressure, the piston area, and the stroke. If that side of the piston to which the rod is attached is under consideration, the area of the piston-rod must be deducted from the cylinder area to obtain the piston area. The force on the piston

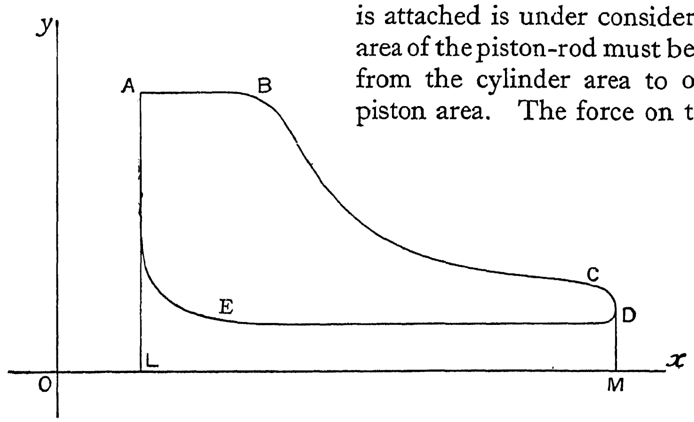


Fig. 36

at any instant is the difference between the pressure forces on its two faces: this is transmitted through the piston-rod, and the work done on the piston is transmitted, less friction losses, through the mechanism to the crank-shaft and hence delivered for use.

Efficiency.—Work or energy is indestructible, the work lost by friction in the transmission taking the form of heat, and that which is transmitted usually serves its end by finally taking the form of heat, although it may do so by devious routes. Thus it may be transformed into electrical energy by means of a dynamo driven by the engine, then into the chemical energy of a storage battery, then again into electrical energy, and then into heat and light in a lamp, the light also finally becoming heat. In the heat form the energy is more or less unavailable for use, and mechanically the heat produced by friction is regarded as lost. Such losses should be a minimum in a good machine, and the ratio of work used to work delivered to the machine is termed its efficiency. In any machine or system designed to transmit energy the efficiency should be predetermined, as it is usually of paramount importance.

Velocity.—The velocity of a point, being measured by the ratio of the distance moved through to the time taken, is determined, when variable, by consideration of the happenings in a short time, and hence is $\frac{ds}{dt}$ where s is measured along the path and t is the time. Velocity, being a vector quantity,

is resolved into its components, and compounded according to the vector laws as are forces. Algebraically, considering the motion of a point x, y in a plane, its velocities parallel to the Cartesian co-ordinate axes are $u = \frac{dx}{dt}$, $v = \frac{dy}{dt}$. In polar co-ordinates the velocity of a point r, θ along the radius to the point is $\frac{dr}{dt}$, and perpendicular to it is $r\frac{d\theta}{dt}$. In mechanism r is frequently constant, so that $r\frac{d\theta}{dt}$ is the total velocity; and if its motion in space is to be discussed the velocity parallel to the z axis is $\frac{dz}{dt}$. In mechanisms and engineering generally the path of the moving point is usually definite, and then,

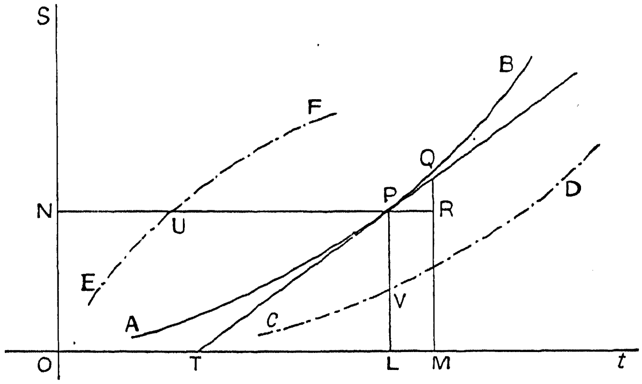


Fig. 37

although the curve may be tortuous, the position and velocity of the point can be shown on a diagram. A position-time diagram is given in fig. 37, where the position of the point along its path, or the distance s, is set up to scale at a series of times, the length OL representing the time and PL the value of s. The velocity at time L is then found by taking a small time LM $= dt$, and drawing the ordinate MQ ($= s + ds$) to the curve, the velocity being $\frac{ds}{dt} = \frac{\text{RQ}}{\text{PR}} = \tan\psi$ (where ψ is the angle which the tangent at P makes with the time axis) in the limit. Setting up LV to represent $\tan\psi$ on any convenient scale, and continuing the process, a curve CVD of velocity-time is obtained. Frequently the velocity of the point at any position in its path is needed: such a velocity-space curve is obtained in fig. 37 by drawing PN perpendicular to OS and setting off NU to represent v or $\tan\psi$, and continuing the process, EUF being the curve so obtained.

Space-time and Velocity-time Diagrams.—If the velocity-time diagram be given (fig. 38), the distance travelled in time LM $= dt$ being vdt; or PL $\times$ LM is represented by the area of the shaded strip, so that the distance travelled between any two times OH $= t_0$ and OK $= t_1$ is the area between HK, the curve, and the ordinates at H and K.

If the space-velocity diagram (fig. 39) be given, and the time of arriving at

any position be required, since $dt = \frac{1}{v}\, ds$, it will be given by finding the area under a curve whose ordinates y represent $\frac{1}{v}$ upon some scale. It is then necessary to first construct this curve by taking points P on the velocity curve,

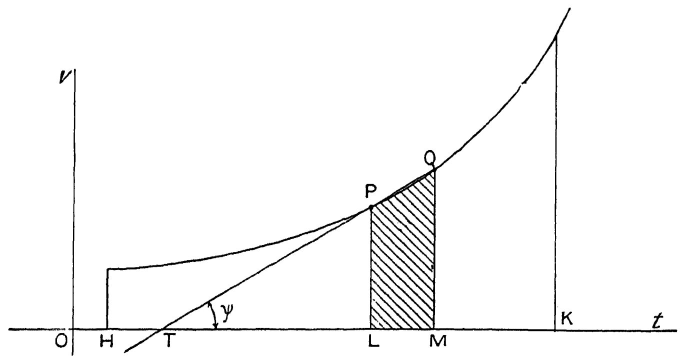


Fig. 38

drawing ordinates PLQ, setting off a constant length LJ, and drawing JQ at right angles to PJ to meet PL in Q. The time to reach any position W is then given by the area between the space axis and the Q curve up to the ordinate at W.

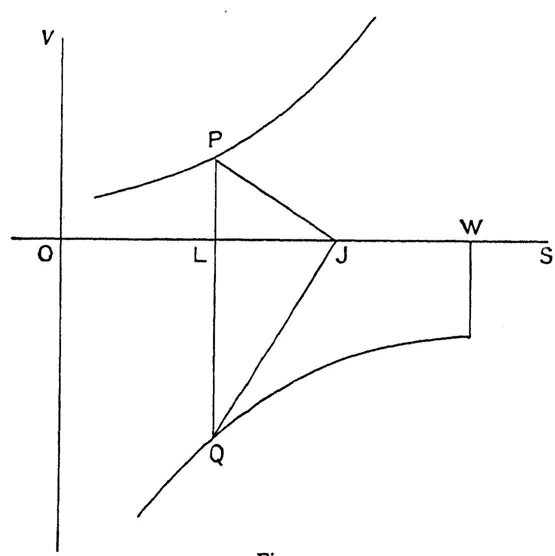


Fig. 39

Acceleration.—The rate of change of velocity or the acceleration of a point (xy or $r\theta$) moving in two dimensions is expressed in Cartesian co-ordinates of the components $\frac{du}{dt}$ or $\frac{d^2x}{dt^2}$ parallel to ox, and $\frac{dv}{dt}$ or $\frac{d^2y}{dt^2}$ parallel

to oy, while the equivalent expressions in polar co-ordinates are $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$ along the radius, and $\frac{1}{r}\frac{d}{dt}\left(\frac{r^2 d\theta}{dt}\right)$ perpendicular to the radius. In mechanism r is frequently constant, so that these expressions become $-r\left(\frac{d\theta}{dt}\right)^2$ and $r\frac{d^2\theta}{dt^2}$. If the point be moving along a plane curve its accelerations along the tangent and the normal respectively are expressed by $\frac{d^2s}{dt^2}$ and $\frac{v^2}{\rho}$ where ρ is the radius of curvature of the path at the point considered.

Since the acceleration f along a curve is $\frac{dv}{dt}$ while its velocity is $\frac{ds}{dt}$, the acceleration-time curve is found from the velocity-time curve by the process whereby the latter was found from the space-time curve, and conversely.

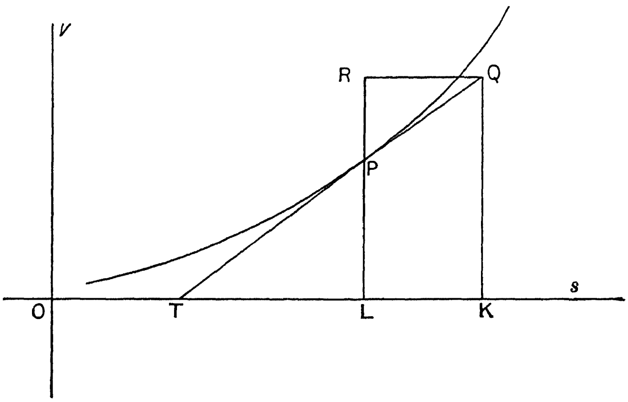


Fig. 40

The slope (tan ψ in fig. 38) in the velocity-time curve gives the acceleration, and the area in the acceleration-time curve gives the velocity.

Acceleration Curves.—Analytically the acceleration f is $\frac{dv}{dt}$, and so is $\frac{dv}{ds}\frac{ds}{dt}$ or $v\frac{dv}{ds}$ or $\frac{1}{2}\frac{d}{ds}(v^2)$, and therefore in a space-velocity curve, fig. 40, the tangent of ψ, the inclination of the curve, is to be multiplied by the ordinate v in order to obtain the acceleration. The construction therefore is to set off from the point T, in which the tangent at any point P cuts the space axis, a distance TK equal to the velocity PL, so that if PT cuts the ordinate at K in Q, the length QK represents the velocity. Drawing QR parallel to Os, RL represents the acceleration at the position OL.

If the acceleration-space curve is given the velocity is obtained from it, since $d(v^2) = 2f ds$, by finding twice the area $\left(\int f ds\right)$ under the curve and taking the square root of it, after adding any constant necessary, such as the square of the velocity of the body as it enters the part of the curve under consideration.

Acceleration and Force.—Mass, force, and acceleration being connected at any instant by the equation $P = mf$, in which P and f are vector quantities, this, with a knowledge of certain data concerning the velocity or positions at definite times, serves for the investigation of any particular problem. In Cartesian co-ordinates, the expressions for the accelerations parallel to the axes are $\frac{d^2x}{dt^2}$ and $\frac{d^2y}{dt^2}$, giving corresponding equations of motion, while in polar co-ordinates the expressions become $\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2$ along the radius and $\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right)$ perpendicular to it. In the usual case of r being constant the central acceleration becomes $r\left(\frac{d\theta}{dt}\right)^2$ towards the centre, requiring an applied force $mr\left(\frac{d\theta}{dt}\right)^2$ or $m\frac{v^2}{r}$, where m is the mass of the particle, to act towards the origin, this force being usually termed the centrifugal force.

The unit of mass being the pound, the unit of force implied by these equations is the poundal, and to obtain the equivalent force in lb. weight it is necessary to divide by g, the value of which at London is 32·2 ft. per second per second. In unfamiliar investigations concerning the forces involved in dynamical cases the investigation is usually carried out in these absolute units, but when the problem is thoroughly understood a variable mass may be taken as a unit and the calculation of forces worked in lb. weight.

Centrifugal Force.—When the path of a point-mass m is known, its acceleration along the path is $\frac{d^2s}{dt^2}$, and the graphical calculations reduce to those connecting positions, velocity, and acceleration given above. There will also be a normal force $\frac{mv^2}{\rho}$, where ρ is the radius of curvature of the path, necessary to keep the mass in the given path, and this can be found from the velocity diagram.

Thus consider a carriage (fig. 41) of mass m, at a velocity v, running round a curve of radius r. In order to produce the accelerating force $\frac{mv^2}{r}$ normal to the track without depending upon the wheel flanges, the track must be banked as shown. This radial force is then produced by the horizontal component of the reaction R, and can be obtained from a force diagram by supposing $\frac{mv^2}{r}$ to be reversed and so in equilibrium with the remaining forces—the weight mg and the reaction R. This gives the relationship $\frac{mv^2}{r} = mg \tan\alpha$, so that $\tan\alpha = \frac{v^2}{rg}$. The angle of banking depends upon the velocity, so that at other velocities the excess or defect of the radial force must be supplied by the action of the wheel flanges on the rails—or in the case of a road by friction perpendicular to the direction of movement.

Momentum.—The vector product mv is termed the momentum, and,

being the time integral ($\int Pdt$) of the force, is a convenient summation when the precise action of the force and its variation is either unknown or not required. Thus a blow or impact is so measured, although the force at any instant of the duration of the blow may be unknown. Conversely the force is measured by the rate of change of momentum. Thus if a uniform jet of water, having a velocity v and area of section A, strike a surface without a rebounding splash, the momentum destroyed in time t is that of a column of water of length vt, or $Avt\rho \times v$, where ρ is the density of water, and the rate of destruction of the momentum is $Av^2\rho$, which accordingly is the force on the surface. If the jet be 3 in. diameter, and v that due to falling from a height $h = 500$ ft., the force $Av^2\rho$ is $A \cdot 2gh \cdot \rho$ or $\frac{\pi}{4} \cdot (\frac{1}{4})^2 \cdot 2g \cdot 500 \times 62{\cdot}5$ poundals or 3060 lb.-wt.

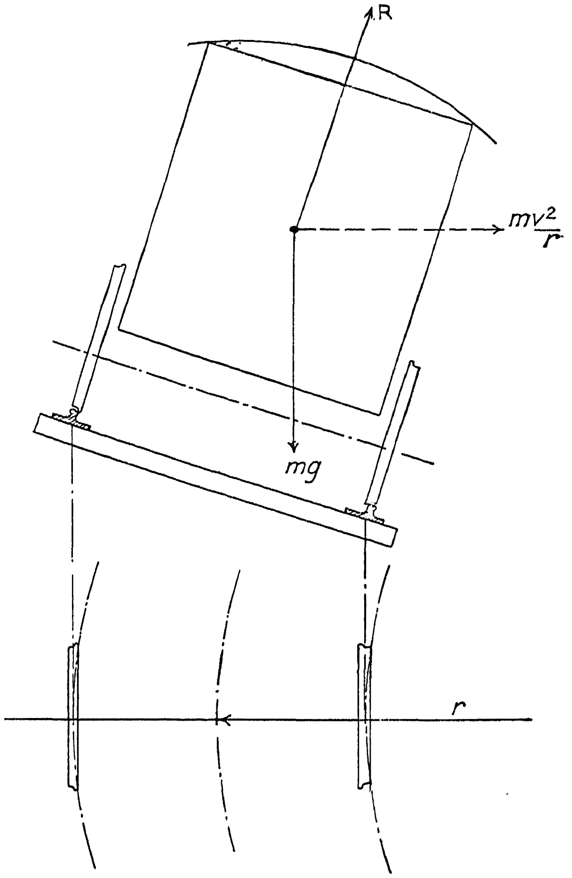


Fig. 41

Kinetic Energy and Power.—The work done by the accelerative forces takes the form of kinetic energy, and in the case of a single mass particle is $\int Pds$ or $\int m\frac{d^2s}{dt^2}\,ds$, which is $\frac{1}{2}mv^2$. The rate of doing work is power, and the unit usually employed is the horse-power, equivalent to 33,000 ft.-lb. per minute. Thus if a load of 1 ton is being raised vertically (whether by a direct lift or by an inclined path) at a uniform rate of 250 ft. per minute, the work done per minute is 2240 lb.-wt. raised through 250 ft., or 560,000 ft.-lb., and this rate is 17 h.p. If, however, the speed of the load is being increased a greater force and greater power is required, the additional force being measured by mf, where f is the acceleration, or mf/g lb.-wt. and the horse-power by the (varying) quantity $m(1 + f/g) \times \frac{v}{33{,}000}$, where v is the

velocity in feet per minute. Thus if after 5 seconds the velocity be doubled, the value of f will be 250/5 or 50 feet per sec. per sec., and the total tension in the hoisting rope will be $2240\left(1 + \frac{50}{g}\right)$ or 5720 lb.-wt. The power required at the instant that the velocity reaches the value of 500 ft. per sec. will be $5720 \times 500/33{,}000$ or 87 h.p.

Angular Velocity.—Corresponding to the three co-ordinates (e.g. x, y, and θ) of a body in a plane, are the velocity components of a point of the body and the angular velocity of the body, which is measured by $\omega = \frac{d\theta}{dt}$, where θ is the angle which a line in the body makes with a fixed line in space. Rate of rotation expressed in revolutions (n) per minute or second is related to the angular velocity by the equation $\omega = 2\pi n$. The rate of increase of angular velocity is termed angular acceleration.

The Instantaneous Centre.—A body in a plane can be brought from any position to any other position by a partial rotation about a certain point, hence when the body is moving in any manner its movement from one position to that which it occupies immediately afterwards may be regarded as a turning motion about a particular centre, which hence is termed the "instantaneous centre". The position of this point changes continuously (in general) both in space and also as regards its position in the body. If the directions of motion of two points of the body (not moving perpendicularly to the line joining them) are known, the instantaneous centre can be found as the intersection of lines drawn through the points perpendicular to the directions of their motion.

Thus in fig. 15, since the connecting-rod BD is moving so that the gudgeon-pin B is moving along line BC, the instantaneous centre I is in the line BI drawn perpendicular to BC. Also since the point D is moving perpendicularly to CD, I lies in CD, and therefore is where CD and BI intersect. If the engine is making n r.p.m., the velocity of D is $2\pi n$CD, and therefore the angular velocity ω of the connecting-rod is $2\pi n \frac{\text{CD}}{\text{ID}}$. Hence the velocity of the crosshead being BI . ω is $2\pi n \frac{\text{CD} \cdot \text{BI}}{\text{ID}}$.

The Motion of a Body in Space.—The movement of a body in space can be regarded as composed of the motion of a particular point of the body and of angular velocities about three lines through the point, usually taken to be fixed as to their directions. In correspondence to the preceding view of the instantaneous motion of a body in a plane, the motion of a body in space at any instant may be regarded as composed of a velocity along a particular line, combined with a definite rate of rotation about this line, that is it may be considered as having movement along a particular screw.

Angular Momentum.—The moment of the momentum of all the particles of a body about any axis in space is termed the angular momentum of the body about that line, and its rate of increase is equal to the moment about this line of the forces applied to the body. Dynamically the motion

of the centre of gravity of a body and its rotation are independent, so that the motion of a body is determined by two sets of considerations: (i) that the rate of change of its linear momentum in any direction (which is equal to that of the whole mass of the body supposed collected at its centre of gravity) is equal to the sum of the components of the applied forces resolved in that direction, and (ii) that the rate of change of the angular momentum of the body about any fixed line is equal to the sum of the moments of the applied forces about that line.

Motion about a Fixed Axis.—The simplest case is that in which rotation takes place about a fixed axis, when the angular momentum about this axis is $\int r\frac{d\theta}{dt}\,.\,dm \times r$ or $\frac{d\theta}{dt}\int r^2dm$, and hence is $\mathrm{I}\,\frac{d\theta}{dt}$, so that the equation of motion is $\mathrm{I}\,\frac{d^2\theta}{dt^2} = \mathrm{L}$ where L is the moment of all the forces about this axis, and I is the moment of inertia about this axis or is $\mathrm{M}(k^2 + h^2)$, where k is the radius of gyration about a parallel axis through the centre of gravity and h is the distance of the centre of gravity from the fixed axis.

Suppose that a disc fly-wheel, 60 in. diameter by 6 in. thick, weighing 4600 lb., be started from rest by the action of a uniform torque of 1000 lb.-(wt.)-ft. Then $\mathrm{I} = 4600 \times \frac{2{\cdot}5^2}{2}$ and the equation of motion becomes $\frac{d^2\theta}{dt^2} = \frac{1000 \times 32{\cdot}2 \times 2}{4600 \times 2{\cdot}5 \times 2{\cdot}5}$, which gives on integration $\frac{d\theta}{dt} = 2{\cdot}24t$, so that the angular velocity at any time is known. At the end of a quarter of a minute it would be 33·6 radians per second or 322 r.p.m.

When the fly-wheel shaft receives a torque L_1 and delivers a torque L_0, the fly-wheel movement is determined by $\mathrm{I}\frac{d^2\theta}{dt^2} = \mathrm{L}_1 - \mathrm{L}_0$, so that when, as is usual in an engine, these torques are not equal the fly-wheel does not run quite uniformly, but continually exceeds and falls below its mean speed, by an amount which varies inversely as the moment of inertia of the fly-wheel.

The Kinetic Energy of Rotation.—The kinetic energy of a rotating body is $\int \frac{1}{2}v^2dm$ or $\frac{1}{2}\mathrm{I}\omega^2$, and represents the work $\mathrm{L}\theta$ (or $\int \mathrm{L}d\theta$) done by the torque in giving angular velocity to the wheel, and this energy is available as work (e.g. is employable in a punching machine) or must be " destroyed " (e.g. by being turned into heat by friction) in bringing the fly-wheel to rest.

Any application of work to the fly-wheel, as by an excess of work W transmitted by the connecting-rod to the crank-shaft of an engine, above the work taken from the engine will increase the fly-wheel speed from ω_0 to ω_1, where $\frac{1}{2}\mathrm{I}(\omega_1^2 - \omega_0^2) = \mathrm{W}$. In most cases engines are designed to run closely to a particular speed so that the change $\delta\omega$ in ω is not great, and is given with sufficient accuracy by (differentiating the work equation) $\mathrm{I}\omega\delta\omega = d\mathrm{W}$, or $\frac{\delta\omega}{\omega} = \frac{\delta n}{n} = \frac{\delta \mathrm{W}}{2\mathrm{E}}$, where n is the revolutions per second and E the mean energy of the fly-wheel.

The Balancing of Rotating Discs.—Usually the centre of gravity of the fly-wheel is upon the axis of rotation, but if it be not, a centrifugal force of $\frac{4\pi^2n^2hM}{g}$ lb.-wt., directed inwards along the radius to the centre of gravity, whose distance from the axis is h ft., will be necessary, and will be furnished by reactions at the bearings, superposed upon the reactions supporting the weight of the wheel and shaft. The directions of these reactions change, rotating round the axis as the centre of gravity does, and will tend to cause vibration of the frame of the engine. The fly-wheel is then " out of balance ", and for satisfactory running this must be corrected, especially carefully where the rotation speeds are high, as the forces given by the above expression, where n is the revolutions per second, depend upon n^2. Small parts are balanced (fig. 42) by being mounted so as to be capable of rotation about a pin A, set at right angles to a lever pivoted at B, and carrying a weighing scale at C. If the centre of gravity G be at a distance h

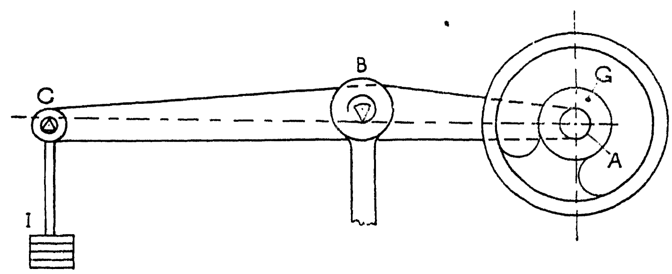


Fig. 42

from the axis, the balancing weights in the pan at C will vary by $2mh$ (where m is the mass of the part being balanced) for the various positions of G, so that the amount and position of the alteration necessary to bring the centre of gravity of the body to the axis is easily found. Larger bodies are tested by being placed upon level straight ways, and the balance weights adjusted until there is no tendency to roll along in any position.

At high speeds the effects are apt to be so great that, although the design may call for a rotor of simple shape symmetrical about the axis, the variation of density in the material may be such as to bring the centre of gravity so far off the axis as to cause trouble. What the variation in the density of forged steel even can be is well known to all who have been engaged in shell manufacture.

The Balancing of Rotors.—In the above the part to be balanced has been regarded as being comparatively thin (a fly-wheel or a propeller), but when the mass is distributed along the axis (as in the case of dynamo construction) there may still be a dynamical cause of vibration although the centre of gravity be upon the axis. Suppose that a rotor, axis xx' (fig. 43), with bearings at A and B, is perfectly in balance, except for two masses m_1 and m_2 situated as shown. The effect of m_1 at C will be to cause forces

$m_1 r_1 \omega^2 \dfrac{b}{a+b}$ at A, and $m_1 r_1 \omega^2 \dfrac{a}{a+b}$ at B in the direction along Az and Bζ indicated, the various dimensions being shown in the figure. If m_1 were the only out-of-balance mass, a mass m_0 at a distance r_0 from AB, satisfying the condition $m_0 r_0 = m_1 r_1$ would make the centre of gravity of the whole to lie on the axis, wherever this mass were placed in the extension of the plane ζBAz beyond the axis. This is termed statical balance, and the usual statical tests would not indicate any want of balance. Unless, however, the single mass m_0 were placed exactly opposite to C—the position of which is unknown in any actual case—the forces at the bearings would not be balanced and the cause of vibration trouble would not be removed. Usually discs, here indicated by the shaded planes at E and F, are provided for the reception of the balance weights, and by suitably adjusting the

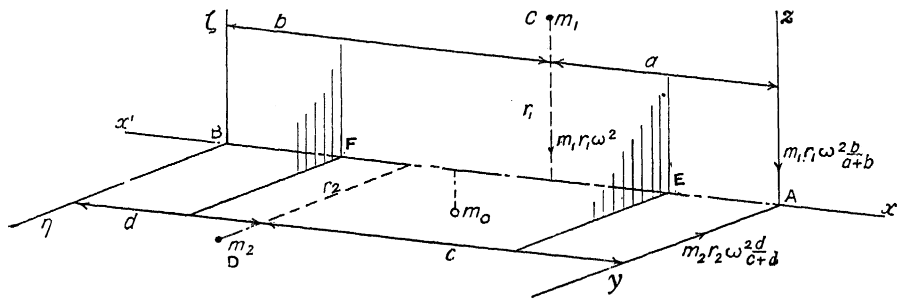


Fig. 43

amounts at E and F the action of C can be balanced dynamically. The amounts of these balance weights must be determined by dynamic trials.

In like manner the mass m_2 at D, here shown in a plane ηBAy perpendicular to the plane through ABC, would produce forces $m_2 r_2 \omega^2 \dfrac{d}{c+d}$ and $m_2 r_2 \omega^2 \dfrac{c}{c+d}$ at the bearings A and B. There will thus be another pair of forces, one at each journal and out of phase with those due to the mass m^1, but rotating with the shaft. This is typical of any irregular mass distribution, and as it can be balanced by masses in planes E and F any case of out-of-balance can be so rectified. The phase angles at which the balance weights are to be set are determined by mounting the rotor in spring-controlled bearings, running it, and marking the shaft as it vibrates and revolves. Owing to the effect of friction on the phase angle, trials are made with each direction of rotation. The balance weights are adjusted by trials until the result is sufficiently near.

Principal Axis.—In "balancing" a body, the axis of rotation is made to be a "principal" axis, the condition for which is that $\int yz\,dm = \int zx\,dm = 0$ where Oz is the axis of rotation. Henceforth parts will be considered with

regard to their principal axes only, and, except where high speeds are employed, the bodies may be regarded as constructed of materials of uniform density. Expressions such as $\int yzdm$ are termed products of inertia, and when principal axes are employed are all zero. As the shape and distribution of mass of a body do not enter into the dynamical equations except through their influence on the moments of inertia, bodies which have the same mass and the same moments of inertia will behave in the same way under equal systems of forces acting alike as regards the position of the centres of gravity and principal axes. Such bodies are therefore said to be dynamically equivalent. Thus in determining the motion of one part of a machine and its effect upon the other parts, it may be replaced by any convenient system of heavy particles or simple bodies dynamically equivalent to it.

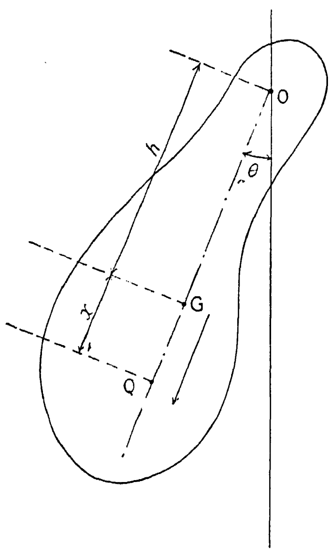


Fig. 44

The Centre of Oscillation.—A small mass suspended by an indefinitely fine string so that it can swing under gravity is termed a simple pendulum, but an actual pendulum is a mass in three-dimensional space arranged to swing about a horizontal axis. Such a pendulum will, however, move in exactly the same way as any dynamically equivalent mass system. As the motion is simply (partial) rotation about the axis, two point masses, one—m_0—placed at the axis and the other—m_1—placed on the perpendicular from the centre of gravity of the body to the axis, will constitute such a system provided that

$$m_0 + m_1 = \mathrm{M}, \qquad m_1 x = m_0 h, \qquad \text{and } m_0 h^2 + m_1 x^2 = \mathrm{M}k^2,$$

where M is the mass of the pendulum, k the radius of gyration about an axis through its centre of gravity parallel to the axis of swing, and h the distance from the centre of gravity G to the axis, as indicated in fig. 44. From these

$$m_1 = \frac{\mathrm{M}h}{x+h} \text{ and } x = \frac{k^2}{h}.$$

The second particle is, therefore, at a distance $\frac{h^2+k^2}{h}$ from the axis, and the motion of the actual pendulum will therefore be the same as that of a single particle at the end of a fine cord of length $\frac{h^2+k^2}{h}$, which is accordingly termed the "simple equivalent pendulum". The position Q on the

body corresponding to the mass m_1 is termed the centre of oscillation as regards the axis O.

The equation of motion for these will take the form $\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$, where θ is the inclination to the vertical, and l being the length of the simple equivalent pendulum. Such an equation, therefore, corresponds to a vibratory motion. If the arc of vibration be small so that θ may be written for $\sin\theta$, with sufficient exactitude the first equation becomes

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0,$$

where ω^2 is written for $\frac{g}{l}$.

Harmonic Motion.—The solution of this equation is of the form $\theta = A \sin(\omega t + \alpha)$, containing two constants A and α, determinable by the conditions of the motion. The quantity A, which is the maximum value of θ reached in the oscillation is termed the amplitude, while the value of α depends only upon the instant from which the time is measured. The value of θ repeats itself after time intervals of $\frac{2\pi}{\omega}$, which is accordingly the complete "period" of the vibration. The inverse of the period gives the number of vibrations in unit time and is termed the frequency.

Thus the simple pendulum will have a period T equal to $2\pi\sqrt{\frac{l}{g}}$ seconds, so that the length of a simple pendulum oscillating once per second is $\frac{g}{4\pi^2}$ ft. or 9·78 in., and a pendulum beating seconds, i.e. whose period of oscillation is 2 sec., will be 39·1 in. long.

The motion given by an equation such as the above is termed "harmonic". If a mass system in equilibrium is slightly displaced it usually tends either to return to the position of equilibrium—in which case it is said to be stable—or to depart widely from it, when it is termed unstable. In the former cases the restoring forces usually vary as the displacement, and the equation of motion takes a form similar to the above, and indicates that the system vibrates harmonically. The period in which it vibrates under the action of the forces tending to restore equilibrium is termed the natural period of vibration, or the free period.

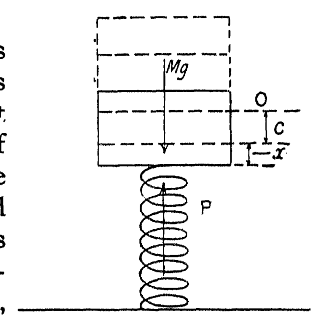


Fig. 45

Elastic systems usually have a number of modes of vibration with corresponding periods. Suppose that a spring, shown in fig. 45 as of the compression type, is to support a load of M lb., and compresses a distance c under the load. One mode of oscillation is vertical, and the equation of motion,

where x is the vertical displacement, is $\mathrm{M}\dfrac{d^2x}{dt^2} + \mathrm{P} - \mathrm{M}g = 0$ where P is the restoring force due to the compression of the spring. This is $\dfrac{c+x}{c} \cdot \mathrm{M}g$, since this force is proportional to the total displacement $c + x$, and hence the equation of motion becomes $\dfrac{d^2x}{dt^2} + \dfrac{g}{c}\,x = 0$. Thus the natural period of vibration is the same as that of a simple pendulum of length c equal to the initial compression of the spring.

The presence of a little friction reduces the amplitude of the oscillations so that the body comes to rest after a series of swings. If such friction is proportional to the velocity the amplitudes diminish exponentially. A considerable amount of such friction prevents the body from oscillating, and it approaches the position of rest in one direction only, and the movement is termed dead beat.

Vibrations.—Any periodic movement or action of frequency n may be regarded as the result of the superposition of a series of harmonic motions of frequencies n, $2n$, $3n$, . . . , the coefficients being whole numbers, so that the treatment of complex repetition motions can be reduced to the summation of harmonic terms.

Thus if in fig. 15 the crank length be r, and the connecting-rod length l, and angle DCB be ωt, ω being the (uniform) angular velocity of the crank,

$$\begin{aligned} \mathrm{BC} &= r\cos\omega t + (l^2 - r^2\sin^2\omega t)^{\frac{1}{2}} \\ &= r\cos\omega t + l\left(1 - \frac{r^2}{l^2}\sin^2\omega t\right)^{\frac{1}{2}} \\ &= r\cos\omega t + l\left(1 - \tfrac{1}{2}\frac{r^2}{l^2}\sin^2\omega t\right), \text{ retaining only the 2nd term;} \end{aligned}$$

$$\text{i.e. } \mathrm{BC} = r\cos\omega t + l - \frac{r^2}{4l} + \frac{r^2}{4l}\cos 2\omega t, \text{ since } \sin^{-2}\omega t = \frac{1 - \cos 2\omega t}{2}.$$

If we put x for $\left(\mathrm{BC} - l + \dfrac{r^2}{4l}\right)$, we get

$$x = r\cos\omega t + \frac{r^2}{4l}\cos 2\omega t + \ldots\ \Big\},$$

where each term represents a harmonic motion having as its period a simple fraction of the period $\left(\dfrac{2\pi}{\omega}\right)$ of revolution of the engine. The acceleration of the piston is therefore given by

$$\frac{d^2x}{dt^2} = -\,\omega^2 r\left\{\cos\omega t + \frac{r}{l}\cos 2\omega t + \ldots\right\},$$

so that the periodic force $\left(\mathrm{M}\dfrac{d^2x}{dt^2}\right)$ necessary to produce it can be regarded as composed of a series of harmonic terms of the above nature, the first two terms giving sufficient accuracy for ordinary purposes.

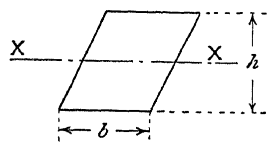
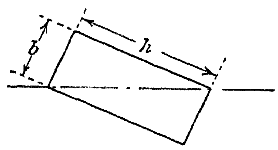
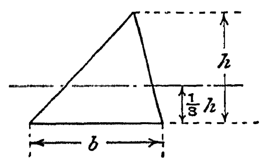
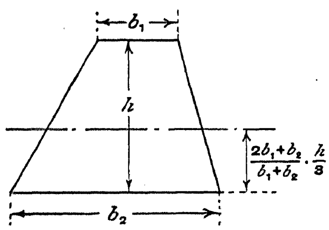
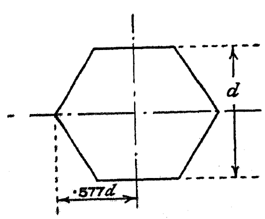
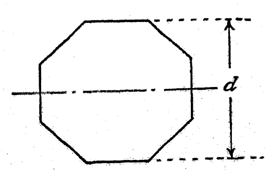
The action of a periodic force, such as the above, is to produce a vibratory motion in the system concerned, with a period corresponding to that of the force, and in this the various harmonic components may be regarded as acting separately. The amplitude of the ensuing vibration, which is the matter of chief importance, is not proportional directly to the coefficient of the harmonic component, but depends upon the nearness of the coincidence of the natural period of vibration of the part or system with the period of one of the harmonic constituents of the disturbing force. Thus if vibration occurs, some disturbing force having a period equal to, or a simple multiple of, that of the vibration is to be sought for and removed, or the period of the system is to be changed (usually by loading it) so that there is no longer any near coincidence of the periods.

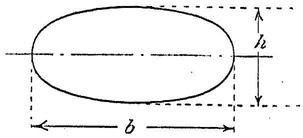
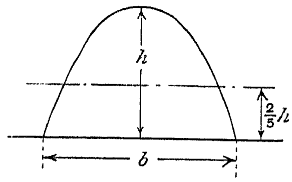
Dynamic Equivalence.—The principle of dynamic equivalence enables many problems to be simplified and in other cases approximations to be made which lessen the necessary calculations. In considering the motion of a connecting-rod we theoretically study the motions of its centre of gravity and of its angular motion about the centre of gravity, but it is usual in practice to consider the effect of the connecting-rod as equivalent to that of two masses, m_1 at the gudgeon-pin and m_2 at the crank-pin, having together the same mass and same centre of gravity as the connecting-rod has. These masses are then treated as added to the piston and to the gudgeon-pin respectively, thus simplifying the work. If the moment of inertia of the masses about the centre of gravity were the same as that of the connecting-rod, they would be dynamically equivalent and would have the same effect, but in general there is a difference, and the accuracy of the approximation is indicated by this difference of moments of inertia.

TABLE I

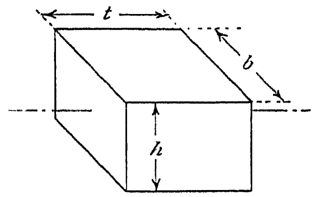
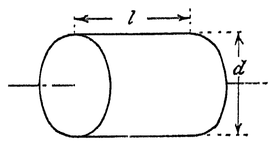
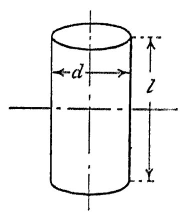
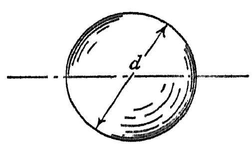
Data concerning areas and solids.
The axis XX is drawn through the centre of gravity.
Areas and moments of inertia of hollow figures are obtained by subtraction.

AREAS

		Area.	Maximum Ordinate.	Moment of Inertia.
Squares Rectangles Parallelogram		bh	$\frac{h}{2}$	$\frac{1}{12}bh^3$
Rectangles about diagonal		bh	$\frac{bh}{\sqrt{b^2+h^2}}$	$\frac{b^3d^3}{6(b^2+d^2)}$
Triangles		$\frac{1}{2}bh$	$\frac{2}{3}h$	$\frac{bh^3}{4}$
Trapezium		$\frac{b_1+b_2}{2}h$	$\frac{b_1+2b_2}{b_1+b_2}\frac{h}{3}$	$\frac{b_1^2+4b_1b_2+b_2^2}{36(b_1+b_2)}h^3$
Hexagon		$\cdot866d^2$	$\frac{d}{2}$	$\cdot06d^4$
Octagon		$\cdot828d^2$	$\frac{d}{2}$	$\cdot055d^4$

		Area.	Maximum Ordinate.	Moment of Inertia.
Circles and ellipses		$\frac{\pi}{4} bh$	$\frac{h}{2}$	$\pi \frac{bh^3}{64}$
Parabola		$\frac{2}{3} bh$	$\frac{3h}{5}$	$\frac{8}{175} bh^3$

SOLIDS.

		Volume.	k^2.
Rectangles		hbt	$\frac{b^2 + h^2}{12}$
Cylinders		$\frac{\pi d^2}{4} l$	$\frac{d^2}{8}$
Cylinder		$\frac{\pi}{4} d^2 l$	$\frac{d^2}{16} + \frac{l^2}{3}$
Sphere		$\frac{\pi d^3}{6}$	$\frac{d^2}{10}$

ELASTICITY AND STRENGTH OF MATERIALS

BY

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Elasticity and Strength of Materials

Physical Basis.—The physical properties of a material, including all those concerned with its yield or fracture, being supposed known, in order to determine the suitability of the design of a part of a machine or structure, it is necessary to ascertain the stress system at any point of the part, which is produced by the applied forces. To carry out such calculations it is, in most cases, necessary to consider the accompanying strains, which are related to the stresses according to certain laws. The relationship was first investigated by Hooke, who propounded, in the case of tension, the law that the extension was proportional to the force producing it, and his name is given to the similar, but more general, law, which is accepted in nearly all theoretical analysis, that the stresses at a point are linear functions of the coexisting strains. Thus the fact that the twist of a shaft is proportional to the torque producing it is regarded as being a particular case of the general law.

Accuracy of Hooke's Law.—Since this relationship, Hooke's Law, forms the basis of analytical stress investigations, it is well to observe that it, like physical laws generally, is approximate only, and that the results of the dependent calculations will be in error to a corresponding degree. Most steel obeys the law exceedingly closely, although the ascending and descending parts of the stress-strain curve do not, even at low stresses, coincide, but enclose a hysteresis area. Other materials depart more widely from the law, the amount being indicated by their stress-strain curves.

Accuracy of Calculations.—Furthermore, it is to be remarked that the calculations are not in general rigorous mathematically, but the cases considered are simplified somewhat from the actual cases. Accurate mathematical analysis has proved that the solutions obtained under such approximations are sufficiently accurate for practical purposes, except in the neighbourhood of the application of a load, and with this proviso they may be accepted.

The linear relationship does not hold beyond the yield point, so that the calculations given, except where otherwise specified, hold only up to the point at which the material takes a permanent set. Thus design can logically be based only upon the yield point and not upon the ultimate stress.

Although failure ultimately is caused by excessive stress or strain, it may

normal to the axis, be taken at an inclination of $\frac{\pi}{2} - \theta$, as in fig. 2. The load P is now carried over the area A $\sec\theta$, so that the stress is $p = \frac{P}{A \sec\theta}$ $= p_o \cos\theta$, where p_o is the value of the stress on the cross section. This stress is inclined to the surface across which it acts, and is termed an oblique stress.

The force P, supposed acting at the section, could be replaced by a component P $\cos\theta$ normal to the section, and a component P $\sin\theta$ along the section, producing a normal stress $p_n = p_o \cos^2\theta$ and a tangential stress $p_t = p_o \sin\theta \cos\theta$. Tangential stresses are usually termed shear stresses.

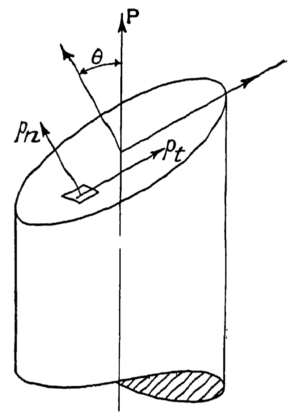

Fig. 2

As the angle at which the section is supposed to be made changes, the normal and shear stresses change, the former from p_o, when the section is normal, to zero when it is taken parallel to the axis, while the latter (being equal to $\frac{1}{2}p_o \sin 2\theta$) increases from zero to $\frac{1}{2}p_o$ when the section is taken at 45°, afterwards diminishing to zero.

Thus the maximum value of the shear stress in the tie-bar considered is 2·835 tons per square inch, and occurs along all sections inclined to the axis at 45°. That steel specimens broken in tension usually show a conical fracture inclined at approximately this angle is suggestive of the effect of this stress action.

Compression.—In parts subjected to compression the state of stress is similar but of an opposite nature. Thus if a short length, say 6 in., of the round shaft previously considered carry a compressive load of 16 tons, across sections normal to the axis there would be a compressive stress of 5·67 tons per square inch, and the maximum shear would again be 2·835 tons per square inch across the same surfaces but in the opposite direction across them. The value of E being the same, the strains would be of the same amount, the total shortening being $\delta l = 6 \times \frac{5 \cdot 67}{13{,}000} = 0 \cdot 0026$ in.

Specimens for tests should be of sufficient length-diameter ratio to permit planes of maximum shear to exist completely across the specimen, and hence should be decidedly longer than their diameters, particularly as there is always an irregularity in the stress distribution in the neighbourhood of the ends.

Thin Tubes under Internal Pressure.—If a thin cylindrical vessel or tube, of radius r, be subjected to internal fluid pressure p_o, a state of tensional stress is produced in the material. Suppose that a length l of the tube between two planes perpendicular to the axis be taken and then divided by a diametral plane. The forces acting upon it and its contents perpen-

primarily be induced by instability of shape of the part. In such cases the physical properties involved are those of the elasticity, and not the strength, of the material.

Tension.—If a pull P be exerted along the axis of a round tie-bar of area A, it produces a tensile stress p over all sections perpendicular to the axis of the value $p = \frac{P}{A}$, stress being defined as force per unit area. If the two faces A, B, fig. 1, of the section be considered, the stress on each acts normally outwards from the surface, there being a double aspect of the same stress. This double aspect can of necessity be taken of any stress, whether normal to, inclined to, or along the area on which it acts.

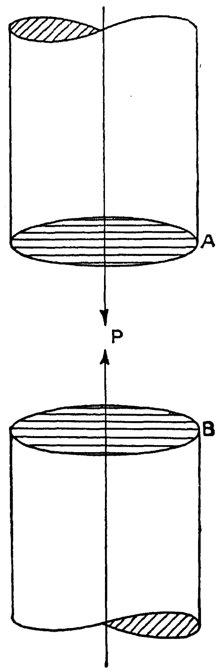

Fig. 1

Young's Modulus of Elasticity.—Under the load, the tie-bar, of length l originally, extends to a length $l + \delta l$, where (by Hooke's Law) δl is proportional to P. To express the distortion in terms of the material and not of the shape of the member, the extension is reckoned by the rate per unit length, $\frac{\delta l}{l}$, which is termed the strain. It is proportional to the stress, the relationship being given by the equation

$$p = \mathrm{E}\frac{\delta l}{l} = \mathrm{E}\epsilon,$$

where the constant E, which is of the nature of a stress, is known as Young's modulus of elasticity.

These relationships may be illustrated by the case of a tie-bar $1\frac{1}{2}$ in. diameter by 20 ft. long, carrying a load of 10 tons. The stress is given by

$$p = \frac{P}{A} = \frac{10}{1{\cdot}77} = 5{\cdot}67 \text{ tons per square inch,}$$

and the elongation in 20 ft. is, if E for the steel be 13,000 tons per square inch,

$$\delta l = l\frac{p}{\mathrm{E}} = 0{\cdot}1045 \text{ in.}$$

Poisson's Ratio.—Simultaneously with this longitudinal extension a lateral contraction takes place in the bar of a proportional amount $\sigma\epsilon$, where σ is a numerical constant for the material of the bar, known as Poisson's ratio. Its value for steel is approximately $\frac{1}{3}$. Thus the diameter of the bar considered would be reduced by the amount $\frac{1}{3}\frac{p}{\mathrm{E}} \times 1{\cdot}5$ in. or 0·00022 in.

Oblique Stress.—Suppose that the section of the bar, instead of being

dicular to the diametral plane are $p_o.2rl$ due to the fluid pressure, and f_1tl at each rectangular section of the tube (fig. 3), where f_1 is the "circumferential" tension in the material and t the thickness of the tube. Hence $f_1t = p_or$.

Now suppose that the vessel be cut by a plane perpendicular to the axis (fig. 4), and the equilibrium of one position—the end of which may be considered as closed—considered. There is a (different) tensile stress f_2 per-

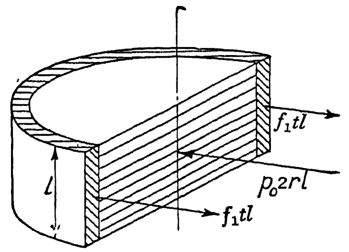

Fig. 3

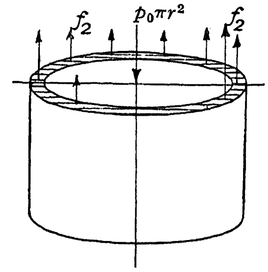

Fig. 4

pendicular to the tube section, giving a total force $2\pi rt.f_2$, which balances the total force πr^2p_o arising from the internal pressure. Hence $2f_2t = p_or$, so that f_2 is of half the intensity of f_1.

Thus if a cylindrical vessel, 6 ft. diameter, of plates $\frac{1}{2}$ in. thick, be subjected to an internal pressure of 120 lb. per square inch, the circumferential stress will be $f_1 = 120 \times 36 \times 2 = 8640$ lb. per square inch, and the longitudinal stress will be half this or 4320 lb. per square inch. This state of stress is indicated in fig. 5. Both these stresses act normally to the surface across which they act.

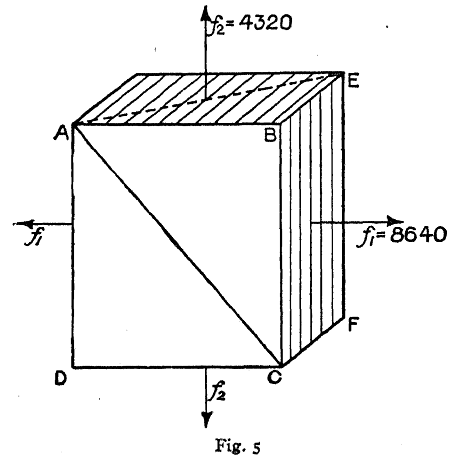

Fig. 5

Thin Spherical Shells.—In the material of a thin spherical shell of radius r and thickness t, subjected to internal fluid pressure p_o, there will be tensile stress of a similar nature, but by symmetry the stress in any direction is the same. Consideration of the equilibrium of a hemisphere and its contents gives as the value of the stress $2ft = p_or$, so that the stress is the same as the longitudinal stress in the material of a pipe of same diameter and thickness.

Principal Stresses.—Whatever be the state of stress at a point it is always equivalent to three stresses acting normally to mutually orthogonal surfaces. These stresses are termed the principal stresses at the point, and constitute the simplest view of the state of stress there, so that their determination for any given case of complex stress is a primary matter. From a knowledge of them the greatest shear stress and the strains can be easily

deduced. The cases of most interest are those in which one principal stress is zero, so that the matter appears to be one of two dimensions only. As the third principal stress in the case under consideration varies from boiler to atmospheric pressure, it is small compared with the other stresses, and, being neglected, this case comes under the above type.

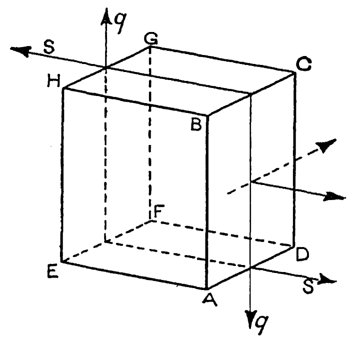

Fig. 6

The shear stress involved is here also a maximum on surfaces inclined to the principal stresses at 45°; the amount across AC (fig. 5), taking a square ABCD and considering the equilibrium of ABC, is $\frac{1}{2}(f_1 - f_2)$. If, however, the element be sectioned by a plane AEF, the shear stress over this plane (found by considering the equilibrium of the wedge ABEFCD) is $\frac{1}{2}f_1$, and is therefore greater than that across AC, so that the case must be viewed in its entirety and not as two-dimensional merely.

Shear Stress.—If a shear stress occurs over a surface of any orientation at a point, there exists a shear stress of equal intensity over a surface perpendicular to the first surface and to the direction of the shear on it. For consider the rectangular element shown in fig. 6, in which there is a shear stress q over the face ABCD, in a direction perpendicular to the edge BC, and therefore one as indicated on the opposite face EFGH, and suppose that there is a shear stress s on the faces BCGH and ADFE. Then by the equation for moments about a line through the centre of the element and parallel to BC, the forces due to any normal stresses, and to any shear stresses in other directions which may exist, have no moment, so that

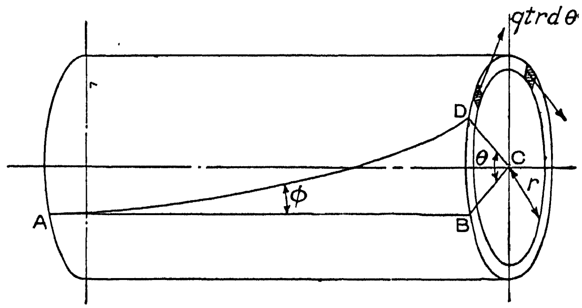

Fig. 7

$$\text{HB.BC.}s \times \text{AB} = \text{AB.BC.}q \times \text{HB},$$
$$\text{so that } s = q.$$

A state of stress consisting of such complimentary shear stresses only is termed a pure shear.

Thin Tube under Torque.—Such a condition exists in the material of a thin tube under a couple L acting round the tube axis. Consider the tube as fixed at the end A and the couple or torque applied at the other end. Taking a cross section at a distance AB ($=l$) from the fixed end, by symmetry each element is stressed in the same manner, which can only be by a shear

stress acting on each element perpendicularly to the radius from the centre C of the section, as is indicated in fig. 7. The moment of the stress forces is equal to the applied torque, so that $L = 2\pi r^2 t.q$.

There is also therefore an equal shear stress q acting parallel to the axis over the sections made by radial planes, as is shown in fig. 8, which gives a view of an element of the tube. The existence of this longitudinal shear stress is suggested by the weakness and behaviour of a tube which has been cut longitudinally.

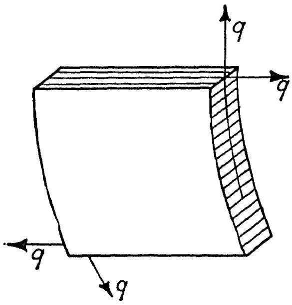

Fig. 8

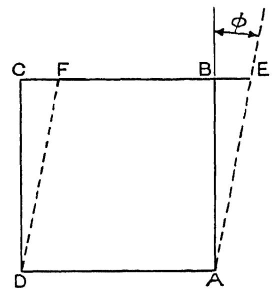

Fig. 9

Shear Strain.—Under the action of a shear stress a square element ABCD distorts into a rhombus AEFD, the amount of strain being measured by the angle BAE, fig. 9. The strain ϕ is proportional to the stress, so that

$$\phi = \frac{q}{C},$$

in which C, a constant of the nature of a stress, is termed the modulus of rigidity.

Under the torque, the section of the tube at BCD, fig. 7, will twist round in its own plane, so that a radius originally at CB turns through an angle θ, into the position CD, the corresponding generator AB becoming a helix AD, the angle (DAB) of which is given by $\phi = \frac{q}{C}$. From the geometry of the figure

$$r\theta = BD = l\phi,$$

so that the twist under any given stress or torque can be found. Thus a tube 2 in. diameter by $\frac{1}{16}$ in. thick, would be stressed to $q = 7650$ lb per square inch by a torque of 250 lb.-ft., and would receive an angle of twist of $12 \times \frac{7650}{11{,}000{,}000} = 0{\cdot}00835$, or $0{\cdot}48°$ per foot length, the modulus of rigidity of the material being 11,000,000 lb. per square inch.

Principal Stresses for a Shear.—If the edges of the square element ABCD of thickness t (fig. 10) be subjected to shear stress q as indicated, the consideration of the equilibrium of the part ABC only shows that there can be no tangential stress on the face AC, and that the normal tensile stress f_1 is given by the equation

$$f_1 \times AC.t = 2q \times BC.t \times \frac{1}{\sqrt{2}},$$

$$\text{so that } f_1 = q.$$

Similarly, considering the equilibrium of the part ABD only, it is seen that the stress on the face BD is a normal compression, $f_2 = q$. Hence the principal stress system shown in fig. 11 represents the same state of stress as does the shear stress of fig. 10.

Elastic Constants.—Under the shear stress q the square ABCD (fig. 10) distorts into the rhombus DA′B′C′, the elongation of the diagonal DB being $\phi.\frac{AB}{\sqrt{2}}$; the elongation strain in this direction is $\frac{1}{2}\phi$ or $\frac{q}{2C}$. The elongation in this direction can also be found from the equivalent principal stresses of fig. 11 by superposing the stress-strain systems, which is a legitimate procedure, provided the strains be small. The tensile stress q produces an elongation strain $\frac{q}{E}$, and the compressive stress adds to this the amount $\sigma\frac{q}{E}$, so that the total elongation is $(1 + \sigma)\frac{q}{E}$. Equating these values of the strain gives

$$\frac{1}{2C} = \frac{1 + \sigma}{E}.$$

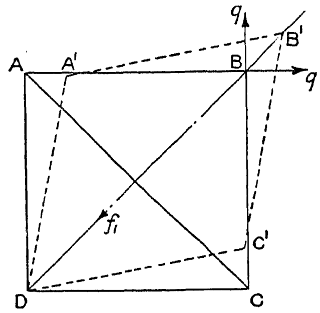

Fig. 10

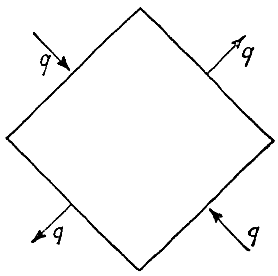

Fig. 11

These elastic constants (E, σ, and C) are the most generally useful, but the primary elasticities of an isotropic material are those referring to its change of shape (C) and to its change of volume (K) under fluid pressure. An elementary cube, side a, exposed to fluid pressure p, is shown in fig. 12, the equal principal stresses being p. The edges will all contract equally by an amount comprised of a contraction $\frac{pa}{E}$ due to the stress parallel to the edge, and by two elongations $\sigma\frac{pa}{E}$ due to the pair of perpendicular stresses. Thus the lateral strain in each direction will be $(1 - 2\sigma)\frac{p}{E}$, the corresponding volumetric strain being three times this amount. Since the volumetric strain is (by the definition of K) here $\frac{p}{K}$, this gives

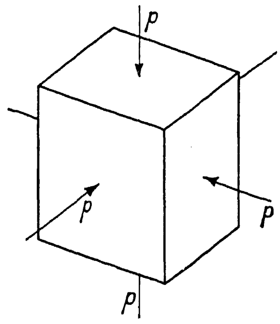

Fig. 12

$$\frac{1}{K} = \frac{3}{E}(1 - 2\sigma).$$

Any pair of these elastic constants specify the elasticity of the material those

most easily determined experimentally for metals being E (by 1 bending) and C (by torsion).

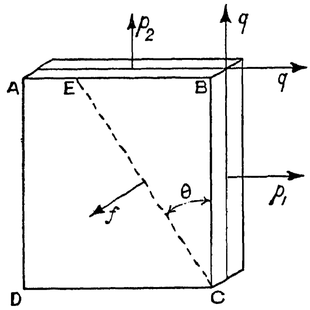

Fig. 13

Combined Stresses.—When th stress is given in a complex manner, cipal stresses of the system have to in order to obtain simplest presenta1 important case is that in which the stress over one surface, the two orthog faces having normal stresses p_1 and shear stress q over them, as indicated Suppose that a section CE is such stress f over it is entirely normal. the angle BCE be θ, taking the leng unity, the forces upon BE are p_2 $q\sin\theta$, upon CB, $p_1\cos\theta$ and $q\cos\theta$, and upon CE the force is f. along BE and CB the equations of equilibrium are

$$f\cos\theta = p_1\cos\theta + q\sin\theta,$$

$$f\sin\theta = p_2\sin\theta + q\cos\theta,$$

$$\begin{array}{ll} \text{giving} & f - p_1 = q\tan\theta \\ \text{and} & f - p_2 = q\cot\theta \end{array}\Bigg\}, \ldots\ldots\ldots\ldots\ldots\ldots$$

the product of which pair gives

$$f^2 - (p_1 + p_2)f + p_1p_2 - q^2 = 0.$$

Hence there are two values for f, namely

$$f_1 = \frac{p_1 + p_2}{2} + \sqrt{\tfrac{1}{4}(p_1 - p_2)^2 + q^2}$$

$$\text{and} \qquad f_2 = \frac{p_1 + p_2}{2} - \sqrt{\tfrac{1}{4}(p_1 - p_2)^2 + q^2},$$

which are the magnitudes of the principal stresses. The directions found by equating the values of f in equations (A), so that

$$p_1 + q\tan\theta = p_2 + q\cot\theta,$$

$$\text{or} \qquad \tan 2\theta = \frac{2q}{p_1 - p_2}.$$

The maximum shear stress, on planes bisecting those of these stresses, being equal to half the difference of these stresses, is

$$s = \sqrt{\tfrac{1}{4}(p_1 - p_2)^2 + q^2},$$

but the shear stresses on planes inclined at 45° to the surface ABCD to be considered for the absolute maximum.

In the case where one normal stress, p, coexists with a shear stress q over the same face, the principal stresses and shear are, from the above,

$$f = \tfrac{1}{2}p \pm \sqrt{\tfrac{1}{4}p^2 + q^2} \qquad \text{and} \qquad s = \sqrt{\tfrac{1}{4}p^2 + q^2}.$$

Limiting Stress.—Where two or more principal stresses exist, calculations upon the strength of the material are now usually based upon Rankine's Law, that the material gives way when the maximum tensile stress reaches a particular value, which is applicable to brittle materials; or upon Guest's Law, that yield takes place in ductile materials at a specific shearing stress. The degree of accuracy of the latter for mild steel is about 6 per cent, dependent partly upon the mode of determination of the limiting shear stress.

Stress Variation.—In the above the stress at a point is regarded as uniform over the space considered, but in many cases (e.g. across the section of a loaded beam) it varies from point to point. In the theoretical discussion, however, the size of the elementary areas and volumes may be taken as indefinitely small, so that the stress is uniform over the space considered, and hence the relationships obtained are true. Actually, however, the metallic structure is not indefinitely fine (see p. 171), which has occasionally (p. 172) a bearing on the matter.

Initial Stress.—Material is sometimes severely strained in the process of manufacture (e.g. cold drawing), and a system of self-restrained stresses are left in the material. These would be superposed upon the stresses due to any loading, and in important machine parts the material should be freed from initial stresses by suitable heat treatment.

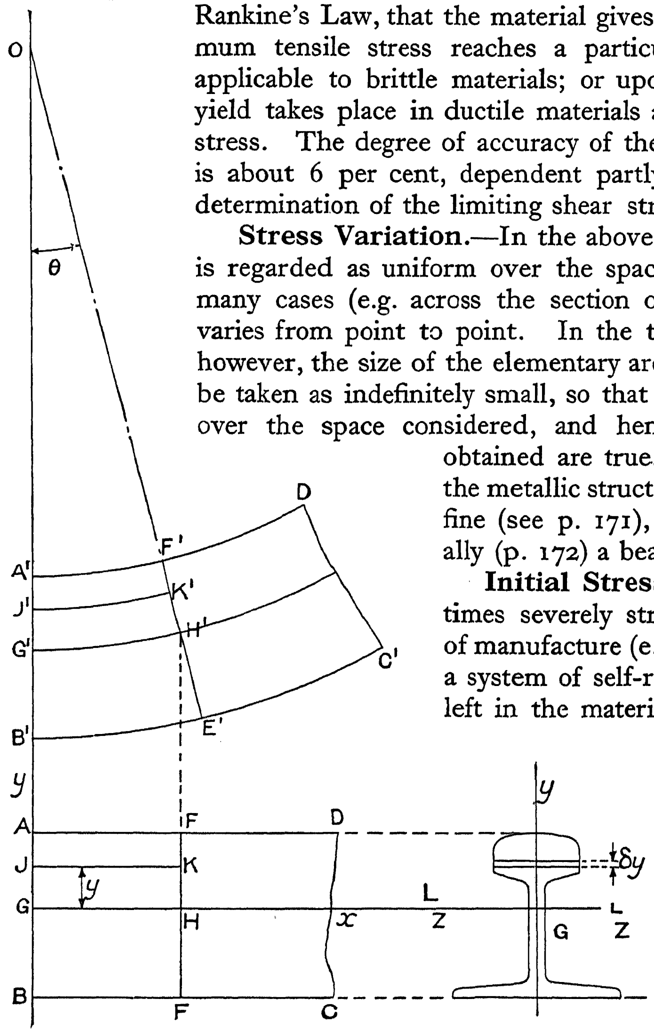

Fig. 14

Bending.—The general force action transmitted across any section of a body has been analysed (p. 94 in "Applied Mechanics") into an axial force (already treated), a bending moment, and a shear force. A part regarded as transmitting or resisting a bending moment is usually termed a beam, and the stress arising can be sufficiently accurately determined from considerations of the strain which arises from the action of a

pure couple only, although usually the bending moment varies along the beam. If the beam be of uniform cross section, symmetrical about the plane of bending, as shown in fig. 14, it will bend uniformly, since the bending moment is uniform, and all longitudinal "fibres" will bend into circles having a common axis, all plane cross sections straining into planes passing through this axis. Thus a beam ABCD, initially straight, under a pure bending moment M, will bend so that its central longitudinal section takes the form shown much distorted at A′B′C′D′, strained cross sections such as A′B′ and E′F′ intersecting in an axis at O. If EHF be a section close to AB, strained to position E′H′F′ at an angle θ to A′B′, there will be some fibre GH (usually within the material) whose length will be unaltered, those above it (as sketched) being shortened, and those below elongated. The plane through GH perpendicular to the longitudinal section is termed the neutral plane, and it intersects the plane of the cross section AB in the "neutral axis".

A fibre JK at a distance y from the neutral axis contracts an amount HK.θ or HK.$\frac{G'H'}{R}$, where R is the radius of the strained neutral plane, so that the strain is $\frac{y}{R}$. The longitudinal stress p therefore is constant over the parts of the cross section when y is constant and

$$p = \frac{y}{R}.E.$$

The maximum stress p_o will occur at the part where y is a maximum, y_o —either at A or B. Thus

$$\frac{p}{y} = \frac{p_o}{y_o} = \frac{E}{R}.$$

Position of Neutral Axis.—Taking GH, GAO, and the neutral axis GL as axes of co-ordinates x, y, and z, the force on an elementary strip δy of the cross section is $pz\delta y$, or $\frac{p_o}{y_o}.z\delta y.y$. The total axial force is therefore $\frac{p_o}{y_o}\int zydy$, taken over the section. If there is no axial force this integral, which is equivalent to $\frac{p_o}{y_o}A\,\bar{y}$ (A being the area and $\bar{y}$ the ordinate of the "c.g." of the cross section), is zero, so that $\bar{y}$ is zero, showing that the neutral axis passes through the "centre of gravity" of the section.

Stress Moment.—The moment (M) transmitted is equivalent to the total moment of the stress forces or

$$M = \int pz\delta y.y = \frac{p_o}{y_o}\int y^2 zdy = \frac{p_o}{y_o}I,$$

where I is the "moment of inertia" of the cross section about the neutral axis. Thus the maximum stress caused in the beam is

$$p_o = \frac{My_o}{I} = \frac{M}{Z},$$

where Z, termed the modulus of the section, depends upon the section only, so that tables of its value for given sections can be compiled.

The above equations can conveniently be written in the form

$$\frac{p}{y} = \frac{M}{I} = \frac{E}{R},$$

p being the stress corresponding to the value of y.

Stress Calculation.—Suppose that a beam is to carry a distributed load of $1\frac{1}{2}$ tons over an effective span of 6 ft., so that the maximum bending moment, occurring at the centre, is 13·5 in.-tons, the weight of the beam itself being omitted or considered as included in the $1\frac{1}{2}$ tons.

(1) If a steel bar of $2\frac{1}{2}$ in. square section, weighing $23\frac{1}{2}$ lb. per foot run be used, since $Z = \frac{1}{6}bh^2 = 2{\cdot}60$ in. units, the maximum fibre stress is

$$p = \frac{13{\cdot}5}{2{\cdot}60} = 5{\cdot}2 \text{ tons per square inch.}$$

(2) If a standard I section $4\frac{3}{4}$ in. × $1\frac{3}{4}$ in. weighing $6\frac{1}{2}$ lb. per foot run be used, the stress will be

$$p = \frac{M}{Z} = \frac{13{\cdot}5}{2{\cdot}8} = 4{\cdot}83 \text{ tons per square inch,}$$

the value of Z being either calculated or taken from the tables.

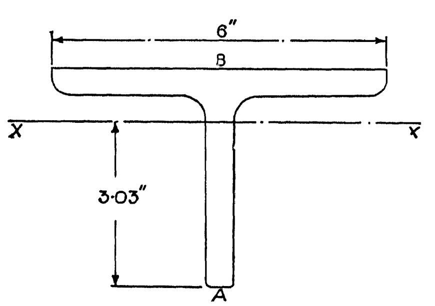

Fig. 15a

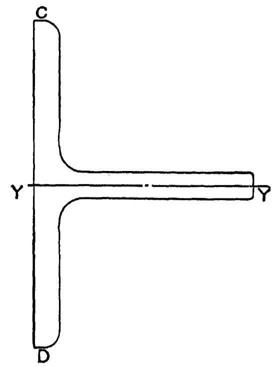

Fig. 15b

(3) If a standard 6 in. × 4 in. × $\frac{1}{2}$ in. T section weighing 16·22 lb. per foot run be used in the position given in fig. 15*a*, where the neutral axis XX through the c.g. is 3·03 from the point A, the stress at A is given by

$$p = \frac{13{\cdot}5 \times 3{\cdot}03}{6{\cdot}07} = 6{\cdot}75 \text{ tons per square inch tension,}$$

and at the upper surface B it will be

$$p = \frac{13{\cdot}5 \times (4 - 3{\cdot}03)}{6{\cdot}07} = 2{\cdot}16 \text{ tons per square inch compression.}$$

If the beam be placed as shown in fig. 15*b*, the stress (equal) at C and D will be

$$p = \frac{13{\cdot}5 \times 3}{8{\cdot}62} = 4{\cdot}71,$$

where the moments of inertia about XX and YY are taken from the tables.

(4) If the 6 in. × 4 in. × ½ in. standard angle weighing 16·15 lb. per foot run, shown in fig. 16, be substituted for the above standard T, the difference being little except that the vertical part is at one side instead of central, the principal axes are UU and VV, at an angle of 23·5°, given, with other data taken from the tables, in fig. 16. The usual procedure is to superpose the stresses arising from the components (M cosθ and M sinθ) of the bending moment M = 13·5. Thus the stress at the corner A, being the sum of the tensions, is—if the line BC be placed horizontally—

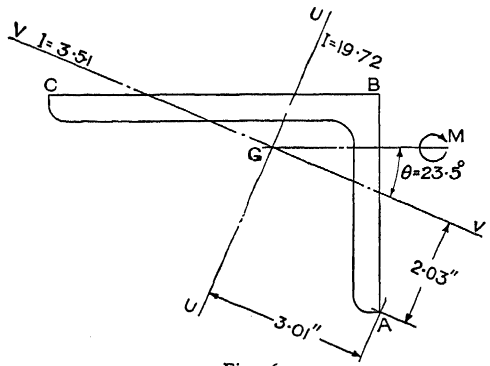

Fig. 16

$$p = \frac{\mathrm{M}\cos\theta . y_1}{\mathrm{I_{vv}}} + \frac{\mathrm{M}\sin\theta . y_2}{\mathrm{I_{uu}}} = 13{\cdot}5\left\{\frac{2{\cdot}03 \times {\cdot}917}{3{\cdot}51} + \frac{3{\cdot}01 \times {\cdot}399}{19{\cdot}72}\right\}$$
$$= 8{\cdot}17 \text{ tons per square inch.}$$

At corners B and C the stresses will be the difference of the stresses produced by M cosθ and M sinθ, as these produce there stresses of an opposite sign.

General Considerations.—From (1) and (2) above, the advantage of distributing the metal to resist a bending moment is evinced, sections of I or hollow section being generally light and economical. The result in (3*a*), where the c.g. is unequally distant from the top and bottom of the section, suggests that where the metal (e.g. cast iron) is unequally strong in tension and compression, the flanges of a beam should be of unequal area. In case 3*b* the beam section is unsymmetrical about the vertical axis; under the strain the plane sections warp, but the calculation is sufficiently accurate. The same applies to (4) when the components of M act; it will be noticed that the beam will deflect under the load not only downwards, but also to one side horizontally. In the theory given the fibres are supposed to stretch independently of one another, while actually they are connected, so that the lateral contraction and expansion under the longitudinal stresses has not been considered. The effect of these is to contract the width of the tension side of a beam and increase that of the compression side. Under this action the

cross section of a rectangular beam becomes convex on the top where the longitudinal section is hollow. The longitudinal stress is only affected slightly, but the stiffness is increased, and is appreciable when the section (as in some springs) is wide and shallow. The allowance may be made by considering the value of E as raised, the limiting amount being $\frac{1}{1-\sigma^2}$ E.

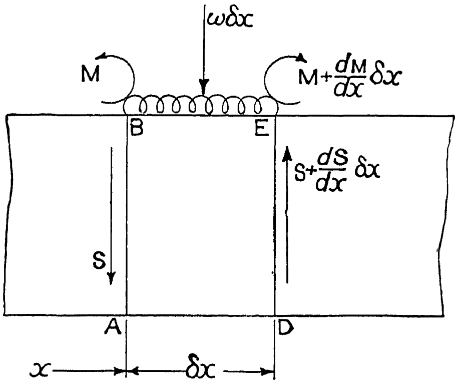

Fig. 17

Shear Stress in Beams.—Taking a cross section AB, at x, of a beam and an adjacent section DE at $x + \delta x$, the shear force and bending moment at x being S and M, the actions on the part ABED are, using the customary notation, as given in fig. 17. Resolving vertically and taking moments gives the equations:

$$\frac{dS}{dx} = w$$

and $$\frac{dM}{dx} = S,$$

so that $$\frac{d^2M}{dx^2} = w.$$

The shear stress over the section AB is not distributed uniformly, being clearly zero at A and B. The value q of the shear stress at a point C of the section, which is shown in fig. 18, distant y_1 from the neutral axis G, can

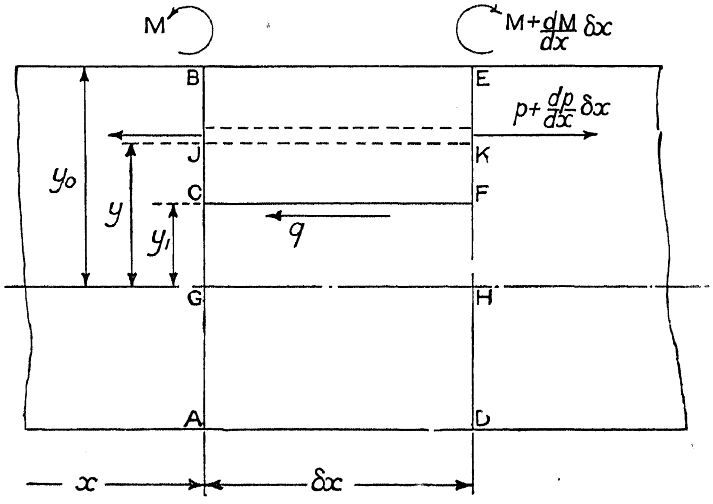

Fig. 18

be found by considering the effect of the associated horizontal shear stress on the section CF. The horizontal force at J on AB, where GJ $= y$, on an elementary strip of the section of depth δy and width z is $pz\delta y$ or $\frac{My}{I}z\delta y$, so that the difference of the horizontal forces on the two ends of JK of an element of the beam is $\frac{dM}{dx}.\delta x.\frac{yz}{I}.\delta y$. The summation of this being equal to the horizontal force on CF (of width z_1), due to shear stress q, gives

$$qz_1\delta x = \int_{y_1}^{y_0}\frac{dM}{dx}.\delta x\frac{yz}{I}dy,$$

$$\text{or} \qquad q = \frac{S}{Iz_1}\int_{y_1}^{y_0} yzdy,$$

where y_o is the ordinate GB.

If the section is rectangular, $q = \frac{3}{4}\cdot S\,\frac{y_0^2 - y_1^2}{z_1 y_0^3}$, so that the distribution is parabolic, being zero when $y_1 = y_o$ and $\frac{3}{2} \times$ (mean value) at the centre.

In I sections practically the whole of the shear stress is carried by the web, over which it is nearly equally distributed.

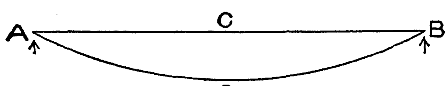

Fig. 19

Deflection of Beams.—If $\frac{M}{I}$ is constant in a beam, the value of R is constant and the beam ACB bends into a circular arc ADB (fig. 19), its centre C deflecting to D where CD $\times$ (2R $-$ CD) $=$ CB^2.

Writing u for CD, l for AB, and neglecting CD in comparison with the radius 2R $\left(= \frac{2EI}{M}\right)$, this gives,

$$u = \frac{l^2}{8}.\frac{M}{EI}.$$

Although M/I is seldom constant (except for built-up girders of constant depth where the lengths of the flange plates are stopped off, keeping the flange stress approximately constant), nevertheless this expression may be used for rough estimates.

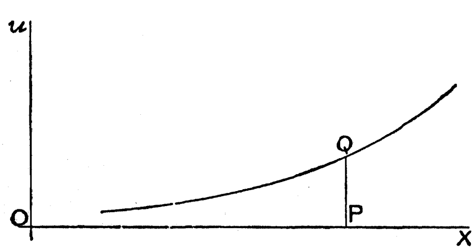

Fig. 20

If, as indicated in fig. 20, the axis of the beam is taken as that of x and the small vertical deflections as u, the value of $\frac{1}{R}$ is $\frac{d^2u}{dx^2}\left\{1 + \left(\frac{du}{dx}\right)^2\right\}^{-\frac{3}{2}}$ which may be taken as $\frac{d^2u}{dx^2}$, as in all practical

cases the slope $\frac{du}{dx}$ is so small that its square may be neglected. When the loading is known the value of M is known at all points along the beam, and the deflection can be found from the equation

$$\frac{d^2u}{dx^2} = \frac{M}{EI}.$$

Cantilevers.—Taking a cantilever of uniform section fixed at A and projecting a length l to B (fig. 21), where it carries a load W, the bending

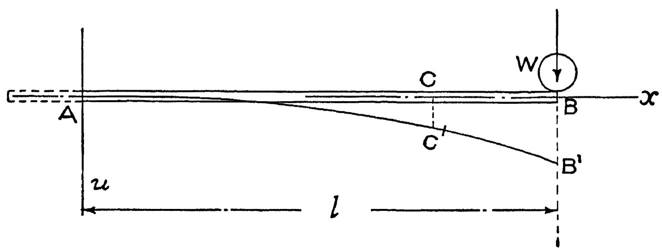

Fig. 21

moment at any point C (x) is W ($l - x$), measuring x from A and the deflection u vertically downwards. The equation then is

$$EI\frac{d^2u}{dx^2} = W(l - x).$$

Integration gives $$EI\frac{du}{dx} = W\left(lx - \frac{x^2}{2}\right) + A,$$

and since $\frac{du}{dx} = 0$ when $x = 0$, the value of the constant A is zero. The second integration gives $EIu = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + B$, and since $u = 0$ when $x = 0$ the constant B is also zero, so that the equation to the curve into which the beam axis deflects is found. The end deflection, putting $x = l$, is

$$u = \frac{Wl^3}{3EI}.$$

If the cantilever carry a uniformly distributed load of w per unit length instead of the single end load the value of M is $\frac{w}{2}\left(l - x\right)^2$, so that

$$EI\frac{d^2u}{dx^2} = \frac{w}{2}\left(l - x\right)^2.$$

Integration gives $$EI\frac{du}{dx} = \frac{w}{2}\left(l^2x - \frac{lx^2}{1} + \frac{x^3}{3}\right) + A.$$

The constant is zero, and $$EIu = \frac{w}{2}\left(\frac{l^2x^2}{2} - \frac{lx^3}{3} + \frac{x^4}{12}\right) + B,$$

where the constant is again zero. The terminal deflection is now

$$u = \frac{w}{2EI}\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{12}\right)l^4 = \frac{wl^4}{8EI}.$$

Beams.—In a similar manner, or by considering a beam as analysed into a pair of cantilevers, the deflection for a uniform beam, loaded with a single load W at the centre, is found to be $\frac{1}{48}\frac{Wl^3}{EI}$, and for one loaded uniformly $\frac{5}{384}\frac{wl^4}{EI}$.

Indirect Method.—Another mode of treatment of this equation is to multiply by x,

$$x\frac{d^2u}{dx^2} = \frac{Mx}{EI},$$

when the integration (by parts) gives

$$\left[x\frac{du}{dx} - u\right]_c^d = \frac{1}{E}\int_c^d \frac{Mx}{I}dx.$$

The limits are then to be so selected that the left-hand side of the equation reduces so as to give the desired information. If the deflection is needed, x is to be zero at one of the limits, u and $\frac{du}{dx}$ at the other. It will be noticed that the right-hand integral is the moment of the area of the curve $\frac{M}{I}$ (or bending moment curve if I be constant) between the limits, taken about the origin. Thus the deflection can be at once obtained if the bending moment curve be simple.

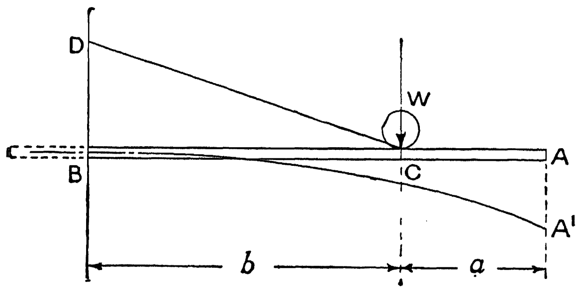

Fig. 22

Thus in the case of the cantilever AB loaded with W at C (fig. 22), the bending moment curve is CD, where BD = Wb. Taking the origin at A, the left-hand side reduces to the end deflection u = AA′, and is equal to the moment of the area of the figure CDB taken about A, divided by EI, so that

$$u = \frac{1}{2EI}.Wb^2(a + \tfrac{2}{3}b).$$

Beams with Fixed Ends.—Suppose that a beam has its ends A and B firmly fixed and carries a uniform load w per unit length. If the ends were simply supported the bending moment M would be zero at the ends and $\frac{wl^2}{8}$ at the centre, the bending moment curve being the parabola shown in fig. 23. The fixing prevents the inclination of the ends causing an additional

moment μ, which will be uniform along the beam, and this, being superposed on the bending moments of the freely supported beam, will give the actual moment transmitted across a section. Thus the actual bending moments are given by the ordinates across the shaded areas. Now

$$\mathrm{EI}\frac{d^2u}{dx^2} = \mathrm{M} - \mu,$$

so that $$\left[\mathrm{EI}\frac{du}{dx}\right]_o^l = \int_o^l(\mathrm{M} - \mu)dx;$$

but $\frac{du}{dx}$ is zero both at A and B, and therefore $\int_o^l \mathrm{M}dx = \mu l$, or μl = area of the parabolic curve $= \frac{2}{3}.\frac{wl^2}{8}.l$.

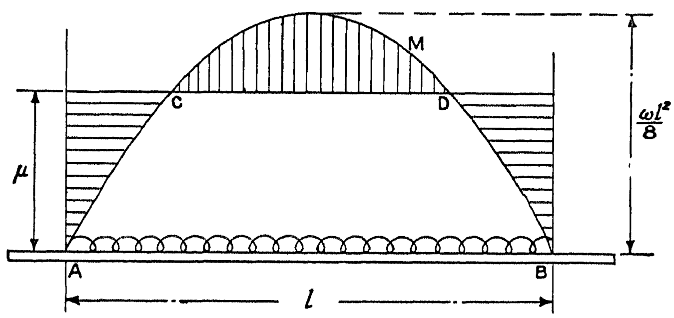

Fig. 23

Hence $\mu = \frac{1}{12}wl^2$, so that the maximum moment is now at the fixing points and is $\frac{2}{3}$ of the maximum moment in the freely supported case, while the central moment is half the amount of the fixing moments. The sign of the transmitted moment changes at the points C and D, so that the beam is convex upwards at the ends and concave upwards in the centre.

Deflections by Graphical Methods.—Since the deflection u is related to the bending moment M by the equation $\frac{d^2u}{dx^2} = \frac{\mathrm{M}}{\mathrm{EI}}$, while M is related to the loading per unit run w by the similar equation $\frac{d^2\mathrm{M}}{dx^2} = w$, the deflection can be found by applying to a curve giving the value of M (or $\frac{\mathrm{M}}{\mathrm{I}}$ if I be variable) those methods (pp. 99, 100, 104, "Applied Mechanics") whereby M can be found from the curve for the loading w.

Calculation.—Suppose that a steel beam, section $3\frac{1}{2}$ in. wide × 2 in. deep of 54 in. effective span, carries a central load W. If the permissible stress be 20,000 lb. per square inch the corresponding load would be 3460 lb., under which the deflection would be given by

$$u = \frac{\mathrm{W}l^3}{48\mathrm{EI}} = \cdot 16 \text{ in.},$$

taking E as 30,000,000 lb. per square inch.

If the thickness had been $\frac{1}{2}$ in. instead of 2 in. the load for the same stress would have been 216 lb., being to 3460 in the ratio of the depth of the beam squared. The deflection will be inversely proportional to the thickness of the beam (the maximum stress being given) and will therefore be ·32 in.

In the above consideration of deflection the effect of the shear stress has been omitted, as it is only of appreciable magnitude (relatively to that due to bending) in short beams.

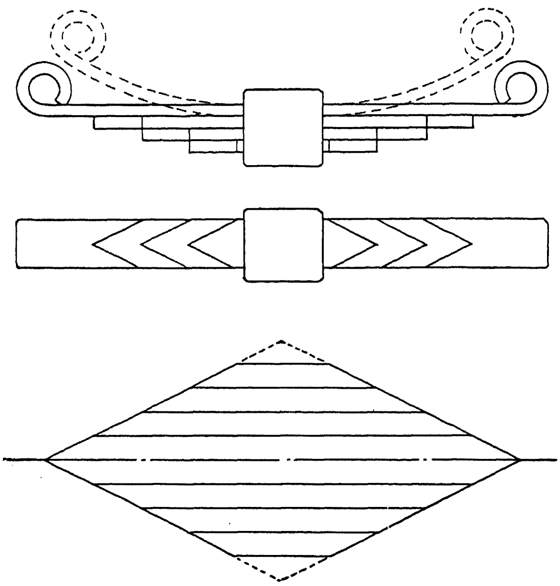

Fig. 24

Springs. — Springs, which essentially must give considerably under the load, are frequently made of such thin pieces acting as beams, a number of leaves being employed in order to carry the requisite load.

The energy which can be stored in a spring without the allowable stress being exceeded is usually termed its resilience (although this term also defines a property of material), and is a matter of importance. In a beam under a bending moment, the strain energy of length δx at a section is the sum of the energy stored in the various layers into which the beam may be divided by planes parallel to the neutral surface. Using the customary notation, the total resilience is

$$E = \int \frac{p^2}{2E} . z dy dx = \frac{p_o^2}{2E y_o^2} . \int y^2 z dy dx,$$

the integral being taken over the cross section. Hence

$$E = \frac{p_o^2}{2E} . \frac{Ak^2 dx}{y_o^2} = \frac{p_o^2}{2E} . \frac{k^2}{y_o^2} \times \text{volume}.$$

Thus a certain fraction of the possible resilience under direct stress only can be stored, depending upon the shape of the cross section. This is $\frac{1}{3}$ for rectangular and $\frac{1}{4}$ for circular sections. Effective employment of material therefore depends upon p_o being constant along the spring when loaded, so that a single beam or cantilever cannot be employed efficiently unless the section varies from point to point of its length.

In leaf springs this can be arranged by employing leaves of the same section and tapering the ends of each, so that the whole may be considered as parts cut off from a rhombus, as is indicated in fig. 24. If the leaves fit

in the working position (here supposed to be when they are straight) they must be equally bent originally as indicated by the broken lines. The exterior and interior radii being different a small space exists between the leaves at their middle points; the leaves are nipped together by a buckle, which closes this gap and introduces some initial stress into the leaves.

The deformation in many springs is so great that the deflection is not exactly proportional to the load, and its effect should be allowed for if close calculation is required. It should also be noted that spring leaves are rolled hollow in order to retain some lubricant, so that the section is less than the nominal rectangle.

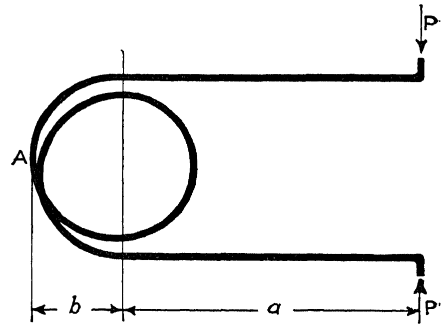

Fig. 25

Spiral springs, and those made of a few coils of (usually square) wire loaded at the ends, as shown in fig. 25, act by bending. The greatest stress in this case is at A, where each coil carries the moment $P(a + b)$. A reasonably close approximation to the deflection is obtained by considering the arms as cantilevers and the n central coils as under a uniform moment Pa, giving

$$u = \frac{Pa^2}{EI}(\tfrac{2}{3}a + 2\pi nb).$$

Axial Load and Bending.—Since both axial loads and bending produce longitudinal stress, these are to be superposed when such loading occurs. The usual cases are where a load parallel to the axis of a member is applied eccentrically, that is not through the c.g. of the cross section. Thus the load P, applied as shown in fig. 26, is equivalent to an axial load P and a bending moment Pc, which are transmitted by the section AB, giving, at a distance y from the neutral axis, a stress p where

$$p = \frac{P}{A}\left(1 + \frac{cy}{k^2}\right).$$

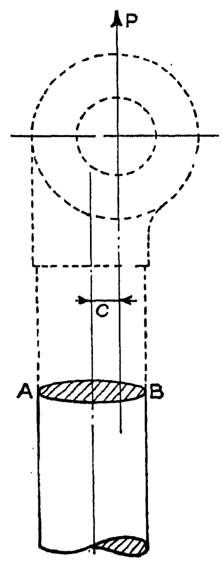

Fig. 26

The load P, applied along a line through the c.g. of the cross section AGB, produces a uniform stress $\frac{P}{A}$, which may be (see fig. 27) represented by the line CDE parallel to AB. As P is displaced the effect is to tilt the line, the ordinates to which give the stress, into a position such as FDH, the angle of tilt increasing with the displacement of the force P. When the tilting is such that the line is ADJ, there is no

stress at A, and when P is further displaced the stress (tension as sketched) changes sign near A, the line taking a position such as KDL. With unsymmetrical sections and sufficient eccentricity of loading the stress AK may exceed BL. In masonry no tension is permitted, so that ADJ is the limiting case, and for rectangular section this gives

$$1 + c\frac{y}{k^2} = 0, \qquad \text{or} \qquad c = -\frac{1}{6}h,$$

or the displacement c of the force is confined to the "middle third" of a wall.

Struts.—While a tension member tends to preserve its straightness under its load, a compression member is liable to buckle, a long strut being

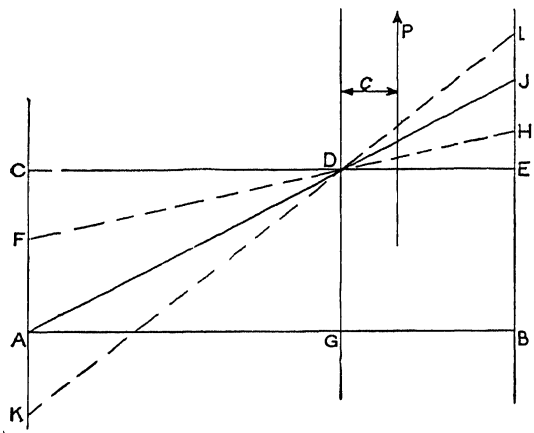

Fig. 27

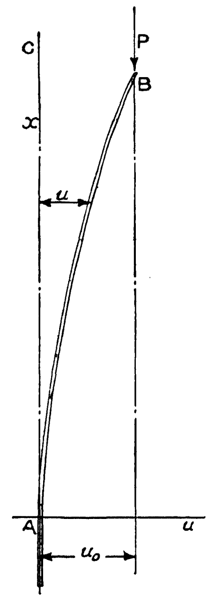

Fig. 28

unsafe under a load which a short member of the same section can carry safely. The buckling load of a strut depends upon the manner in which the ends are secured. Suppose the strut shown in fig. 28 to have the end A rigidly fixed, and to be loaded at the end B, which is free, by a force P coaxial with the strut, and that the end B be then displaced by a small amount u_0 from the initial centre line AC. Then the strut has a bending moment at each cross section varying from zero at B to Pu_0 at A, and the curvature may increase, leading to instability and collapse, which thus depends primarily on elastic resistance to initial displacement and not upon the strength of the material. Taking AC as axis of x, the equation for bending becomes

$$\frac{d^2u}{dx^2} = \frac{P(u_0 - u)}{EI}$$

where u is the displacement at x. Solving gives

$$u = u_0 - A\cos\sqrt{\frac{P}{EI}}.x + B\sin\sqrt{\frac{P}{EI}}.x,$$

in which, since $u = 0$ when $x = 0$, the value of A is u_o, and, since $\frac{du}{dx} = 0$ when $x = 0$, the value of B is zero; hence

$$u = u_o\left(1 - \cos\sqrt{\frac{P}{EI}}.x\right). \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

At the free end $x = l$ and $u = u_o$, so that $\cos\sqrt{\frac{P}{EI}}l = 0$. From which

$$\sqrt{\frac{P}{EI}}l = \frac{\pi}{2},\ \frac{3}{2}\pi,\ \frac{5}{2}\pi,\ \text{\&c., \&c.},$$

the least value of P being given by the first, so that

$$P = \frac{\pi^2 EI}{4l^2}.$$

Under the load the strut will bend to the curve given by (1), straightening itself under a lesser load, but collapsing under a higher.

The higher values $\frac{3}{2}\pi$, $\frac{5}{2}\pi$, indicate that the strut will sustain higher loads provided that it bends into several undulations as in fig. 29, instead of into a single curve as in fig. 28. Considering the strut shown in fig. 29 as loaded with P at F, a strut of the length AE, where E is a similar point to A, if fixed in direction at each end, would support the same load as a strut AF with the bending shown, so that if $AE = l_1$, the load would be

$$P_1 = \frac{\pi^2 EI}{4(\frac{5}{4}l_1)^2}.5^2 = \frac{4\pi^2 EI}{l_1^2}.$$

Also, if the strut AD be loaded at D, the portion BD hinged at both ends would carry the same load as the strut ABCD with the bending shown. Putting $BD = l_2$ thus gives

$$P_2 = \frac{\pi^2 EI}{l_2^2}.$$

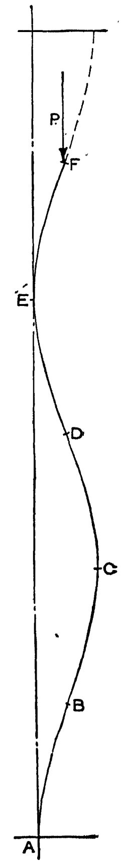

Fig. 29

Euler's Formula.—This expression, which is due to Euler, gives the buckling load for a strut which is hinged at each end, and is the most usual type; the load for the same strut is four times as great if it be fixed at each end, and a quarter of this amount if fixed at one end and free at the other.

The stress (uniform) over the cross section (area A) of the strut is therefore

$$p = \pi^2 E\left(\frac{k}{l}\right)^2,$$

where k is the least—as the strut is to be safe against buckling in any plane—radius of gyration of the cross section. For very short lengths this will give a value to p greater than the crushing strength of the material, so that two conditions must be satisfied for security in a strut.

Rankine's Formula.—These were combined by Professor Rankine into the working formula

$$P = \frac{f_c A}{1 + a\left(\frac{l}{k}\right)^2},$$

which agrees with the Euler formula for long struts and gives the direct crushing load for short struts, f_c being the strength of the material under compression and a being $\frac{f_c}{\pi^2 E}$, and therefore a constant for the material. In practice the values of the constants are modified to meet effects of such deviations from axial loading as are unavoidable. It is further to be noted that ideal fixing of an end does not occur in practice, and imperfections in fixing should be allowed for. In built-up struts the stability of each portion, in addition to the stability of the strut as a whole, is to be considered.

If the strut is loaded laterally (giving a bending moment) in addition to the end load P, a close approximation to the stress produced is obtained by finding the deflection u under the lateral loading only, and taking the bending moment as the sum of Pu and that due to the lateral loading only.

The stability of the web of a girder may be estimated by considering a diagonal strip inclined at 45°, of unit width, supposing it to be loaded at its ends with the force due to the compressive principal stress of the shear.

Collapse of Thin Cylinders.—In the material of a thin cylindrical vessel under external pressure there is a circumferential compressive stress given by the same equation as that for the tensile stress due to internal pressure, $f_1 t = p_o r$, but the tube is liable to collapse. Long tubes collapse into two lobes (the section being somewhat as the figure 8), but shorter tubes into three or more owing to the restraint of the ends. Considerations of the equilibrium of the nearly circular ellipse into which the cross section distorts under pressure give as that causing collapse of a long tube

$$p = \frac{2E}{1 - \sigma^2}\left(\frac{t}{d}\right)^3,$$

where d and t are the diameter and thickness of the tube. Further calculations and experiment suggest that below a certain length (proportional to $d^{\frac{3}{2}}t^{-\frac{1}{2}}$) the collapsing pressure may be taken as inversely proportional to the length. Owing to such causes as want of uniformity of tube thickness or radius, the collapsing pressure will be lower than that given above, the factor being about 50,000,000 lb. per square inch.

Thick Cylinders.—In discussing the stresses caused in a tube under pressure, the tube has been supposed so thin that the stress is sensibly uniform

over the thickness, but the variation is considerable when the ratio of thickness to radius is not small. In fig. 30 is shown the cross section of a cylinder, ABC being the exterior and DEF the interior surface, the latter being exposed to a pressure p_o. The cylinder axis is at O, and the internal and external radii being r_1 and r_2. The fluid pressure will produce a normal (radial) compressive stress varying from p_o to zero (or atmospheric) pressure at the exterior, combined with a circumferential tensile stress. Taking unit axial

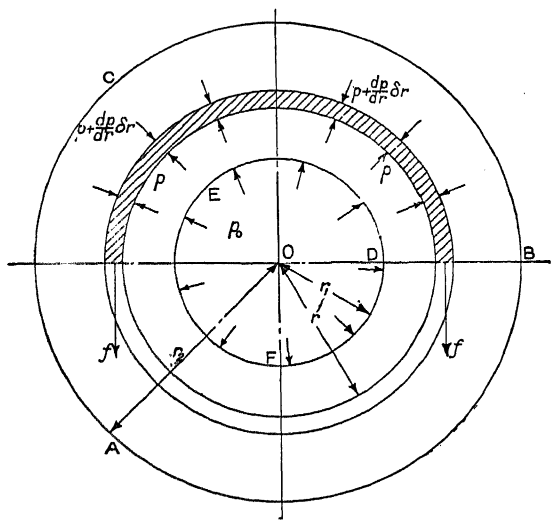

Fig. 30

length, the condition for equilibrium of the semicircular strip, radii r and $r + \delta r$, under radial compressive stresses p and $p + \frac{dp}{dr}.\delta r$, and the tangential tensile stress f at the ends of the strip, gives

$$2(p + \frac{dp}{dr}\delta r)(r + \delta r) - 2pr + 2f\delta r = 0,$$

$$\text{or } f + p + r\frac{dp}{dr} = 0.$$

It is usually taken that any longitudinal stress acting normally to the section (fig. 30) is uniformly distributed, and since the longitudinal strain must be constant all over the section in a long tube, the amount contributed to it by the stresses f and p must be constant all over the section, this amount being $\frac{\sigma(f-p)}{E}$. Hence $f - p = 2A.$

Substitution in the former equation gives

$$r\frac{dp}{dr} + 2(p + A) = 0,$$

giving on integration $\qquad \log(p + A) + \log r^2 = B,$

$$\text{or } p + A = \frac{C}{r^2},$$

the constants A and C being found from the values of p at the exterior and interior surfaces. Hence in the case given

$$A = \frac{C}{r_2^2} \quad \text{and} \quad p_o + A = \frac{C}{r_1^2}$$

so that
$$C = p_o \frac{r_1^2 r_2^2}{r_2^2 - r_1^2} \quad \text{and } A = p_o \frac{r_1^2}{r_2^2 - r_1^2},$$

and
$$p = p_o \frac{r_1^2 r_2^2}{r_2^2 - r_1^2}\left(\frac{1}{r^2} - \frac{1}{r_2^2}\right),$$

$$f = p_o \frac{r_1^2 r_2^2}{r_2^2 - r_1^2}\left(\frac{1}{r^2} + \frac{1}{r_2^2}\right).$$

Both the circumferential and radial stresses are a maximum at the internal surface, where the values are

$$f = p_o \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} \qquad \text{and} \qquad p = p_o.$$

The maximum shear stress for this pair of principal stresses (in planes cutting the inner surface at 45°) is

$$p_o \frac{r_2^2}{r_2^2 - r_1^2}$$

Thus if the shear stress q in a steel cylinder be limited to 9000 lb. per square inch, if the internal diameter be half the external, the internal pressure must not exceed the value given by

$$p_o = \frac{r_2^2 - r_1^2}{r_2^2} q = 6750 \text{ lb. per square inch,}$$

and if the internal diameter were one-third of the external, this pressure could be raised to 8000 lb. per square inch.

To increase the resistance of cylinders (such as guns) to internal pressure, without increasing the ratio of the radii, an initial compressive stress may be set up in the interior at the expense of an initial tensile stress at the exterior. This may be done by building up the cylinder of a series of tubes, shrinking each on in turn, the initial radii of the tube surfaces being arranged to produce the desired initial (and therefore final upon the application of the internal pressure) state of stress. The same end is attained by winding wire under tension on the inner tube, an outer tube being finally fitted. An entirely different method is to raise the yield point of the material at the inner surface by the application of high internal pressure, which furthermore leaves an initial state of stress throughout the cylinder.

Forced Fits.—An important case of such stress distribution is that of the hub part of a forced fit. Since there is a uniform radial pressure p_o over the shaft surface, there is uniform (two-dimensional) compressive stress p_o throughout the shaft, giving a radial contraction $(1 - \sigma)\frac{p_o}{E}r_1$. The increase of hole diameter, being at the same rate of increase as the circumference of the hole $\left(\text{which is } \frac{f + \sigma p_o}{E}\right)$, is $\frac{f + \sigma p_o}{E_1}d_1$, taking a different value of E as the materials may be different. Hence, if x be the diametral allowance for the fit,

$$(1 - \sigma)\frac{p_o}{E} + \frac{f + \sigma p_o}{E_1} = \frac{x}{d_1}.$$

For hub and shaft of the same material this gives $f + p_o = \frac{x}{d_1}E$. Now $\frac{1}{2}(f + p_o)$ is the shear stress q, so that, accepting the writer's rule that this is the limiting factor (to an accuracy sufficient for engineering purposes), permissible allowances for ordinary steel forced fits are given by

$$x = \frac{2q}{E}d_1.$$

For hubs of brittle material (e.g. cast iron, hardened steel) f is the limiting factor; the allowance is (eliminating p_o between the equation for x and the equation $f = p_o\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}$) given by the equation

$$x = \left\{\left(\frac{1 - \sigma}{E} + \frac{\sigma}{E_1}\right)\frac{d_2^2 - d_1^2}{d_2^2 + d_1^2} + \frac{1}{E_1}\right\}fd_1.$$

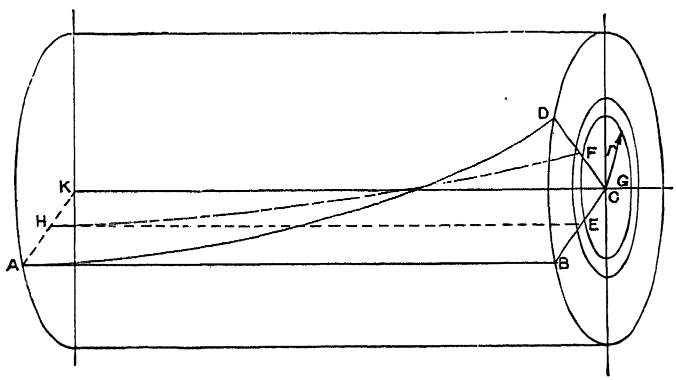

Fig. 31

Torsion of Shafts.—In a thick tube or solid shaft of circular section, the application of a torque produces a shearing stress varying uniformly from zero at the axis to a maximum q_o at the outside. Suppose, as shown in fig. 31, that a shaft of outer radius r_2 and inner r_1 be fixed at A, and a cross

section BCD be taken of which C is the centre, and let the generator initially at AB strain to the helix AD. Then an elementary tube EFG of radius r will strain so that E twists to F, the initial generator HE of this tube becoming the helix HF. The strain angle θ of this helix is then given by

$$l\phi = r\theta,$$

where AB $= l$ and angle BCD $= \theta$. If ϕ_o be the strain angle at the exterior, $l\phi_o = r_2\theta$, so that eliminating θ

$$\phi = \frac{r}{l} \cdot \frac{l}{r_2}\phi_o = \frac{r}{r_2}\phi_o,$$

and the shear stress q in the elementary tube EFG will be given by

$$q = C\phi = C\frac{r}{r_2}\phi_o = \frac{r}{r_2}q_o,$$

and each elementary tube will contribute (see p. 142) the part $\delta L = 2\pi r^2 \delta r q$ to the total torque L, which accordingly is

$$L = 2\pi\int_{r_1}^{r_2} qr^2 dr = 2\pi\int_{r_1}^{r_2} q_o\frac{r^3}{r_2}dr = \frac{\pi}{2}q_o\frac{r_2^4 - r_1^4}{r_2} = \frac{\pi}{16}q_o\frac{d_2^4 - d_1^4}{d_2},$$

where d_1 and d_2 are the internal and external diameters.

If the shaft be solid this gives $L = \frac{\pi}{2}q_o r^3$, where r is the shaft radius, or $\frac{\pi}{16}q_o d^3$ where d is the shaft diameter. The twist θ of the shaft in a length l will be given, as for a thin tube (p. 142), by the equation

$$r\theta = l\phi_o = l\frac{q_o}{C}$$

The energy stored in the shaft will be $\frac{1}{2}L\theta$, which, in the case of a solid shaft, becomes $\frac{1}{2} \times \frac{\pi}{2}q_o r^2 l\frac{q_o}{C}$ or $\frac{1}{4}\frac{q_o^2}{C} \times$ volume of shaft.

Thus the torque which a hollow shaft 9 in. external and 5 in. internal diameter will carry without the shear stress exceeding 9000 lb. per square inch is given by

$$L = \frac{\pi}{16} \times 9000 \times \frac{6561 - 625}{9 \times 12} = \text{lb.-ft.} = 97{,}000 \text{ lb.-ft.}$$

A solid shaft to carry the same torque, with the same stress limit. would have a diameter d given by

$$\frac{\pi}{16}q_o d^3 = \frac{\pi}{16}q_o\frac{d_2^4 - d_1^4}{d_2}$$

$$\text{or } d^3 = \frac{6561 - 625}{9} = 660$$

$$\text{so that } d = 8{\cdot}71 \text{ in.}$$

The ratio of the weight of the solid shaft to the hollow one would be given by

$$\frac{d^2}{d_2^2 - d_1^2} = \frac{75 \cdot 8}{56} = 1 \cdot 35.$$

Square Shafts.—When the section of a shaft is other than circular the conclusions drawn above from the symmetry of the cross section no longer hold, and the section becomes warped. The problem was investigated by St. Venant, who found that the maximum shearing stress generally occurs at that part of the surface which is nearest to the axis, and calculated the stress in a number of cases. For a shaft of rectangular section $a \times b$ the maximum stress occurs at the middle of the longer side a, and is given approximately by the equation

$$L = \frac{ab^2}{3 + 1 \cdot 8b/a} \cdot q,$$

which becomes in the case of a square shaft $L = 0 \cdot 208a^3q$; and the angle of twist by

$$\theta = \frac{40k^2}{A^3} \cdot \frac{Ll}{C},$$

where k is the radius of gyration of the section about the axis of the shaft.

Re-entrant Corners and Cracks.—He also found that at re-entrant sharp corners the stress theoretically became infinite, so that yielding must take place over a small area of the section. Thus a sharp-edged keyway is a source of danger even although the conditions are quite different if the key is a complete fit. Also any longitudinal cracks or scratches may be of an importance depending upon the nature of the material, and whether or no the stress be frequently repeated (p. 171).

Shafts under Torsion and Thrust.—When a shaft—such as a propeller shaft between the screw and the thrust block—sustains an axial force P in addition to transmitting a torque L, the state of stress is to be found by combining the stresses which would be produced by the actions separately, that is the normal (uniform) stress $\frac{P}{\pi r^2}$ with the shear, taking the maximum value $\frac{2}{\pi} \cdot \frac{L}{r^3}$, where r is the shaft radius, for the limiting conditions. The result (p. 144) is that the principal stresses are

$$\frac{P}{2\pi r^2}\left\{1 \pm \sqrt{1 + \frac{16L^2}{P^2r^2}}\right\}$$

and the maximum shear stress is $\frac{P}{2\pi r^2}\sqrt{1 + \frac{16L^2}{P^2r^2}}$.

It can be shown that the torque has little effect upon the stability of the shaft under the thrust.

Shafts under Bending and Torque.—Many shafts, in addition to

transmitting a torque L, sustain forces perpendicular to their length which cause bending moments and shear forces. Usually the most important stress action is that at the outer surface, when the shear stress q due to the torque is a maximum, and at those points of it where the longitudinal stress p due to the bending is a maximum. Stresses will be equal but reversed at opposite sides of the shaft and so of an alternating nature. If a simultaneous axial load is carried, the stress due to it is to be superposed upon the direct bending stress. If the bending moment be M, the stresses are $p = \mathrm{M}\frac{4}{\pi r^3}$ and $q = \mathrm{L}\frac{2}{\pi r^3}$, so that the principal stresses are

$$f = (\mathrm{M} \pm \sqrt{\mathrm{M}^2 + \mathrm{L}^2})\frac{2}{\pi r^3},$$

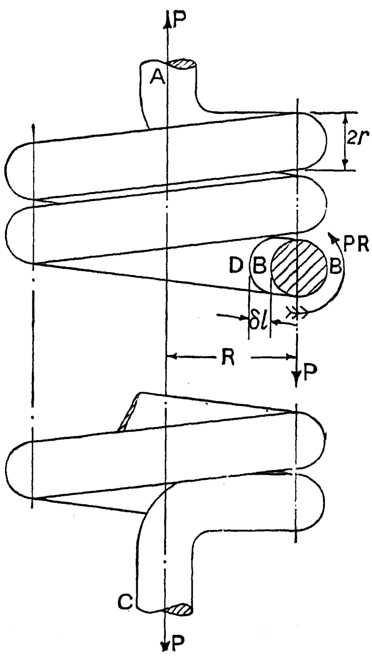

Fig. 32

the maximum of which is the same as if the shaft were subjected to a single bending moment of the value $\frac{1}{2}(\mathrm{M} + \sqrt{\mathrm{M}^2 + \mathrm{L}^2})$ and no torsion, so that this expression, Rankine's formula for the equivalent bending moment, can be applied directly the values of M and L are known.

The maximum shear stress is given by $q = \frac{2}{\pi r^3}\sqrt{\mathrm{M}^2 + \mathrm{L}^2}$, so that it can be similarly obtained by regarding the shaft as subjected to a torque only, of the value $\sqrt{\mathrm{M}^2 + \mathrm{L}^2}$. This, Guest's formula for the equivalent torque, is applicable to mild steel, while Rankine's is to be used for materials of a brittle nature.

St. Venant's formula for the equivalent bending moment,

$$\tfrac{3}{8}\mathrm{M} + \tfrac{5}{8}\sqrt{\mathrm{M}^2 + \mathrm{L}^2},$$

is derived from the assumption that the maximum elongation strain is the determining factor in yield, and is favoured by some authorities.

Helical Springs.—If an axial force P be applied at A to a close-coiled helical spring ABC (fig. 32), any section, such as B, transmits a torque PR where R is the mean radius of the coil, and a shear force P; but as the effect of the latter is small it may be neglected to a near approximation. The maximum stress q is given by

$$\mathrm{PR} = \frac{\pi}{2}qr^3,$$

where r is the radius of the wire from which the spring is made. The ex-

tension δx due to any small length BD (δl) of the spring is $R\delta\theta$ where $\delta\theta$ is the angle of twist in this small length, so that

$$\delta x = R\delta\theta = R\frac{\delta l}{r}\phi = \frac{R}{r}\frac{q}{C}\,\delta l.$$

Hence the total extension is

$$x = \frac{R}{r}\frac{q}{C}\,l = \frac{R}{r}\cdot\frac{2PR}{\pi r^3 C}\,l$$

$$\text{or } x = \frac{2}{\pi}\frac{R^2}{r^4}\,l\,\frac{P}{C} = \frac{4R^3 n}{r^4}\frac{P}{C},$$

since $l = 2\pi Rn$ when n is the number of coils in the spring. Expressed in terms of D and d, the diameters of the coil (mean) and wire, these equations are

$$P = \frac{\pi}{8}\,q\,\frac{d^3}{D}; \qquad x = \frac{8D^3 nP}{d^4 C}.$$

The resilience is the same as that of a shaft under torsion and accordingly (p. 162) is $\frac{1}{4}\frac{q^2}{C} \times$ volume. As the design of a spring usually calls for a certain deflection under the maximum load, this at once gives the volume of material necessary. The adjustment of the particular dimensions is much facilitated by the construction of an alignment or other suitable chart.

If square or flat section wire is to be used the load and deflection can be obtained by the use of the previously (p. 163) given values for the couple and twist.

A spring employed in compression may be arranged to yield first at a definite load by suitably limiting its movement, and a tension spring can be similarly arranged by being wound with an initial tension, the coils closing hard upon one another.

If the spring be not close coiled but has an appreciable angle of helix the wire is subjected to bending as well as torsion, and may be treated by the same methods, but the results can only be well used for checking the designed spring, and not in the original determination of the dimensions.

Flat Plates.—The stresses in flat circular plates have been analysed by Grashof, under assumptions corresponding with those made by Bernoulli for beams. The maximum stress under certain loadings and edge fixing is as follows, in which r is the radius and t the thickness of the plate:

Uniform pressure p applied all over plate—

Edges free: $f = \frac{3}{8}\frac{r^2}{t^2}(1 + 3\sigma)p.$

Fixed edges: $f = \frac{3}{4}\frac{r^2}{t^2}p.$

Central load W, applied over a circle of radius r_o—

Edges free: $$f = \frac{3W}{2\pi t^2}\left\{1 + (1 + \sigma)\log_e\frac{r}{r_o} - (1 - \sigma)\frac{r_o^2}{4r^2}\right\}.$$

Fixed edges: $$f = \frac{3W}{2\pi t^2}(1 + \sigma)\left\{\log_e\frac{r}{r_o} + \frac{r_o^2}{4r^2}\right\}.$$

Fly-wheels.—Owing to the employment of high speeds in machinery, the accelerations involved are such as frequently to produce high stresses; the simplest of such cases is that of simple rotation. Suppose that a ring ABC of radius r, and density ρ, rotate with angular velocity ω, or n revolutions per second, about an axis through its centre perpendicular to its plane.

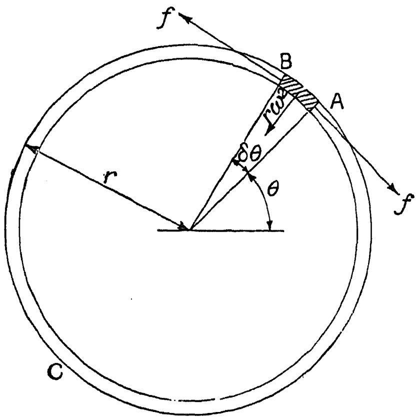

Fig. 33

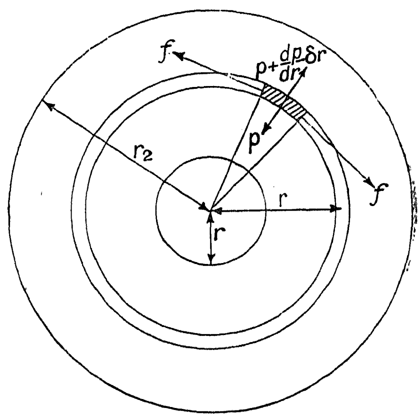

Fig. 34

The equation of motion for an element AB, of length $r\delta\theta$, in the radial direction is (see fig. 33)

$$\rho A r\delta\theta . r\omega^2 = fA\delta\theta,$$

where the area of section A is supposed to be so small that the tangential stress f is uniform over it. Thus

$$f = \rho r^2\omega^2 = 4\pi^2 n^2 \rho r^2 = \rho v^2$$

in absolute units when v is the velocity of the ring, and in gravitational units each of the expressions for f is to be divided by g. Thus the stress depends only upon the material and velocity of the ring, and is independent of its particular radius or sectional area.

For cast iron $\rho = 450$ lb. per cubic foot, so that $f = 0{\cdot}0975v^2$ lb. weight per square inch if v is in feet per second. In design of cast-iron fly-wheels, 80 to 100 ft. per second is a usual speed for rim velocity, but the stress action is then more complicated, the arms lengthening a little and the rim bending outwards midway between them. The stress is considerably higher

than in an ideal ring, and is analysed in Prof. Unwin's *Elements of Machine Design*.

Rotating Discs.—Fly-wheels for high speeds are usually made as discs and of steel for the highest speeds. Suppose that the disc be of uniform (small) thickness t and of density ρ, and that at a distance r from the axis there is a circumferential tension f and a radial tension p, measured in gravitational units, as indicated in fig. 34.

Resolving along the radius the equation of motion for the element $rd\theta dr$ is

$$\frac{\rho}{g}r\delta\theta.\delta r.t\omega^2 r = f\delta r t\delta\theta - \frac{d}{dr}(pr\delta\theta.t)\delta r,$$

which gives
$$f = \frac{d}{dr}(pr) + \frac{\rho}{g}\omega.^2 r^2. \qquad (1)$$

If the radius initially r becomes $r + u$ under the strain due to the rotation, the circumferential strain is $\frac{u}{r}$ and the radial strain $\frac{du}{dr}$. The equations connecting these with the stresses f and p, which by symmetry are principal stresses, are

$$E\frac{du}{dr} = p - \sigma f, \qquad (2)$$

$$E\frac{u}{r} = f - \sigma p. \qquad (3)$$

Differentiation of the latter, followed by substitution for u and $\frac{du}{dr}$, gives

$$(p - f)(1 + \sigma) = r\frac{d}{dr}(f - \sigma p),$$

and the substitution of the value of f from (1) then gives

$$r\frac{d^2p}{dr^2} + 3\frac{dp}{dr} + \frac{\rho}{g}\omega^2 r(3 + \sigma) = 0,$$

or
$$\frac{d}{dr}\left(r^3\frac{dp}{dr}\right) + \frac{\omega^2\rho}{g}.(3 + \sigma)r^3 = 0,$$

and hence
$$\frac{dp}{dr} + \tfrac{1}{4}.\frac{\omega^2\rho}{g}(3 + \sigma)r + \frac{A}{r^3} = 0,$$

and
$$p = B + \frac{C}{r^2} - \frac{\omega^2\rho}{8g}(3 + \sigma)r^2, \qquad (4)$$

the constants of integration being determined by the terminal conditions, one of which is that $p = 0$ when $r = r_1$, the radius of the outside of the disc.

The disc may be bored or solid to the centre. In the former case, if r_2 be the radius of the bore, p is zero when $r = r_2$. From these conditions

($p = 0$ when $r = r_2$ or r_1) the values of B and C are found, and when substituted in equation (4), give

$$p = \frac{\omega^2(3+\sigma)}{8}\frac{\rho}{g}\left\{r_1^2 + r_2^2 - \frac{r_1^2 r_2^2}{r^2} - r^2\right\},$$

from which, by (1),

$$f = \frac{\omega^2}{8}\cdot\frac{\rho}{g}\left\{\left(3+\sigma\right)\left(r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{r^2}\right) - (1+3\sigma)r_2\right\}.$$

The latter stress is a maximum at the bore, and has there the value

$$\frac{\omega^2\rho}{4g}\{(3+\sigma)r_1^2 + (1-\sigma)r_2^2\},$$

while the maximum value of p occurs at $r = \sqrt{r_1 r_2}$, and is there $\frac{\omega^2\rho}{8g}(3+\sigma)(r_1 - r_2)^2$.

If the hole is very small, so that r_2 is negligible, the value of the central circumferential stress is

$$f = \frac{\omega^2}{4g}\rho(3+\sigma)r_1^2.$$

If, however, the disc be solid, without any central hole, the conditions for determining the constants are $p = 0$ when $r = r_1$, and $u = 0$ when $r = 0$. Now

$$u = \frac{r}{E}(f - \sigma p)$$
$$= \frac{1}{E}\left\{B(1-\sigma)r - \frac{1+\sigma}{r}C - \frac{\omega^2}{g}\rho r^3\left(\frac{(3+\sigma)(1-\sigma)}{8} - \frac{3+\sigma}{4} + 1\right)\right\},$$

so that $C = 0$, and hence

$$p = \frac{\omega^2\rho}{8g}(3+\sigma)(r_1^2 - r^2),$$

$$\text{and hence } f = \frac{\omega^2\rho}{8g}\{(3+\sigma)r_1^2 - (1+3\sigma)r^2\}.$$

Thus the circumferential and radial stresses are both a maximum at the axis, where the value of each is

$$\frac{\omega^2\rho}{8g}(3+\sigma)r_1^2,$$

which is half the value of the circumferential stress when there is a small central hole.

This analysis suggests that the stress would be lessened by making the disc of greater thickness at the axis and tapering it towards the edge. Owing

to the importance of the matter in steam turbine work, certain cases have been analysed, and the results have been experimentally tested.

Acceleration forces cause stresses in all moving parts of machinery, and require investigation when the speed is high. The part may be considered to be at rest and loaded with the reversed acceleration forces in addition to the force loading, and investigated by the methods previously given. Thus a side rod can, in the position in which such acceleration forces have their greatest effect, be regarded as acting as a beam with regard to them, while it simultaneously transmits the driving force axially.

Resonance Effects.—High repetition stresses occasionally arise from resonance, occurring (p. 131 "Applied Mechanics") when the running period is nearly coincident with a period of free vibration in the machine or a multiple of such period. If the curve of stress variation be determined without regard to resonance and analysed harmonically, the amplitude of a term having a period near that of the free period is greatly increased (being divided by the square root of unity less the square of the ratio of the periods). This danger is always to be avoided by running the machine decidedly slower or faster than such a speed.

Impact.—High stresses may also be produced by impact. Suppose that a mass W, fig. 35, be supported by the stop A fixed on the bar AB, the radius of which is r. When it is at rest the stress p in the bar will be $\frac{W}{\pi r^2}$, and under it the bar will have extended a distance x, equal to $\frac{p}{E}l$, where l is the length of the bar. If, however, the weight be supported so as just to touch the stop A without causing any load in the bar, and be suddenly released, it will commence to vibrate, and as its final position of rest is at x below its starting position, it will descend a further distance x beyond this final position; thus the total extension of the bar will be $2x$, and the maximum corresponding stress $2p$, or twice as much as the steady stress. The statement that a suddenly applied force produces double the stress that the same force applied slowly would produce, applies only to cases in which mass is concerned in a manner similar to the above instance. If the mass had an initial velocity the bar would stretch further, and the maximum stress would be higher, the amount being determined by the conversion of the original kinetic energy of the mass and its loss of potential energy into strain energy of the bar. Such consideration gives only a lower approximation to the maximum stress which actually occurs; for impact, even in the simple case taken above, implies a more complicated action, a wave of stress running (at 17,000 ft. per second in the case of steel) to and fro along the bar. The harmonic vibration of the mass mentioned previously is not established until the stress flux has settled down into the same phase throughout the bar.

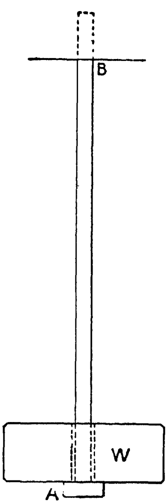

Fig. 35

Overstrain.—If the energy taken up by a body in such an impact is so stored that the material is nowhere overstrained, an elastic vibration ensues, but if the energy to be taken up is so large that the yield point is somewhere exceeded, that part of the energy which causes the overstrain is changed to the form of heat, and is not recovered mechanically. The energy which can be absorbed by a ductile material after passing the yield point is very large compared with its resilence, and, as is evident from a comparison of the areas under the stress-strain curve, the total energy which can be absorbed before fracture greatly depends upon the final elongation—or in cases of torsion upon the angle of strain at fracture. Thus material with a large ultimate elongation has a preference when impact effects of unknown magnitude may occur. The bending moment under which a beam fails, and the torque under which a shaft fails, are not connected with the corresponding stresses by the elastic formulæ previously given, but by expressions based upon the approximate assumption that the stress is uniform over the section. These expressions are

$$\mathrm{M} = \tfrac{1}{4}bh^2p \quad \text{instead of} \quad \mathrm{M} = \tfrac{1}{6}bh^2p \text{ for bending, and}$$

$$\mathrm{L} = \tfrac{2}{3}\pi r^3 q \quad \text{instead of} \quad \mathrm{L} = \frac{\pi}{2}r^3 q \text{ for torsion.}$$

Temperature Effects.—Change of temperature may produce stresses owing to actual differences of temperature occurring in the machine or structure, or to the difference of the coefficient of expansion of different materials employed. The stresses can be found by a consideration of the strains occurring.

Thus, suppose that a steel rod, sectional area A_1, fits inside a brass tube, sectional area A_2, and that collars on the bar are adjusted by nuts just to touch the ends of the tube when the temperature is t_1. If the temperature be raised to t_2, the brass will be in compression and the steel in tension owing to the coefficient of expansion for the steel, α_1, being less than that for the brass α_2. If the stress in the steel be f_1 and that in the brass f_2, since the total force in each is the same,

$$\mathrm{A}_1 f_1 = \mathrm{A}_2 f_2,$$

and the steel will elongate $\frac{f_1}{\mathrm{E}_1}$, and the brass compress $\frac{f_2}{\mathrm{E}_2}$ per unit length, if E_1 and E_2 be the values of Young's modulus for steel and brass respectively. The sum of these amounts will be equal to the difference between the expansions for brass and steel per unit length produced by the rise in temperature, or $(\alpha_2 - \alpha_1)(t_2 - t_1)$, so that

$$\frac{f_1}{\mathrm{E}_1} + \frac{f_2}{\mathrm{E}_2} = (\alpha_2 - \alpha_1)(t_2 - t_1).$$

From these equations f_1 and f_2 can be found.

If the steel be a bar 1 in. diameter, just fitting into a brass tube $1\frac{1}{2}$ in. external diameter, taking

$$\alpha_1 = \cdot 0000067 \text{ per degree F., and } E_1 = 13{,}500 \text{ tons per square inch}$$
$$\text{and } \alpha_2 = \cdot 0000120 \quad ,, \quad ,, \quad E_2 = 5{,}500 \quad ,, \quad ,,$$

a rise of temperature of 200° F. would produce a stress of 3·86 tons per square inch in the brass, and 4·82 tons per square inch in the steel.

Bearing Area.—In many of the cases discussed above, the loading is applied to the part over comparatively small areas, and near these the conclusions arrived at for the stress distribution do not hold. The point of application of a force to a part, usually the "contact" of two parts, is never a geometrical point, but is a small area owing to the elastic distortion. This area may be very small, as in the case of the contact of a ball with its race, or relatively very much larger, as in the case of a bearing in which the total force existing between the two elements is transmitted by oil pressure.

The original analysis of the stresses in the material subjected to a "point load" is due to Hertz, and shows pressure along lines which, arising from the small area of contact, radiate (along curves) into the material, combined with perpendicular stresses which are pressure near the surface and near the area, but change to tension elsewhere. When the load is definitely applied over area, it may still be supposed to be composed of a series of point loads, each contributing to the total stress system. Parts fitted together over defined areas may be regarded as lying between point loads and the cases of lubricated bearings, and include the case of riveted joints, the treatment of which is deferred so as to be discussed in connection with boilers. Owing to this type of stress distribution near the area of load application, provision must be made in design for adequately distributing the incidence of the load into the part considered, e.g. girders must have stiffeners at the points of support and of heavy single loads, the ends of connecting rods must be stiffened, &c.

Regions of High Stress.—Mathematical analysis has indicated the occurrence of stress at rapid changes of section of a part, sharp corners, quick bends, small holes, &c., very much exceeding the average stress in the neighbourhood. Sketches of the possible arrangements of stress and strain lines also indicate this, and the calculations have been confirmed, in two-dimensional cases, by Brewster's method employing polarized light, and in torsion by the analogy of soap film shapes. With the premises of the theory of elasticity (p. 137) it can be shown that the presence of a small spherical flaw in the material causes the shear stress to rise to double its mean value; a small hole drilled normally into a shaft causes the shear stress under torque to be twice what it would be if the hole were not drilled, and, if the shaft be subjected to bending, three times that amount. At the bottom of a groove of shape of a Whitworth thread the stress is about $2\frac{3}{4}$ times what it would be if the shaft were smooth, and in the case of a re-entrant angle with a perfectly sharp corner the stress due to torque would be infinite. However, material is not the ideal substance of mathematical theory; a complex crystalline

structure exists in metals, the size of the crystals being dependent on the history of the material. This being so, the mathematically calculated effect of very small flaws is included in the test of the material. The very high stresses due to notches, cracks, or re-entrant corners cannot be so included if the size of these defects is large compared with that of the structure of the material. In ductile materials subjected to a constant stress they are not usually of serious importance, as the material yields and flows a little, reducing the stress, and at the same time leaving the material in the neighbourhood with a higher yield point. If, however, the part is subjected to an alternating repetition stress the small plastic flow takes place at each stressing, and gradually disintegrates the material, the flaw extending and finally causing failure. In brittle material (cast iron or hardened steel) all such defects or shapes are to be regarded as dangerous.

THE TESTING AND PROPERTIES OF MATERIALS

BY

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The Testing and Properties of Materials

Introduction

The strength and reliability of materials used for constructional purposes are of considerable importance to engineers and designers, an importance which tends to increase with the expansion of modern engineering methods. Not only is there a demand for an extension of the range over which well-known materials can be made applicable, but also for new materials of highly specialized properties. Thus, from time to time, standard methods are called in question, and newer methods developed by the engineer for controlling the products of the metallurgist. With a view to keeping pace with new conditions of service, the specifications of the engineer are drawn up with increasing exactness, and greater vigilance is kept on the methods of testing, prior to putting any material into actual service.

The ultimate aim of testing is of course to determine how materials are likely to behave when exposed to the complex conditions of practical use. It must be remembered, however, that probably no test embodies these conditions. Service may be a matter of years, while obviously in a test the material must be carried to destruction in a reasonably short time. In cases where careful study of working conditions has revealed the magnitude of the stresses and the way in which they are applied, it has frequently been possible to devise tests nearly reproducing the forces or system of forces which a body will be called upon to withstand. In this connection may be mentioned the various types of fatigue tests made in view of the fact that modern machinery is subject to continuous vibration, which induces a marked depreciation in the mechanical value of the metals. Another interesting advance is the examination of metals at temperatures to which they may normally attain in service. On the other hand, some tests, though of great practical convenience, make no attempt to imitate working conditions, and their indications can only be interpreted when they have been correlated with results of actual use. Experience has shown, however, that reliable information may often be obtained by the use of simple and inexpensive methods.

The scientific way of testing would be to measure one property of the material at a time, but as yet the various physical properties which constitute "strength" are not fully understood. From the point of view of practice, testing is a compromise with a large allowance—called a factor of safety—to cover inconsistencies of the test and service. As a rule the actual piece of metal employed for the test is not put into service, and there is always some uncertainty as to the uniformity of the bulk. For this reason, specimens should be thoroughly representative of the material under inspection, and have undergone exactly the same treatment. Exact and careful testing is not enough in itself; there must also be intelligent selection of representative material. As test values are affected by a difference in size and shape of the specimen and the manner of testing, it is important to adopt standard methods of testing. In this connection the work of the British Engineering Standards Committee should be referred to.

The commercial importance of determining the material best suited to the manufacture of various parts is apparent, as it leads to increased reliability, longer life, and reduced cost, as the more suited the material for a given part, the lighter the part can be made. It will generally be found, in fact, that the practice of engineers is in advance of their theories. This arises from the rule of meeting and solving difficulties by experiments limited to the immediate demand. The need is apt to be overlooked, however, for more extended researches on broader lines for the enunciation of laws, which will have a determining effect on the choice and preparation of materials for engineering purposes.

CHAPTER I

Tensile Testing

The Tensile Test.—In addition to being one of the earliest tests to which metals and other materials were put, the tensile test is the most widely used of all mechanical tests. At one time engineers were inclined to rely exclusively upon it. Certainly no other simple test is so comprehensive in the data supplied in a single operation. In this test a piece of metal is firmly held at the two ends and subjected to a tensile force which is increased till rupture occurs. Before this happens, however, a number of well-defined features are discernible which are discussed below.

Tensile Machines.—These are varied in size and form. They range from machines exerting a load of a few pounds for testing very fine wires, to powerful machines for applying loads of 3000 tons. They consist essentially of two main organs or mechanisms. The first, which applies the required force to one end of the test piece and takes up the extension, is generally a hydraulic ram, although in American machines the usual method of straining is by screw gearing driven by variable-speed electric motors. The second

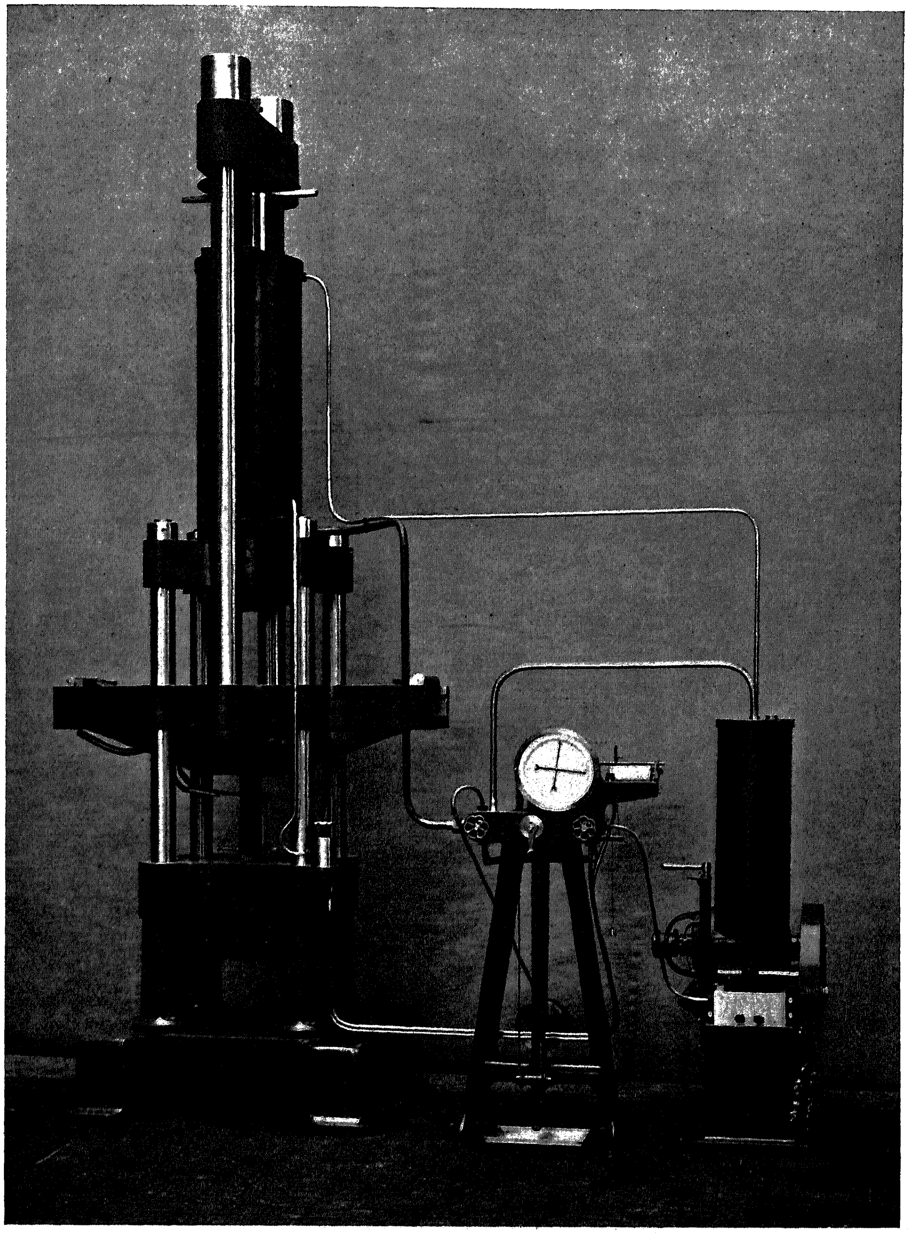


Fig. 1.—100-ton Universal Testing Machine assembled with Standard Pendulum Dynamometer and Motor-driven Oil Pump, 300 atmospheres pressure

determines the magnitude of the force or load applied to the test piece, either by direct counterpoise or some form of hydraulic-pressure instrument. In this country the former is almost exclusively used, though the latter is simpler and more compact. In the dead-load instrument the weight is run out

along a lever, and counterpoises the pull on the test piece due to the hydraulic pressure of the ram. In the second type, the load on the test piece is transmitted to a chamber which is partly composed of a flexible diaphragm. The chamber is filled with water or other fluid in communication with a sensitive pressure gauge, which records the pressure corresponding to the pull on the test piece, any friction of the ram working in its cylinder being allowed for in calibrating the instrument. In another type of instrument, developed by the Swiss firm of Amsler (see fig. 1), the ram serves to apply and measure the load. The pressure is applied by oil, a thin film of which

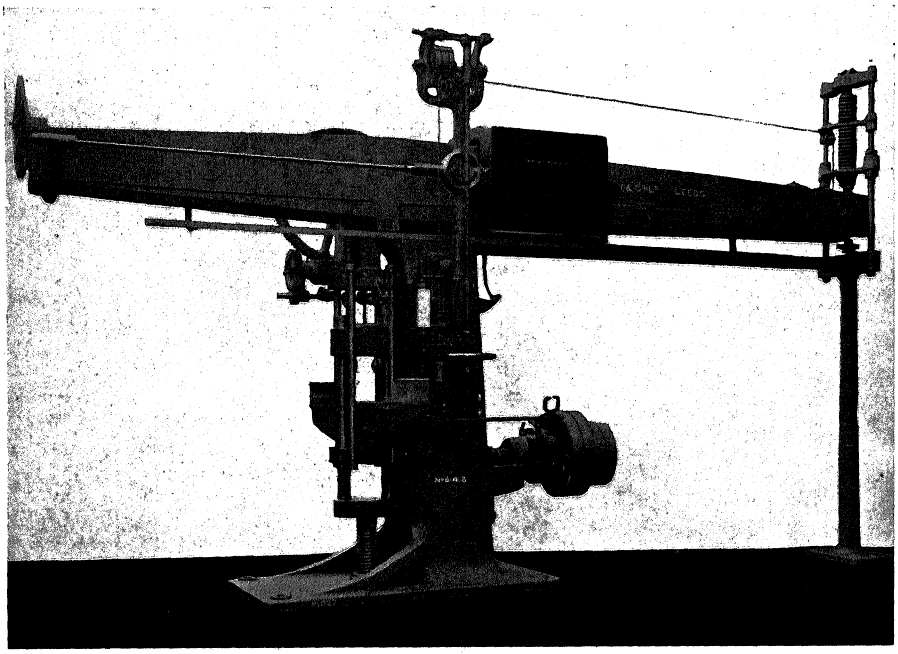


Fig. 2.—50-ton Vertical Testing Machine

continually leaks past the ram, thus rendering its friction negligible. The pressure of the oil behind the ram is measured by the deflection of a heavy pendulum forming a simple gravity pressure gauge.

For full information on the construction and use of the various testing machines on the market, the reader is referred to works on testing, original papers, and the technical press. The following may, however, be mentioned as typical machines.

50-ton Wicksteed Vertical Single-lever Testing Machine (fig. 2).—The loading may be accomplished by the admission of high-pressure water through a controlling valve into a steel hydraulic cylinder, or the machine may be arranged for control by belt pulleys, straining taking place by a screw through gearing as shown in the figure. As the pull on the test piece is increased, the balance of the main lever is maintained by running

out the counterpoise. The figure shows the machine intended for tension, compression, and bending tests, also torsion tests as well. For prolonged tests, where the possibility of leakage in a hydraulic machine would be disadvantageous, an electrically-driven machine is generally preferred This vertical type of machine with its single self-balanced lever is one

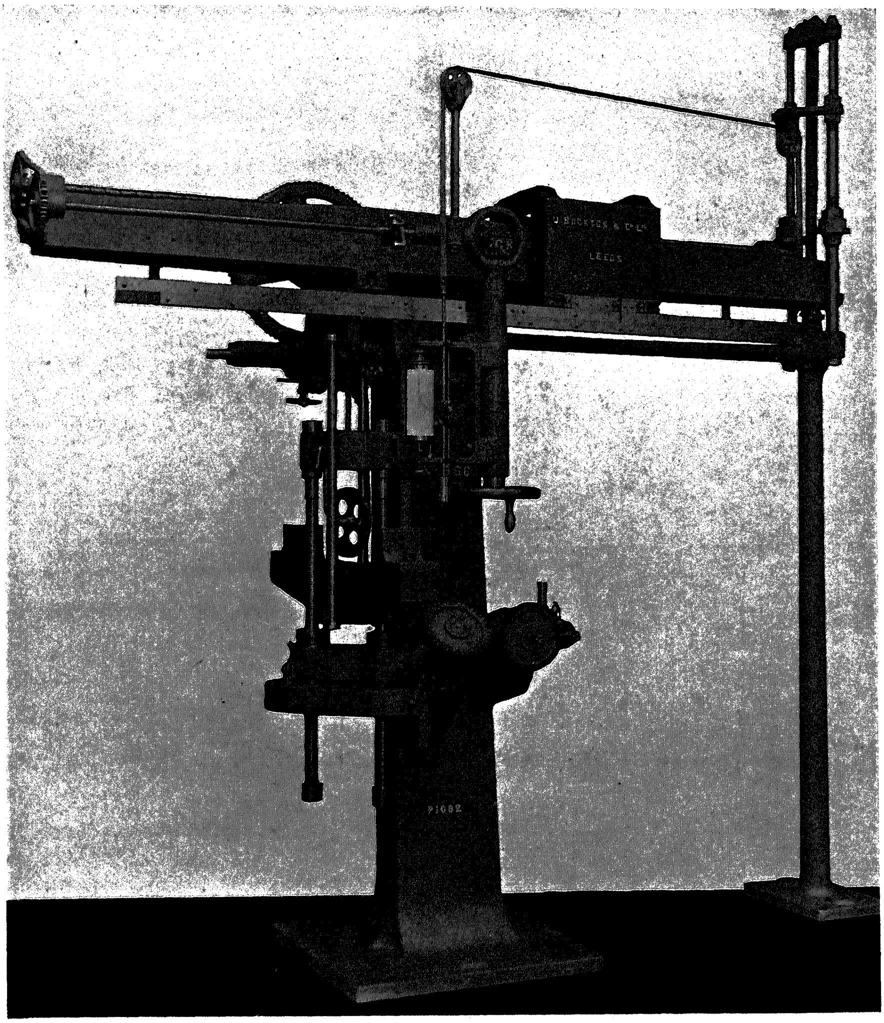


Fig. 3.—5-ton Vertical Single-lever Testing Machine controlled by Hand

of the simplest forms of tensile machines. It can be calibrated at any time by the direct application of standard weights. Unfortunately the lever and jockey weight possess a certain amount of inertia which may introduce errors when making measurements of a delicate nature. In addition the single-lever machines are rather bulky and costly.

5-ton Wicksteed Testing Machine arranged for tension, compression,

bending, and torsion tests. The loading is effected by a large central screw driven through gearing by hand pressure applied to the large hand-wheel (fig. 3).

The knife edges should receive attention, as they are naturally subjected to severe stresses, also shocks, when the test pieces break.

The machine is shown fitted with one of the automatic recorders described below.

Avery's 50-ton Testing Machine (fig. 4).—This machine is of the single-lever type, and has the advantage over the horizontal machine that it can

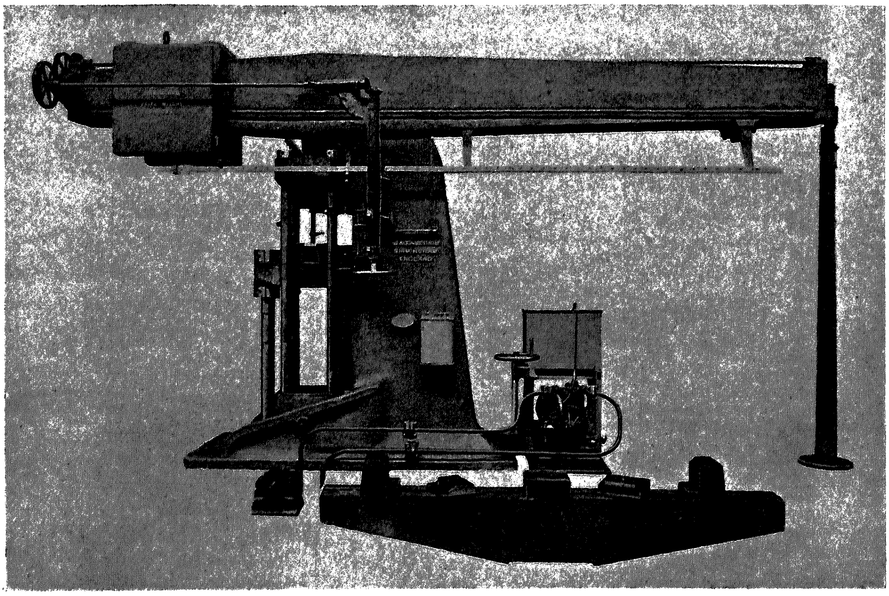


Fig. 4.—Avery 50-ton Testing Machine

be readily calibrated, from time to time, by the suspension of standard weights.

The tension specimen is secured in wedge grips, one pair of which is fitted into the upper weighing crosshead, and the lower pair carried in the straining crosshead, which is connected to a hydraulic ram by means of two tension rods. The upper crosshead is suspended from the load knife-edge of the steelyard, and in this way communicates the load, which is then balanced off by the movement of the poise weight.

The graduated scale is marked from zero to 50 tons by sub-divisions of 1/100 ton.

The compression test is carried out by placing the specimen below the straining crosshead, resting it on the lower weighing platform, the compressive load being communicated from this lower crosshead to the upper one by means of four tension rods.

The bending test is carried out by the provision of a crossbeam, which

is inserted between these rods, and is allowed to rest upon the lower weighing crosshead.

The torsion test is effected by worm gearing at the back of the machine. The axis of the specimen is arranged so as to coincide with the fulcrum knife-edge of the steelyard.

The twisting load applied to the specimen is balanced off by the poise, as in the case of the other tests.

An autographic stress-strain recorder is attached to the frame of the

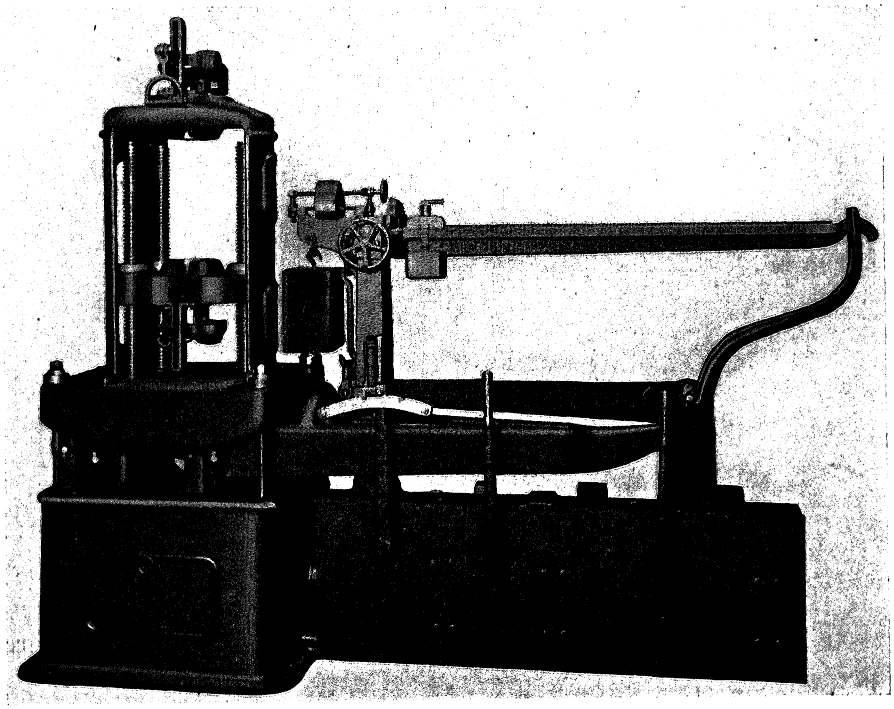


Fig. 5.—Riehlé 100,000-lb. Testing Machine

machine, the rotation of the drum being effected by the elongation of the specimen, while the vertical movement of the pencil is operated by means of gearing, so that it moves in sympathy with the poise weight.

The hydraulic supply is derived from a motor-driven "Hele-Shaw" pump, which feeds oil to either end of the hydraulic cylinder, the speed and direction of the ram being controlled by the hand-wheel of the pump.

For prolonged tests, where the possibility of leakage in a hydraulic machine would be disadvantageous, an electrically driven machine is sometimes preferred.

Riehlé 100,000-lb. Screw Power Testing Machine.—A typical American tensile machine is shown in fig. 5. It is of the variable-speed, compound-lever type. Compression and bending tests can also be made.

Avery's 100-ton Horizontal Testing Machine (fig. 6).—This machine is designed specially for testing full-size members to destruction. It consists of a strong cast-iron frame, made up of four columns, the lever system being at the one end and the hydraulic cylinder and ram situated at the other.

The tension test is effected by securing the specimen at the one end in a crosshead, which is connected to the lever system by means of tension rods, the other end of the specimen being secured in a straining crosshead, by which the load is applied.

The straining crosshead can be keyed in any position in the four sliding racks, each of which is continued through to the crosshead of the hydraulic ram. The movement of the ram causes the straining crosshead to apply the load to the specimen, which is then balanced off by moving the poises upon the steelyard.

The bending and compression tests are carried out by the employment of a long crossbeam, carrying sliding presser feet at each end for beams, and a fixed platen at the middle for columns respectively.

The tension specimen has a maximum stretched length of 15 ft. 6 in. The compression specimen may be any length up to 15 ft.

The knife-edges for the main lever are 20 in. long, and are accurately hardened and ground, and engage in hardened-steel bearings.

The movement of the hydraulic ram is easily controlled, both for speed and direction, by the hand-wheel of the pump.

Calibration of Tensile Machines.—Testing machines must be checked or calibrated at intervals to assure their accuracy. Inaccuracies may be due to worn or broken knife-edges, or to dirt or grease interfering with the sensitiveness, or to the beam or other parts becoming bent or distorted. Calibration may consist in loading with standard weights. For the application of large forces by means of weights, recourse is generally had to the intermediary of levers which multiply the force exerted by the known load. The corresponding reading of the machine will indicate its degree of accuracy. It is usual to check at intervals the zero positions and the sensitiveness of the machine. Recently standardizing boxes have been developed by Messrs. Amsler for calibrating testing machines, both in tension and compression stresses (figs. 7 and 8). When subjected to stress the standardizing box is deformed to a degree proportional to the applied load, a very accurate measurement of which is given by the volume change of the box by means of a micrometer reading. The walls are of a suitably heat-treated special steel, and the interior of the box is filled with mercury extending into a capillary tube in the side of the instrument. On this tube is a mark to which, after the application of stress, the level of the mercury column is always brought back by turning a micrometer screw actuating a plunger.

Data derivable from Tensile Test.—When the load, which should be uniformly applied, is put on a test piece, the latter first undergoes a slight but measurable extension, the stretch being "elastic", or, in other words, the specimen returns to its original dimensions when the load is

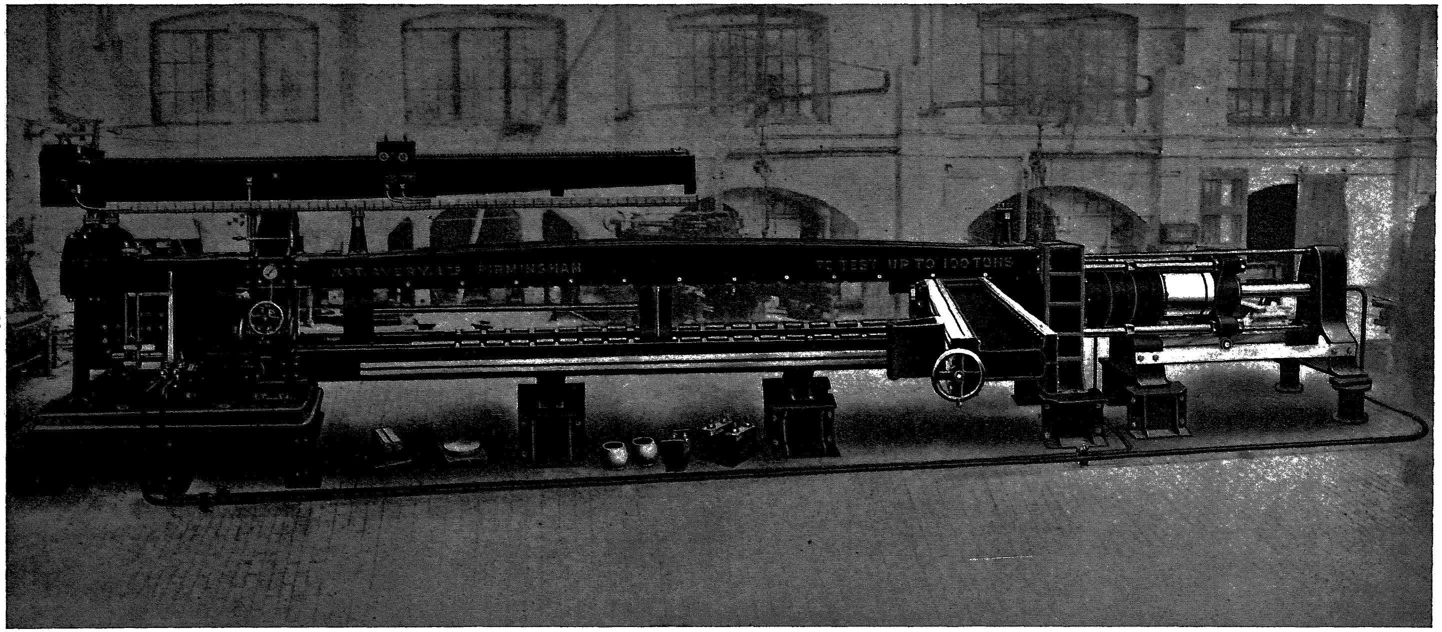


Fig. 6.—Large Horizontal Tensile Testing Machine (100 tons)

removed. In this range the well-known *Hooke's Law*, that strain is proportional to stress, is obeyed. The elastic constant termed *Young's Modulus* is the stress that would suffice to stretch the test piece to twice its original length—if such were possible and assuming Hooke's Law to hold throughout. After a certain point, however, the stretch is no longer proportional to the load. This point is known as the *Elastic Limit* or *Limit of Proportionality* (fig. 9). The elastic limit is the maximum

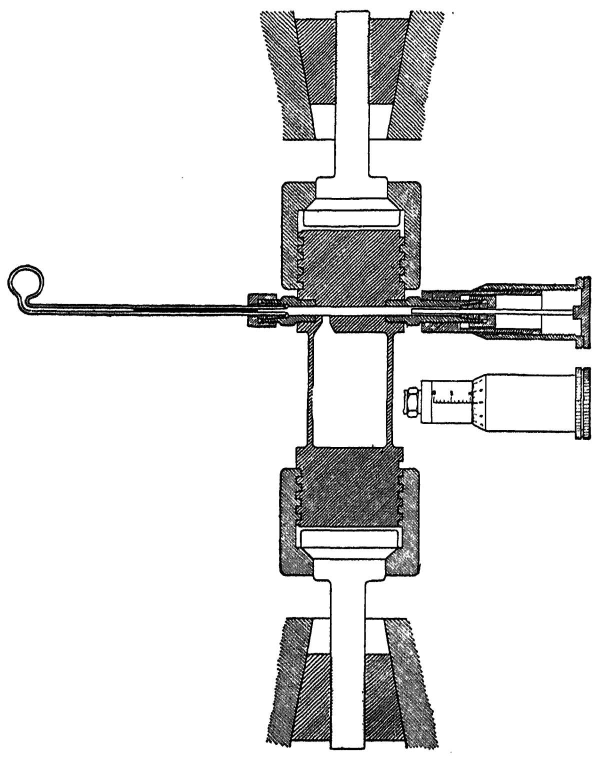


Fig. 7.—Section of Standardizing Box (Inst. of Mech. Eng., 1920)

stress which can be applied to the test piece without deforming it, or after the removal of which no permanent elongation remains. Its position is influenced by the treatment which the material has undergone, being raised by overstrain, cold rolling, or drawing, but lowered when prior working has effected a diminution in length; so that, unless normalized material is under consideration, the elastic limit has no very definite meaning. On the other hand, the elastic limit is a most important physical property. When determinations are accurately made it may be taken as the figure best representing the strength of the material.

After the applied stress exceeds the elastic limit, the tensile strain begins

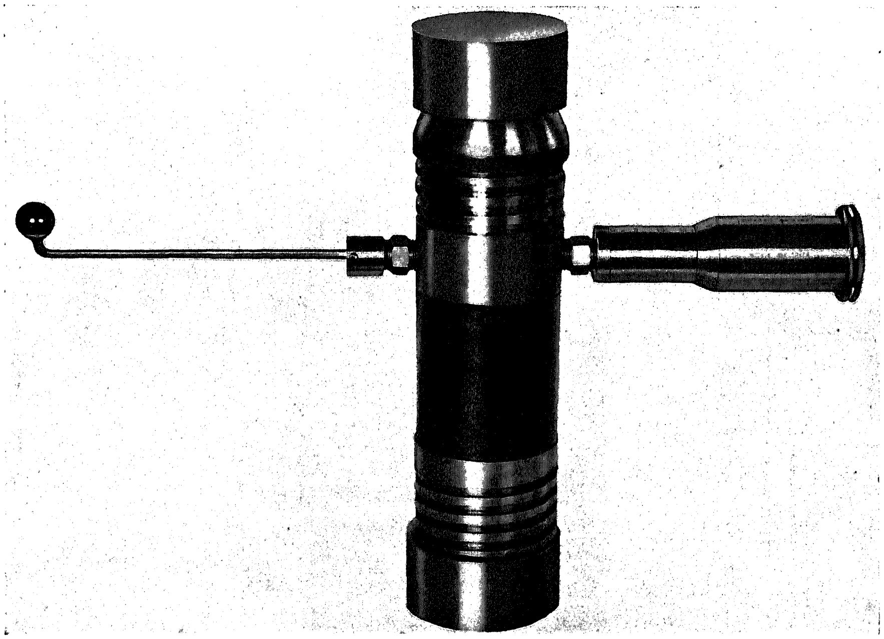


Fig. 8.—Double-purpose Standardizing Box arranged for Compression Tests

to increase more quickly, and continues to grow at an increasing rate as the load is augmented, and the material takes a permanent set. The actual point at which, after removal of the load, the increase in length between the gauge points becomes visible is called the *Yield Point*. In some materials, notably wrought iron and steel, the yield point is indicated by a sudden sharp drop in the load which the material will bear. A marked breakdown occurs, the elongation becoming many times greater than previously with little or no increase in stress. When the yield point is reached the bright surface of the test piece takes on a matt appearance, and generally two systems of parallel curved lines equally and oppositely inclined to the axis appear, indicating elastic failure. If a ductile material has been overstrained—i.e. a permanent set produced—and is immediately reloaded, it will be found to have no true elastic limit, although its yield point will be somewhat raised. If it be allowed to recover by resting several days or weeks (ageing), or is heated for a few minutes at the temperature of boiling

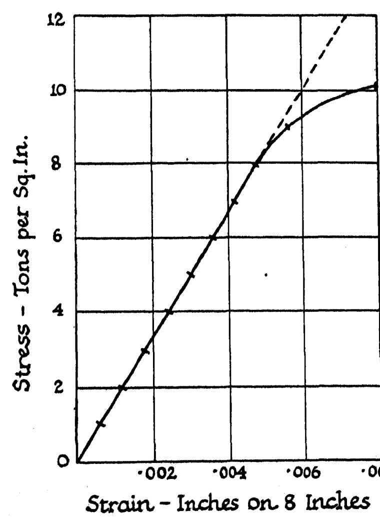


Fig. 9.—Stress-strain Curve showing Elastic Limit

water, a true elastic limit will be found, but higher than in the original material. The yield point will be higher still and the ductility decreased.

After stretching under constant load at the yield point, the material takes up a gradually increasing load, the test piece stretching uniformly along the parallel portion. After the maximum load is reached, a last stage occurs when at a particular point a local necking is formed, the constricted portion experiencing an increased stress until finally fracture occurs (fig. 10). The greater part of the strain at loads above the yield point occurs very quickly, but is followed without any further loading by a small additional extension, which increases with time, but at a diminishing rate. The continued extension from continued application of the same load is called creeping or plastic elongation.

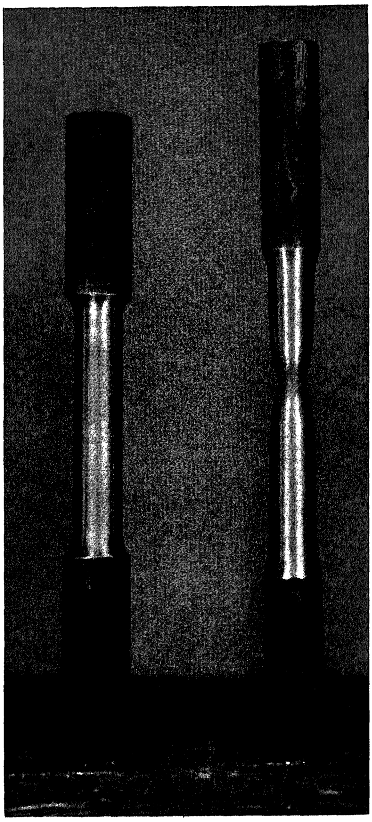


Fig. 10.—Tensile Test Pieces before and after Testing

The maximum load necessary to rupture a specimen in simple tension divided by the original cross-sectional area of the test piece is known as the *Ultimate Strength* of the material or its *Tenacity*. Engineers' units are tons per square inch or pounds per square inch.

When the final load is divided by the reduced area at the point of fracture, the *Final Strength* is obtained. The local reduction in area is such that the load necessary to break the bar at the neck is less than the maximum load on the bar before the local necking takes place. On the other hand, the breaking load divided by the reduced area of section shows that the actual stress intensity is greater than at any previous load.

The fracture of ductile materials, such as mild steel, aluminium, and copper, under tension, generally takes place partially or wholly by shearing or sliding in directions oblique to that of the direct stress. As just previous to fracture a local reduction of section takes place, this tends to an uneven distribution of stress over the fracture.

The *Elongation* is determined by measuring the distance between gauge marks before and after rupture, and expressing the increase in length as a percentage of the original distance. The gauge length should always be stated, as in shorter specimens the necking down occupies a proportionately greater part of the length. In other words, the extension is

made up of two distinct parts, a fairly uniform elongation, and an increased local elongation about the section of fracture.

The *Reduction of Area* is the ratio of the diminution in sectional area at the point of rupture to the original area, and is generally expressed as a percentage, thus

$$R = \frac{a - a_1}{a} \times 100.$$

The elongation and reduction of area indicate the ductility of the material.

Stress - strain Diagrams. — The well - known stress - strain diagrams are generally constructed by measuring the length between two gauge marks on the test piece, and plotting the results.

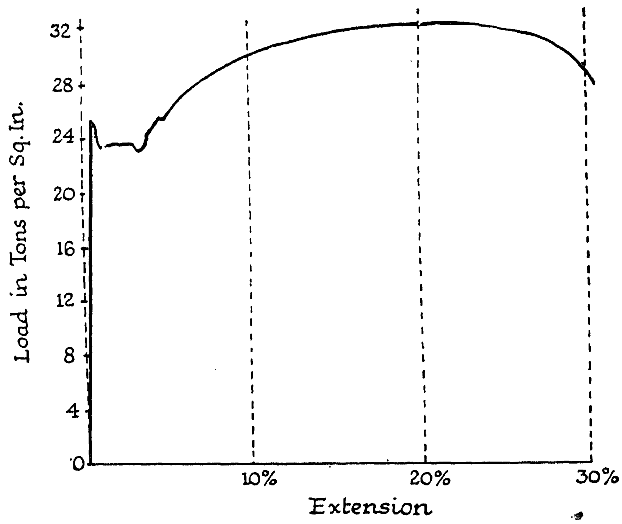
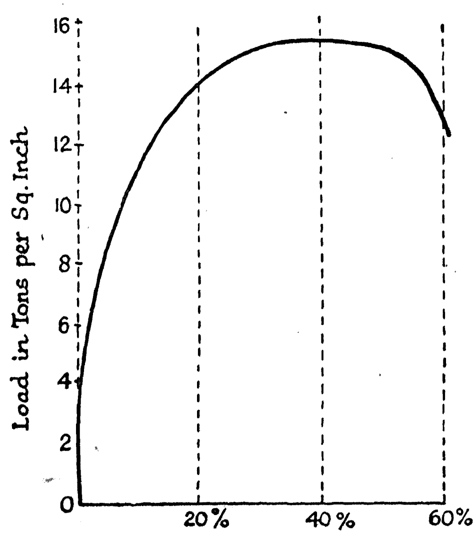


Fig. 11.—Stress-strain Curve of Mild Steel

Note.—The slope of the curve up to the yield point is somewhat exaggerated for diagrammatic purposes.

Beyond the elastic limit the strains of ductile materials may conveniently be measured by a pair of dividers and a steel rule. The elastic extension of test pieces is as a rule too small for direct measurement, and special instruments known as extensometers are used for this work, some of which are described on p. 188.

Fig. 11 shows a typical stress-strain diagram for mild steel. The shape of the stress-strain curve is slightly modified by the rate of loading. The speed of testing principally affects the determination of the yield point.



Extension

Fig. 12.—Stress-strain Curve of Annealed Copper showing Gradual Yielding

In the majority of non-ferrous alloys the stress - strain diagram does not show true porportionality between stress and strain even at low loads (fig. 12) There is something corresponding to the yield point but no drop, or extension under constant

load. In the case of rolled aluminium slowly and continuously loaded, at very low stresses, the strains increase faster than the stresses, and yet practically the whole of the strain disappears after the removal of the stress. Hence the elastic limit cannot be found by inspection of the stress-strain diagram.

Autographic Recorders.—A large amount of information may be missed in stress-strain diagrams unless a continuous curve giving a complete story of the tensile test is obtained. Various types of autographic apparatus have been devised for giving a continuous record of the stress and strain throughout a tension test. The apparatus is usually of the following nature. A pencil is given a movement, proportional to the stress, over a paper wound round a cylindrical drum rotating in a direction at right angles to the movement of the pencil, and at a rate proportional to the strain of the specimen. This arrangement obtains in Wicksteed's Autographic Spring Recorder (fig. 2), in which a helical spring connected to a pencil shortens by an amount proportional to the tension on the test piece. On the other hand, the arrangement may be reversed, and the rotation of the drum made proportional to the stress, as in Gray's Spring Autographic Recorder, fitted to the Riehlé testing machine.

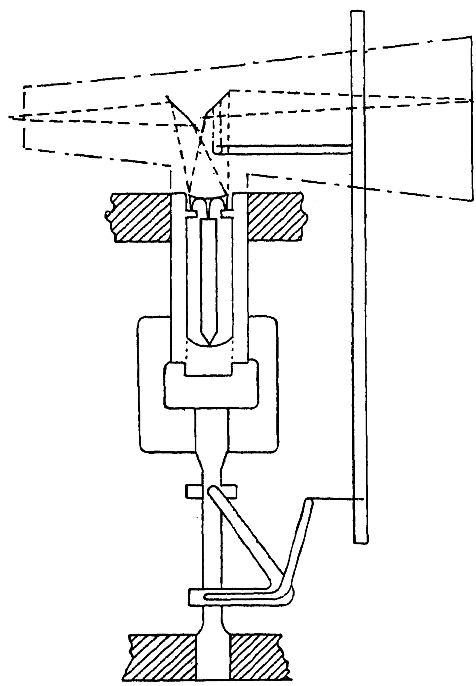


Fig. 13.—Diagram of Dalby Apparatus

A new type of recording instrument has been invented by Dalby (fig. 13). The pull of the machine is measured by the use in series with the test piece of a steel plate of high elastic limit, called a weigh bar, which acts as a perfect spring. By means of an optical device the extension of the weigh bar measuring the applied load deflects a spot of light horizontally, and the extension of the test piece is made to deflect the spot vertically. In this way the spot of light traces out a stress-strain curve, which may be photographically recorded.

Extensometers.—A number of instruments of ingenious design have been devised for the measurement of the elastic stretching of metals. In one type of instrument the movement apart of two clips, fixed on the test pieces at the ends of the gauged length, causes the rotation of a small roller to which a fine pointer is attached to magnify the movement, the pointer moving over a circular scale engraved on a mirror.

Ewing's Extensometer.—In this instrument two clips are attached to the

test bar a definite distance apart. From the upper clip a rod is suspended carrying a microscope. Attached to the lower clip is a small glass scale in the field of view of the microscope. By a system of levers the glass scale is given a movement five times the extension of the test piece across the objective of the microscope. The extension of the test piece is easily measured in units of $\frac{1}{50000}$ in. A modified form of this instrument is used for measuring compressive elastic strains. The length over which the strain is measured is $1\frac{1}{2}$ in., and the unit of compression measurable is $\frac{1}{250000}$ in.

Martens' Extensometer.—In the instrument developed by Martens (fig. 14) a clip is attached to the upper end of the test bar. At the point corresponding to the lower clip, small rectangular prisms are placed between the rods and the test piece, so that any movement of the bar relative to the rods tilts the prisms. The angular movement of the prisms, to which mirrors are attached, is measured by a telescope and scale.

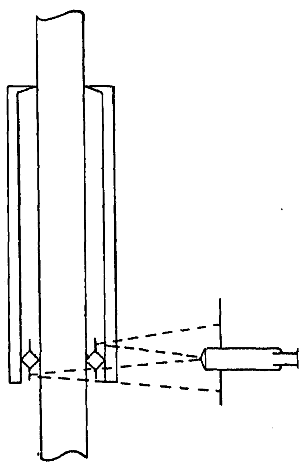


Fig. 14.—Martens' Mirror Extensometer mounted on a Tensile Test Bar

Tensile Specimens.—In some cases the specimen tests may be made on a finished object or section, yet ordinarily this cannot be readily done, and a relatively small sample or test piece is taken, which prevents a large amount of good material from being wasted unnecessarily. If the length of the test piece is parallel to the direction of rolling or extension which the material has undergone, it is called a longitudinal test piece; if perpendicular to this direction, a transverse test piece. Tensile specimens may be either flat or round, and have been standardized in dimensions. The distance between gauge marks is either 2 in. or 8 in. With specimens of 2-in. acting length (diameter 0·564 in.), less material is wasted in cutting them out of the article to be tested, but the percentage elongations computed are greater than those determined from 8-in. specimens. Gauge marks are sometimes placed at equal intervals down the length of the test piece to determine the elongation of different portions. Owing to the effect of necking, with local increase in the elongation, it is customary to eliminate specimens which have not broken in the middle third of the gauge length.

It has been found approximately true that the same results will be secured from two test pieces cut from the same material if they are of identical dimensions, or of similar form. Thus, when standard specimens cannot be obtained, the dimensions of smaller specimens are calculated from the formula:

$$\text{Acting length } (e) = k \times \sqrt{\text{area of cross-section}},$$

which is known as *Barba's Law.*

$k =$ 4 in the British standard specimen
$=$ 4·5 in the United States standard specimen.
$=$ 11·3 in the German standard specimen (in metric units).

The ends of ductile test specimens are generally gripped by serrated wedges which fit into recesses in a socket resting on a spherical seating in the shackle, so as to give a pull as nearly axial as possible. With materials of low stretch special care should be exercised in the gripping of specimens, and it is essential to hold them in ball-and-socket joints. Workmanship on specimens should be of the most careful character, and the surface should be free from nicks and tool marks. In machining test pieces, abrupt changes of section must be avoided, specially in brittle or non-extensible metal, such as cast iron or hard steel, as it will lower the value of the breaking load. In making tests on wire ropes it is important to grip the ends without damaging them. To this end a hard alloy of lead and antimony is sometimes cast on the ends in the form of conical caps, which are received into split conical dies in the shackles of the testing machine.

Determination of Stress Distribution in Tensile Specimens.— Professor Coker has developed an optical method of determining the stress distribution in test bars. The method consists in determining the lines of principal stress by observations on a transparent xylonite model in a field of view illuminated by plane polarized light, and afterwards measuring the differences of these principal stresses by comparison with a standard tension member. It has been proved, theoretically and by experiment, that loaded transparent models with one boundary give distributions of stress precisely similar to those in metals loaded in a similar manner. This method has demonstrated the disturbing effects of large ends, and abrupt changes of section, in producing uneven distribution of stress in tensile bars, also that punch marks, notches, and holes cause regions of concentrated stress. It is particularly valuable in the determination of stress distribution in cases where exact mathematical treatment is practically impossible.

CHAPTER II

Compression Test

The compression test is not a simple or satisfactory test, as pure compression stresses only occur in very short test pieces. Such tests are not generally applied in commercial testing of ductile materials, but to non-ductile metals such as cast iron. In the case of the more plastic materials there is no such thing as an ultimate strength in compression. There is, however, a definite apparent elastic limit. Beyond this the material simply spreads. Metals generally have practically the same elastic limit and modulus of elasticity in direct compression as in tension, but the tension test being much easier to make, it is usual to rely on it as an index of the mechanical properties of nearly all metals. For stresses beyond the elastic limit, hard or brittle materials generally fracture by shearing across some plane oblique to the direct compressive stress. The intensity of shear stress is a maximum

for an angle of 45° to the direction of application of the compressive stress, the fracture sometimes taking the form of two cones with their points together.

For metals the test pieces are cylindrical in shape and usually 2·5 to 4 times the diameter in length (fig. 15). The test specimens taken from materials for building construction are cubic. Tests are carried out in the tensile machine suitably adapted for the purpose. The bearing blocks which transmit the pressure should be truly normal, and one may be supplied with a hemispherical bearing. The angle of fracture and ultimate resistance to crushing are found to vary with the nature of the material in which the specimen is bedded, and receiving the external pressure. The speed of compression should be slow. Up to the point of rupture the strains are small, the modulus of elasticity being determined with a precise compressometer. Several readings should be taken on approaching the proportional limit. The yield point may be noted by the drop of the scale beam. Compression tests are

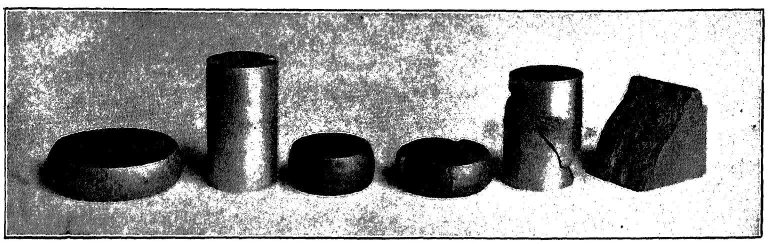


Fig. 15.—Proportions of Metal Pieces used in Compression Tests

sometimes used to determine ductility, in which case they consist in reducing a piece (either hot or cold) a considerable proportion of its original length. Wrought iron cracks longitudinally after much crushing.

For the ultimate compression or crushing strength of various materials the tables on p. 246 and following pages may be consulted.

CHAPTER III

Shear Testing

Shear stress exists between two parts of a body in contact when the two parts exert equal and opposite forces on each other laterally in a direction tangential to their surface of contact. The only method of obtaining pure shear is by torsion of a cylindrical test piece, though even in this case the intensity is not uniform, and above the elastic limit its distribution is not accurately known. In practice, however, shearing tests consist in determining the stress required to shear or cut material such as plates or bars. It is quite possible to obtain results with an apparatus on the principle shown in the diagram (fig. 16). The relative movement of the two parts may be obtained

by thrust, as in a compression test, or by pulling by means of tension shackles. The blocks may be arranged to take round or square bars. A great deal depends, however, on the alignment and closeness of the parts, and the truth and sharpness of their edges. When plastic flow begins, local crushing of the material occurs accompanied by lateral flow, and the conditions become rather vague and complicated. A machine for autographically recording the stress-strain diagram of a static shearing test has been devised by M. Frémont.

It is of special importance to know the relation between the shearing and tensile elastic strengths. Guest's law for ductile materials is to the effect that elastic shear strength is one-half of the elastic tensile strength, but the law is not justified by experimental results. It has been found that the correct value of the elastic shearing strength may be determined from a torsion test of a hollow thin-walled cylindrical specimen, or may be taken as 85 per cent of the elastic strength obtained from a torsion test of a solid cylindrical specimen. The elastic shearing strength is from 55 to 65 per cent of the elastic tensile strength. The breakdown of shearing elastic action of ductile or semi-ductile steel is gradual. The elastic strength in shear is little affected by the direction in which the material has been rolled.

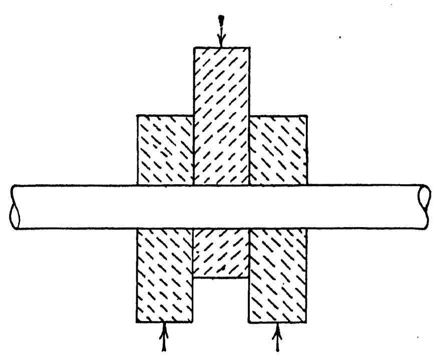


Fig. 16.—Diagram of Shearing Test

The shearing modulus of elasticity is the ratio of the shearing stress to the corresponding angular strain, and is about two-fifths of the modulus of extension.

Punching Test.—This test measures the shearing strength of a material. In one form of this test, flat plates are placed over a hole in die. A plunger which exactly fits the hole is fixed in correct alignment, and placed in a tensile machine arranged for compression test. A disc of the same diameter as the hole is punched out, the load necessary for this being noted. In practice, punching tests usually consist in punching holes in a sample to determine the suitability of the material for this kind of work.

CHAPTER IV

Torsion Tests

The testing of metals in torsion is not very widely practised. When properly designed, however, only pure shearing tests arise up to the point of plastic flow. In solid specimens only the outer fibres are subjected to the maximum stress. For this reason hollow specimens are sometimes used.

In making this test the cylindrical piece is provided with enlarged ends, generally square, which are fixed in the two jaws of the machine, one of which is capable of rotation by suitable gearing. The tendency of the other end to rotate is resisted by means of a spring balance, or counterpoised by a jockey weight moving over a graduated lever, and in this way measured. A diagram showing the relation between twisting moment and

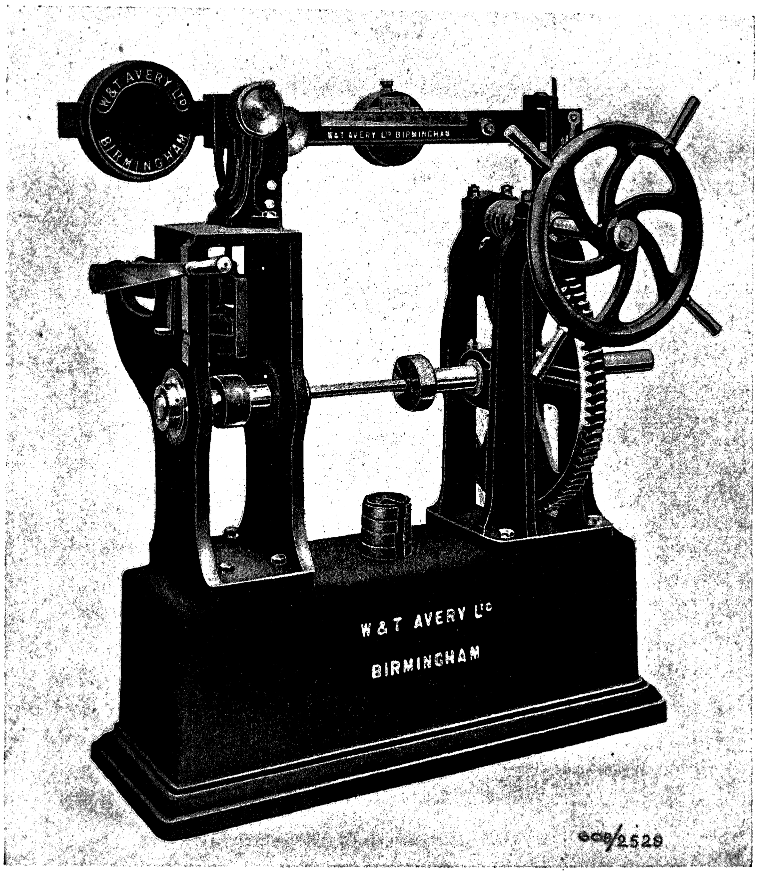


Fig. 17.—Avery's Tortion Testing Machine, up to 10,000 lb. capacity

the angle of twist can be prepared and the elastic limit, yield stress, and breaking stress in shear determined. For great refinement in the measurement of torsional strain, special instruments have been devised, such as Coker's torsional-strain measuring apparatus.

The dimensions of a test piece widely adopted are: overall length 5 in., the parallel portion being $\frac{5}{8}$ in. in diameter and 2 in. long. ends of the parallel portion turned to $\frac{1}{2}$ in. radius.

Wrought-iron specimens $\frac{5}{8}$ in. in diameter may be twisted through four complete rotations in a length of 8 in. before fracture. Cast iron breaks perpendicular to the length; mild steel at about 45°.

In the Avery torsion machine (fig. 17) the angle of twist is at first increased by increments of $\frac{1}{2}°$, and the twisting moment indicated by the machine for each angle observed. A twisting-moment torsional strain graph is plotted up to fracture of the test piece.

To convert the twisting moment into terms of shear stress in twisted shafts, the following formula may be used:

$$\text{T}w\text{.M.} = \frac{\pi}{16} D^3 f,$$

where f = the intensity of shear stress at the skin of the material,
D = the diameter of the parallel portion.

This formula is, however, only strictly applicable to conditions within the elastic range; it does not indicate the real maximum shear. A more likely quantity is obtained from $\text{T}w\text{.M.} = \frac{\pi}{12} D^3 f$

CHAPTER V

Bending Test

A common test for structural steel is the bend test for ascertaining whether a given sample of metal possesses a certain minimum of ductility. The cold bend test is made by bending a bar or strip round a mandrel of specified diameter or flat on itself. The bending may be accomplished either by pressure or blows of a hammer. As by the latter method only common workshop appliances are required, the test is a very common as well as a very good one. To withstand this treatment without fracture or cracking is evidence of the material's ductility, as the outer surface undergoes considerable elongation. Wrought iron or steel which will fold completely through 180° with zero radius of curvature is undoubtedly of high quality. For comparative work the strips or bars must have the same thickness. The requirements of the British Engineering Standards Committee for structural steel is that the test must bend double without fracture over an internal radius not greater than one and a half times the thickness. For aircraft material with a tensile strength up to 60 tons per square inch, the specimen is expected to bend over a radius equal to its own diameter without signs of fracture. Commercial work is discontinued here, but experimentally bending is carried on till the specimen breaks or bends double.

The bend test is often specified to occur at different temperatures. For iron and steel it is sometimes specified at red or blue heat. The temper bend test for structural steel consists in heating specimens to a blood-red heat, and quenching in water below 80° F. (27° C.) previous to testing.

If flat test pieces have been sheared out, it is usual to machine or grind

the sheared edges to cut away the material which may have been hardened by shearing.

As will be seen, the ordinary bend is only qualitative; a more quantitative test is given by using bars of such thickness that they will not bend close without breaking, and comparing the angles of flexure at which fracture occurs. It is particularly applied in the case of relatively brittle or high elastic limit material such as springs. These flexure tests are often carried out in the tensile testing machine arranged for compression. The bars

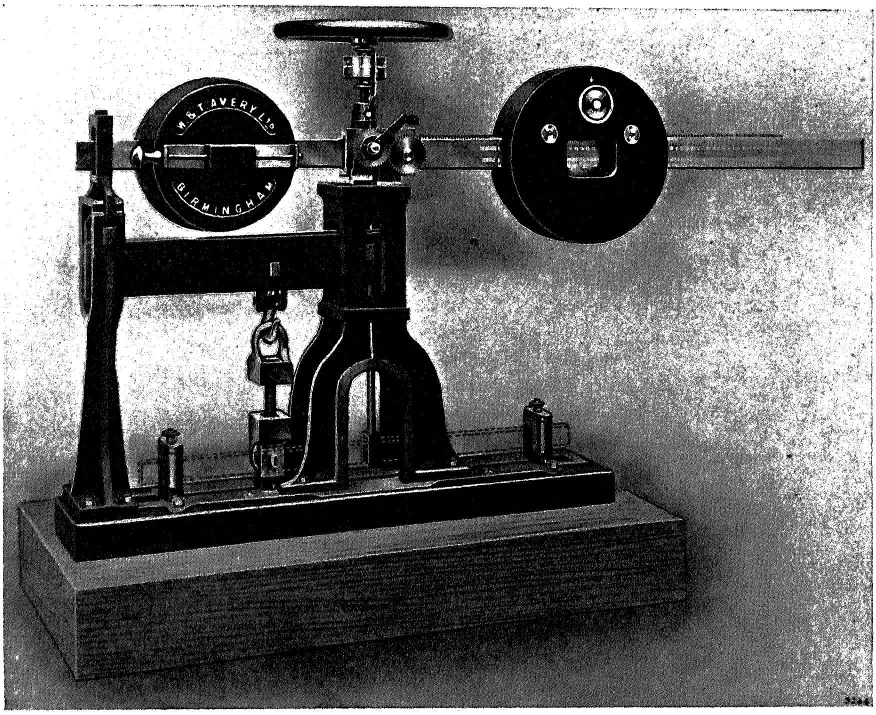


Fig. 18.—Transverse Bar-testing Machine

are placed across supports arranged with a definite span, and loaded at the centre by means of a tool, which is pressed down vertically. For different materials, the size of specimens and the span is varied. In one specification rectangular pieces $\frac{3}{8}$ in. × $\frac{3}{4}$ in. section and about 8 in. long are supported broadside horizontally on rollers of 3 in. diameter at $5\frac{1}{4}$ in. span. The stresses and corresponding deflections are usually plotted in a stress-strain diagram, including any permanent set after each successive load.

A common requirement of cast iron is that a test bar 1 in. wide and 2 in. deep, on a span 36 in. between supports, shall have a central breaking load of not less than 26 cwt. Another is that a bar of 1-in.-square section shall have a breaking load of at least 2000 lb. on a 12-in. span. A small transverse bar-testing machine is made by Messrs. Avery for cast-iron foundry bars of sections up to 2 in. deep and 1 in. broad (fig. 18). The strain is applied by

a screw turned by a hand-wheel which puts an upward pull midway on the beam and a downward thrust on the lever near the fulcrum. The load is measured by balancing the lever by the travelling poise.

A bending-test machine is also made by Messrs. Amsler (fig. 19) for rapidly performing transverse tests on cast-iron bars, and for slow bending tests on steel, using a notched bar. It consists essentially of an A frame, housing the mechanism and supporting the bending beam. The test bar is raised by a plunger actuated by oil pressure, the reaction being supplied

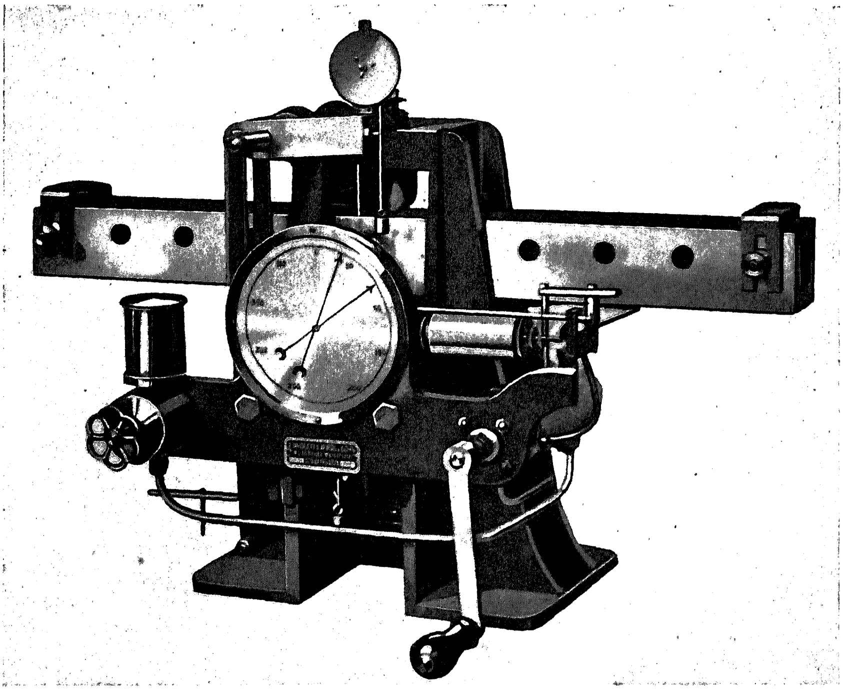


Fig. 19.—2-ton Transverse Testing Machine

by a middle bolster fixed in the crown of the A frame. The load is measured by the pressure of oil actuating a piston in a small measuring cylinder. The piston is connected to an overhung pendulum, the deflection of which balances and measures the load. A loose pointer indicates the maximum load attained, and a recording drum is provided for making an autographic diagram.

Humfrey has devised a machine autographically recording the stress-strain diagram during the static bending of a notched test piece.

CHAPTER VI

Hardness Testing

Another static test of considerable importance, and one in which there is still room for more exact definition and standardization, is that of hardness testing. Hardness is a term that has variously been defined, but it is generally taken as the resistance of a material to permanent deformation, or to local displacement of portions of its substance. The first may be termed indentation hardness, and the latter scratch, or abrasive, hardness.

Indentation Hardness.—In these tests the surface of the material is permanently distorted by the pressure of a hard steel ball, cone, knife-edge, or pyramidal point. Such a test is especially suitable for ductile materials. The weight causing a given depth of indentation, or the reciprocal of the depth of indentation with a given weight, have been taken as the measure of hardness.

Foeppl's method consists in testing the material with itself. Two cylinders are placed at right angles, and pressed together in a testing machine. The pressure per unit of flattened surface is taken as a measure of the hardness.

Dr. Unwin has used a small square bar of hardened steel as an indenting tool, one edge of which is pressed into the specimen in an ordinary testing machine. The hardness number was taken as the load in tons divided by the depth of impression. The ratio (tensile strength)/(hardness number) was found to be nearly constant.

All pointed indenting tools may lose their sharpness, and subsequent results may be considerably affected. Probably a spherical ball offers the best form for the purpose of making the indentation.

Brinell Ball Test.—Of the various hardness-testing methods based on the resistance of the material to indentation, this test has proved the most satisfactory for general purposes, and is of universal application. It is a static test, and consists in pressing a hardened steel ball under a known pressure (3000 Kg. for a ball of 10 mm. diameter) into the surface of the material, and measuring the diameter of the indentation by means of a micrometer microscope. The hardness number is then calculated from the formula

$$\text{Hardness number} = \frac{\text{total pressure}}{\text{curved area of the impression}} = \frac{P}{A}.$$

The hardness number is read off immediately from a table or suitable slide rule.

In recent practice the diameter of the impression is kept the same on every material, and the pressure required is then a measure of the hardness.

Different loads actually give different hardness numbers for the same material; probably a hardness number based on P^n where n is a constant for a

given material would give a constant hardness number for different pressures.

If a ball flattens in service the impressions are not circular, and erroneous readings will result. For this reason the ball should be frequently changed, specially when testing hard material. In the case of very hard material the ball deforms, and above a hardness figure of about 630, or in the region of superhardness as it has been termed, the method breaks down.

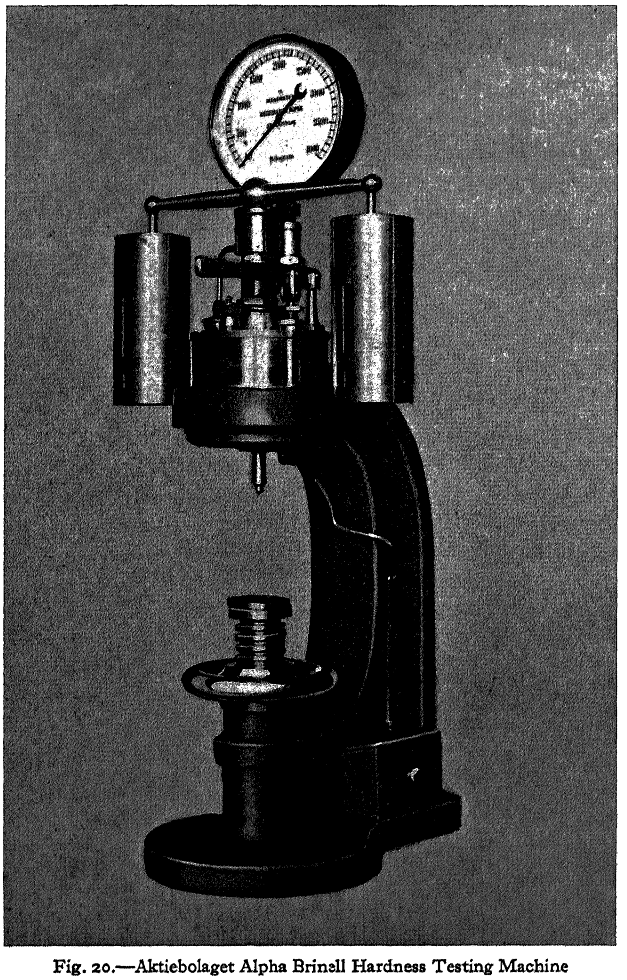


Fig. 20.—Aktiebolaget Alpha Brinell Hardness Testing Machine

In metals having a coarse structure the impressions are also irregular, and a number of measurements of the diameter should be taken to obtain a mean value.

Tests made by balls of different diameter can be rendered fairly comparable by employing Benedick's formula

$$\text{Hardness number} = \frac{\text{load}}{\text{superficial area of indentation}} \times \sqrt[5]{\text{radius of ball}}.$$

The time of application of the pressure in the Brinell ball test does not seem to have much influence on the hardness readings, i.e. after the first few seconds, but it is advisable to keep the pressure on for a fixed time, say half a minute, and maintain this in all tests. The Brinell hardness is not a safe guide to the power of a material to resist abrasion, but is found to bear a definite relation to the ultimate strength determined by tensile tests, in fact the test forms a handy way of getting the approximate tenacity of steel without the cost of preparing an ordinary tensile specimen. The relation between hardness and other properties of steel does not vary if the samples are of the same composition and have received the same heat treatment. To calculate the tensile strength from the Brinell hardness the following formulæ have been proposed as best fitting the results.

$$\text{Tenacity (in tons per square inch)} = \frac{\text{Brinell Hardness Number}}{5} + 6.$$

Tenacity (in kilograms per square millimetre)

$$= \frac{\text{Brinell Hardness Number}}{32} + 10.$$

Only a small amount of material is required for making a Brinell test; on the other hand, it has the disadvantage of testing only a very local surface. To judge the uniformity of a specimen a number of tests must be taken over the surface and through the thickness.

Ball Testing Machines.—Brinell tests are frequently made in the ordinary tensile machine arranged for compression, the ball being held in a suitable holder, and the pressure determined from a balanced dead load or sensitive gauge which is calibrated on intervals.

In the machine made by the *Aktiebolaget Alpha* (fig. 20), the ball is attached to the lower side of a piston above which the necessary oil pressure is applied by means of a small hand pump. As soon as the requisite pressure is reached a small upper piston carrying a cross bar and dead load rises, thus preventing a further increase in pressure.

The *Avery Machine* (fig. 21) is designed on the lever principle so as to maintain its accuracy for an indefinite period. This machine is fitted with a standard 10-mm. ball, which is forced downwards against the specimen, the load being communicated through the lower platen, which rests upon weighing lever, to two steelyards, from the ends of which are suspended balance weights, this corresponding, as desired, to loads of 3000 Kg. and 5000 Kg. on the platen.

A wheel attached to the screw is provided for quick setting, after which the screw is locked by a spring bolt at the top, and the strain applied by the rotation of a hand-wheel at the side of the machine.

The poises are fitted to the steelyards for the purpose of balancing off the weight of the specimen. In operation the weights rise when this load has been reached, and this prevents any greater loading being applied.

Guillery Machine.—This appliance is a modification of the Brinell method,

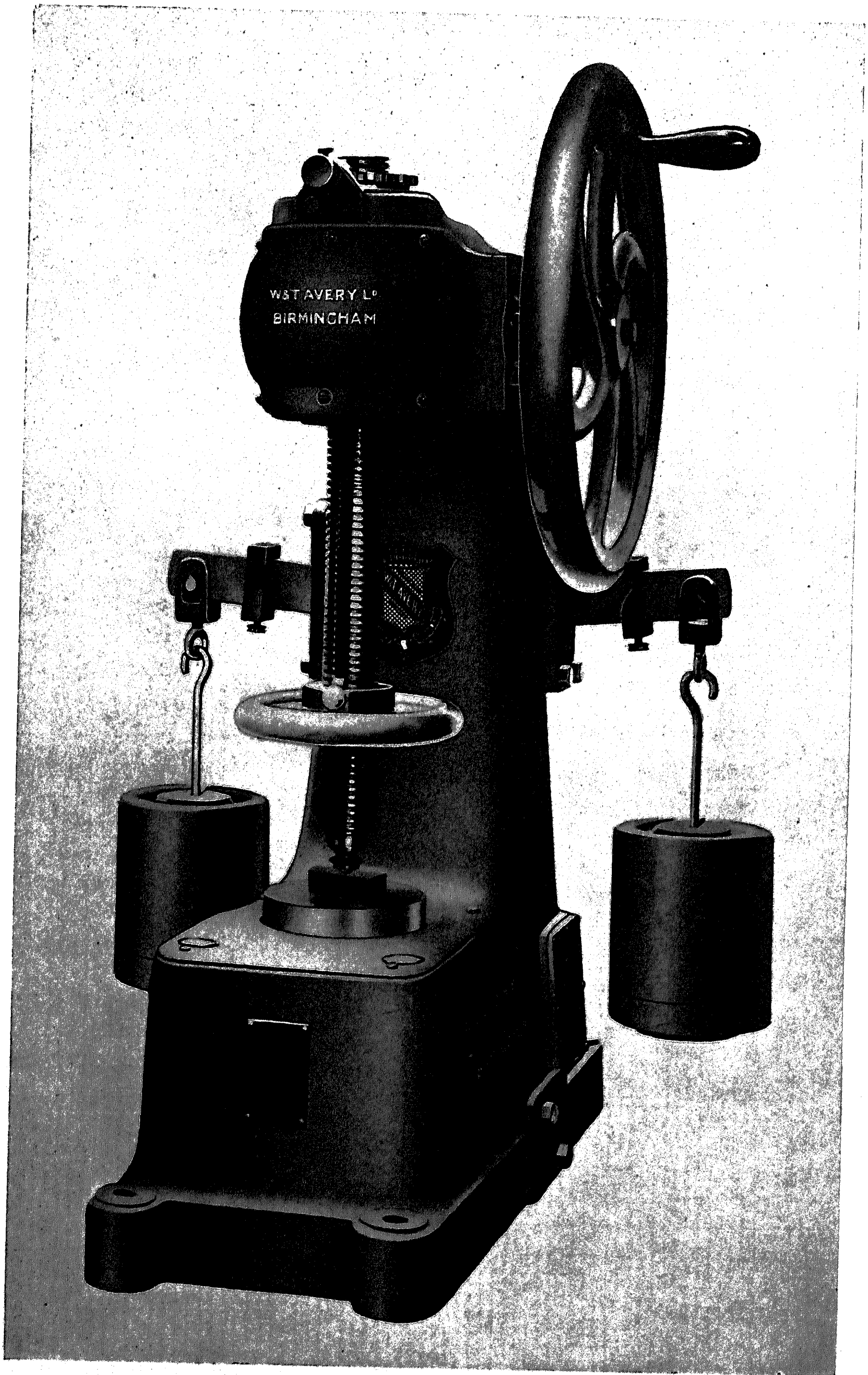


Fig. 21.—Avery Brinell Hardness Testing Machine

the pressure being transmitted upwards by a system of hand levers against a steel ball, above which is placed a column of spring discs calibrated to a maximum load. The case containing the discs and the steel ball below it can be adjusted to receive specimens of different thickness.

One of the strongest objections which may be raised against the common method of getting the Brinell hardness number by the measurement of the imprint diameter, is that the raised ridge caused in a soft piece of metal gives too large a reading, which results in too small a hardness figure. This effect is of course different to the low values obtained on very hard specimens due to the flattening of the ball. To obviate the above objection, means have been devised of measuring the depth of penetration of the ball into the material under test, and the spherical area of impression is calculated from this figure. A depth indicator or depth gauge merely consists of an amplifying or magnifying arrangement for indicating the depth of the imprint. The testing machine to be described depends on the direct de termination of the depth of the ball impression.

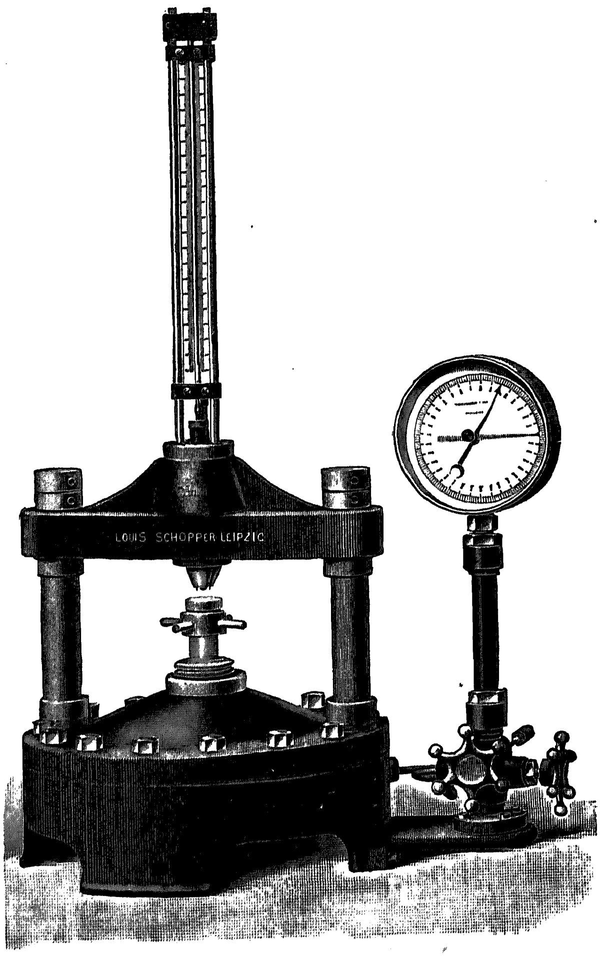


Fig. 22 —Martens' Hardness Testing Machine

Martens' Machine (fig. 22) is designed for obtaining the hardness number by direct reading on a scale. Three steel pins project from the steel blocks holding the ball, on a level with the lowest point of the ball. When the ball is forced into a test piece, the movement of the pins is communicated to a piston working in a reservoir of mercury, which is pushed up a thin capillary tube to an extent corresponding to the depth of penetration of the ball. From this quantity the superficial area of the impression is readily calculated, or if the load is constant the scale can be calibrated in hardness figures.

The *Brinell Meter* (fig. 23) has been designed as a portable workshop instrument. A hardened steel ball is held between the test piece and a specimen of known hardness. A comparison of the diameters of the two impressions—which may conveniently be measured by means of a V-shaped scale engraved on celluloid—enables the hardness of the test piece to be determined from a table, or it can be read off on a suitable slide rule.

The standard Brinell test with a 10-mm. ball cannot be applied to determine the hardness of thin sheets of material, or within a certain distance of the edge of thick specimens. When it is necessary to determine the hardness of thin specimens, machines have been devised using a very small ball and a few kilograms dead load.

Fig. 23.—Brinell Meter Outfit

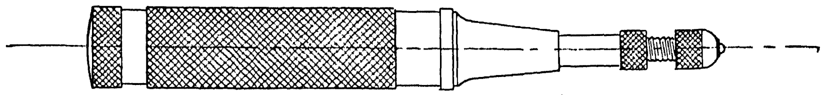
The *Ludwig cone* hardness test is a modification of the Brinell test in which a hardened steel conical point of 90° angle is used in place of a steel ball. There are some advantages in calculating the result from the conical indentation.

A machine made by Amsler-Laffon employs a 90° rounded cone which is forced into the material under constant load, and the depth of impression shown on a graduated scale by means of a needle indicator.

Dynamic Ball Tests.—Brinell designed a hardness-testing apparatus, in which the impression of the ball is made by the impact of a definite mass with a definite height of fall, and found that for harder steels the ratio of the hardness numbers to those obtained by the static test diminished if the ratio for a soft steel were made unity. For a moderate range of low carbon steels, the static hardness number has been found to be approximately proportional to the square of the number found by the ball hardness test under constant impact. It has also been found by experiment that if a hammer is allowed to fall with a known amount of energy, and to transmit its blow to the metal under test through a hardened steel ball, the relation between the impact energy and the diameter of indentation for each metal agrees with the equation: Diameter $=$ (energy)$^{0·25}$ $\times$ a constant (K), so that if hardness is defined as resistance to penetration under an impact blow, the inverse of K is the true measure of the hardness of a metal. Other investigators have found that if

H is the Brinell hardness number obtained in the static test of a metal, and *d* the diameter produced by an impact of 63 in.-lb., the following relation holds good: $H = \frac{7455}{d^3}$.

An *Auto-Punch* (fig. 24) has been developed by the Rudge-Whitworth research laboratories in which a small spring hammer delivers an impulse



Institution of Mechanical Engineers

Fig. 24.—Auto-Punch

to a $\frac{1}{8}$-in. steel ball. This pocket instrument is used for rapidly testing small hardened articles. The diameter of the impression is measured by a transparent V-scale. On reference to a calibration curve, this quantity gives the approximate Brinell hardness number.

Shore Scleroscope—This instrument supplements the Brinell ball machine in hardness testing (fig. 25). It consists essentially of a vertical graduated glass tube containing a small diamond-pointed drop hammer

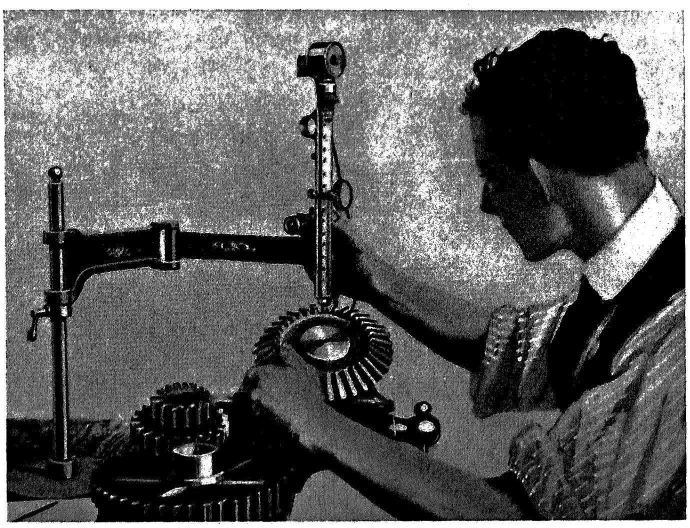


Fig. 25.—Shore Scleroscope in use

weighing 1 gm. By means of a suction bulb a vacuum is produced in the glass tube, and the hammer is drawn up to the top, where it is held by hooks. By further application of pressure to the bulb the valve actuating the suspension hooks is caused to act, and the drop hammer released, the height of fall being 10 in. The sample to be tested is solidly mounted under the hammer, which rebounds from the surface of the test piece up the graduated glass tube, the height of the rebound being noted on the scale. The rebound is of course greatest from a hard surface, for in a softer one

more of the energy of the hammer is spent in the work of indentation. Two hammers are generally supplied with an instrument, one for use on steel and the harder metals, and the other for magnifying the readings on the softer metals. The surfaces of test pieces must be properly prepared. Hard surfaces in particular should be perfectly smooth. In the case of steel any decarbonized surfaces must be removed, otherwise readings will be too low. The hammer should not be allowed to drop more than once on the same place, as the surface is strained at the point struck, and subsequent readings will be too high. This test should not be applied to small or thin specimens, and care should be taken that the falling tup strikes a normal blow. The scleroscope measures a value closely associated with the elastic limit of the material. It is very useful in exploring the whole surface of a specimen with a view to detecting variations, for example gear wheels. Another field of usefulness is in the case of hardened steel, such as cutting tools on which the ordinary ball test makes little impression. A modification of the scleroscope, which has recently been developed, consists in automatically recording the height of rebound by a pointer moving over a circular scale, the pointer remaining stationary at this reading until another test is taken.

Brinell and scleroscope tests have the advantage that they be taken without damage to the material to be used, instead of being vicarious tests on samples chosen to represent the material. Above a hardness figure of 300 Brinell, or 45 scleroscope, materials are difficult to machine. There is a fairly well-defined relation between Brinell hardness and scleroscope hardness for different grades of iron and steel. Recently, using a 10-mm. diamond ball, and reducing the Brinell pressure so as not to exceed the elastic limit appreciably, Mr. Shore obtained very good correlation. The following factors have been given for conversion of scleroscope figures to Brinell figures: steels, 6·67; cast iron and bronzes, 4·6 to 5·25; aluminium, 6; nickel, 7·7.

Scratch Hardness; Abrasive Hardness.—A simple practical method of measuring hardness is by means of a file, the surface of the material being worn away under the cutting action of the file teeth. This is obviously not a scientific means, but in the hands of an experienced man successful results are obtained. The abrasion test by a file has been developed in the Heathcote sclerometer, which seeks to eliminate the variable elements of hand application. The instrument consists of two files hinged together with one placed horizontal. This combination grips a soft material at a greater angle than a hard one. The hardness is determined by the angular position of the files, and is read off on a graduated quadrant.

The earliest scale of hardness is that proposed by the mineralogist Moh, and is based on the fact than an object will scratch one of equal or less hardness, but not one which is harder. Arranged in order of increasing hardness, the scale is: (1) talc, (2) rock salt, (3) calcite, (4) fluorspar, (5) apapite, (6) orthoclase, (7) quartz, (8) topaz, (9) corundum, (10) diamond. A copper coin has a hardness of 3, ductile iron a hardness 4·5, and a file a hardness of 6·5.

A scale applying to metals has been given by Behren, based on the use of sharp-pointed needles for testing; but certain of the materials are too variable for the scale to be anything more than approximate, and only a few are given below.

Lead	1	Gun metal	3·3
Tin	1·7	Iron wire	3·7 to 3·9
Zinc	2·5	Sewing needles	5 to 5·5
Copper	3	Chrome steel	6·2 to 6·5
Brass wire	3·1	Ferro chrome	7 to 7·3

For determining scratch hardness various instruments have been devised, termed sclerometers, but it is doubtful how far they are applicable to metals

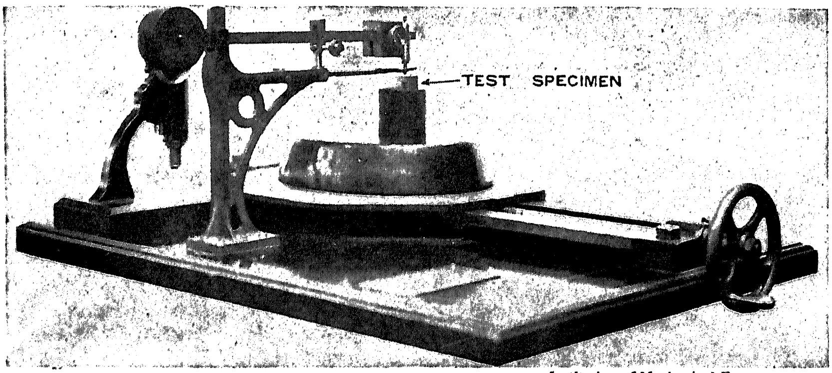


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Fig. 26.—Scratch Testing Apparatus

like modern steel. The apparatus (fig. 26) consists essentially of a balanced lever arm, fitted with a jockey weight carrying at its end a diamond point, which can be drawn along the surface of the metal to be tested, making a scratch. The test piece is traversed on a table by means of a screw, and the width of the scratch is measured by a special measuring microscope.

Turner's method depends on the weight necessary to produce a visible scratch. Martens used two methods: (1) a constant weight was used, and the widths of the scratches produced in different materials were compared; (2) the hardness number was defined as the weight in grammes, producing a scratch 0·01 mm. wide. Several scratches were made and the loads and widths plotted. Using a diamond point ground to 90°, Martens obtained the following hardness numbers: lead 17, copper 34–40, soft steel 71–77, hard steel 138–141.

Abrasion has been defined as the mechanical removal of particles by the rubbing action of another substance. Various methods of testing abrasion have been devised, based on the depth of penetration of a *drill* operating under constant conditions of speed and pressure. In *Keep's test* an autographic record is taken of the progress of the drill. A micro-sclerometer has been constructed which is an instrument consisting of a

small weighted drill provided with a diamond point. The depth of the hole produced by drilling for a certain length of time at constant speed is measured. It is intended to be used with a microscope for determining the relative hardness of the microscopic constituents of metals. In another form of wear-test the specimen is abraded under constant load by means of a grinding wheel or revolving disc covered with a suitable abrasive.

The resistance to wear in the rubbing together of smooth bodies, as in many machine parts, is another aspect of hardness, not identical with resistance to penetration or to scratching with a fine point or points. In tests

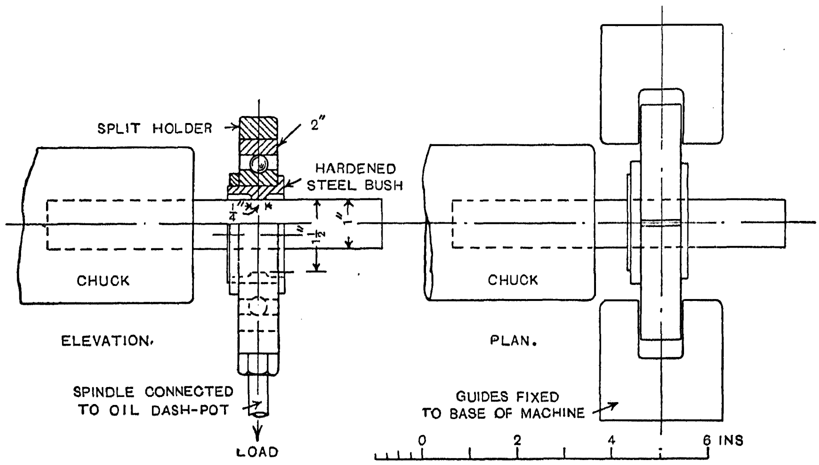

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Fig. 27.—Diagram of Saniter's Machine

of this type of abrasion hardness the particles of the material are torn away by sliding contact with some other substance whose corresponding resistance is so high that its surface remains unimpaired. Machines for measuring the wear on sliding or rolling surfaces have been developed by Saniter, Stanton, and others, in which cylindrical specimens are rotated in contact with other materials.

In *Saniter's machine* (fig. 27) for testing dry rolling friction, a revolving test piece drives by friction on its top side the inner ring of a loaded ball bearing. An essential test condition is the absence of vibration. The results on carbon steels showed a general relation to the corresponding Brinell number, but on alloy steels the relation entirely broke down. That the test is, however, reliable is seen from the best wear number being given by Hadfield's manganese steel, the excellent wearing properties of which are well known.

CHAPTER VII

Dynamic Tests

Engineers were for a long time wont to be content with the information derivable from the static tests already described. Brief consideration will show, however, to what extent they differ from conditions obtained in practice. More nearly to realize these conditions dynamic tests have recently been developed. In the main they are intended to imitate the conditions which engender vibrations or shock, and therefore to give a better index of the ability of the metal to withstand the destructive influences of service.

The power of resisting shocks or alternating loads is not identical or even proportional to the power of resisting a steady load. Materials which have failed in service have often subsequently been found to show satisfactory static tests. The effect of a stress depends largely upon the manner in which it is applied. Thus, a suddenly applied load has a greater effect than a stress slowly and steadily applied. Again, if the stress is repeated, each application being made before the material has been given time to recover from the preceding, it will eventually break, even though the stress is below the elastic limit. Further, if the stresses are not merely repeated, but reversed, the resistance to fracture is less than if the same intensity of only one kind of stress is repeated.

Fatigue.—Fatigue may be defined as the weakening which a material undergoes as the result of repeated application of stress, though it is doubtful if the term gives a correct idea of what actually occurs in the material. It depends on the range through which the stress varies. If the stress is nearly equal to the true breaking load, a few applications suffice to cause fracture. On the other hand, if the stress is below a value termed the fatigue or endurance limit, its repetition may be indefinitely continued. The stress may vary from a higher to a lower value of the same sign, or from positive to negative, i.e. tension to compression. In each case the limiting stress which can be borne depends on the algebraic range of stress, and is almost independent of the actual value of the maximum stress. Much light was thrown on the behaviour of iron and steel under fluctuating stress by an exhaustive series of experiments made by Wöhler, employing tensile compressive and torsional stresses usually with 60 alternations per minute. The forces were applied and measured by the deflection of springs of known stiffness. The results are summarized in *Wöhler's Law*, which may be expressed as follows. Rupture of material may be caused by repeated stresses, none of which attains the breaking limit. The differences of the limiting strains are sufficient for the rupture of the material.

Method of making Wöhler's Test.—Wöhler's alternating stress test is really a bending test (fig. 28). A rod of the material under examination is held at one end in the chuck of a heavy lathe, or specially designed machine, and rotated at a steady rate. By hanging on a yoke support-

ing a weight, any given portion of the test piece is exposed to alternating tension and compression once in every revolution, the maximum stress being experienced by the outer fibres. Vibration of the test bar must be avoided. The number of revolutions required to break the specimen is recorded on a counter, and the machine is automatically stopped when fracture occurs.

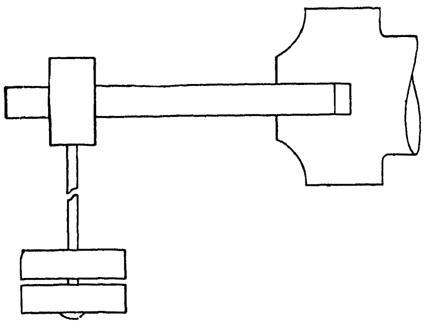


Fig. 28.—Diagram of Wöhler Test

Duplex Fatigue Testing Machine.—Criticism has been raised against the loading of the specimen as a cantilever as in Wöhler's machine, which develops the maximum stress at a single cross-sectional plane of the test piece. A type of fatigue test is sometimes employed, for example, in the

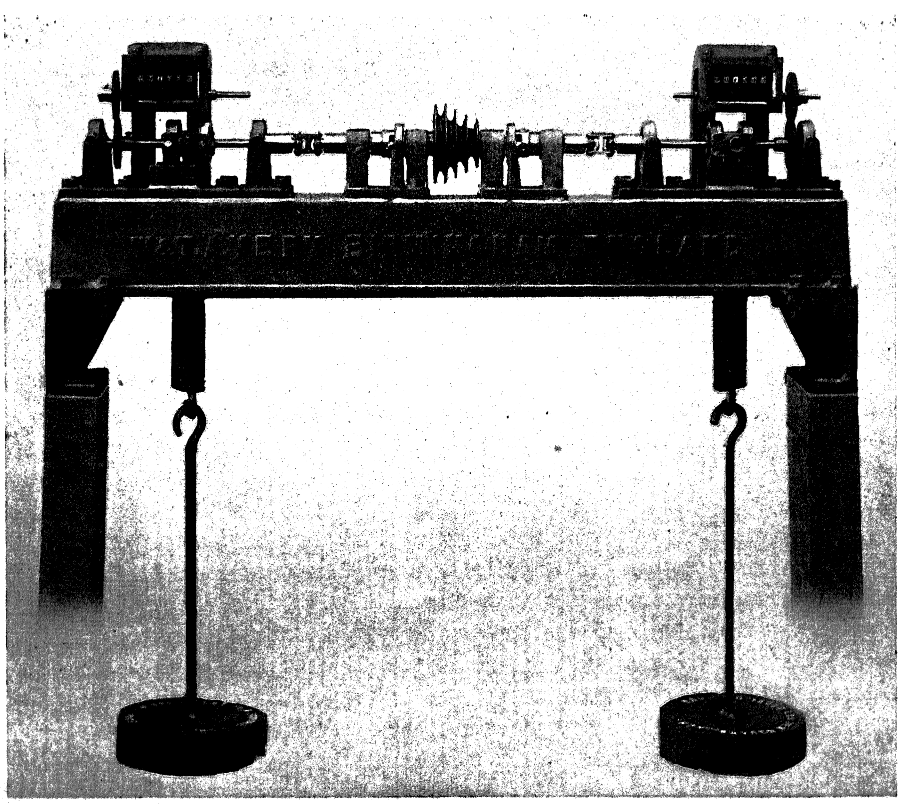


Fig. 29.—Duplex Fatigue Testing Machine

testing of welds, in which a uniform stress is developed throughout the middle portion of the test specimen. To this class of machine belongs

Avery's Duplex Fatigue Testing Machine shown in fig. 29. Each of the two round specimens, which are under test at the same time, is secured at each end in suitable sockets. A load is applied at two points in the centre of the length of the specimen. The two specimens are then caused to rotate by a belt-drive on a cone pulley at the centre of the machine, and the number of revolutions before the specimen fractures is recorded upon consecutive counters.

For practical purposes the safe range of stress is determined, i.e. the

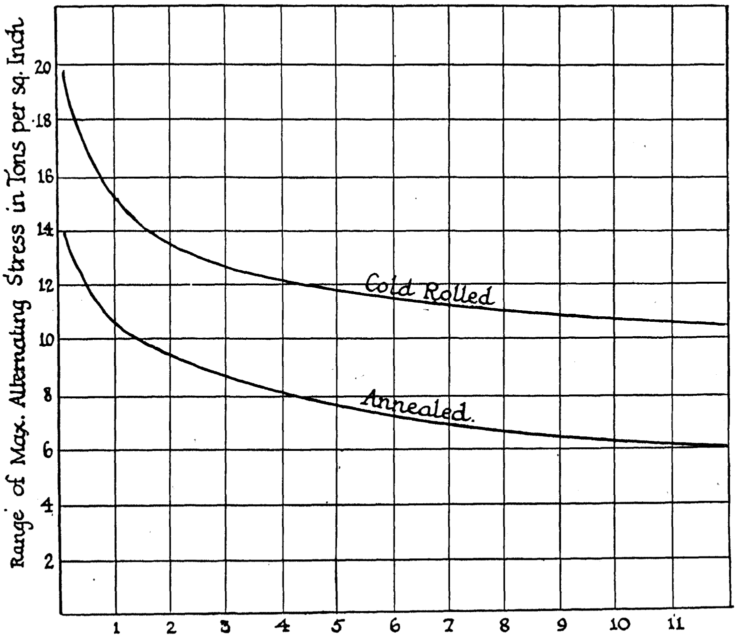


Fig. 30.—Wöhler Test Curves for Brass

intensity of stress which the metal will resist indefinitely (fig. 30). Three or four tests are usually carried out at different intensities of loading and the results plotted. They lie on a parabolic curve, the asymptote to the intensity of stress giving the safe range of the material. The following typical results are taken from Wöhler's figures on a wrought iron for axles, having an ultimate strength of 23 tons and an elongation of 20 per cent.

Maximum Stress (tension).	Minimum Stress (compression).	Range of Stress.	Number of Repetitions before Fracture.
Tons/in.^{2}	Tons/in.^{2}	Tons/in.^{2}	
15·3	−15·3	30·6	56,430
10·5	−10·5	21	3,632,588
7·6	− 7·6	15·2	132,250,000 (unbroken)

The fatigue limit of this iron was therefore a complete reversal of a stress of 7·6 tons per square inch, i.e. a range of 15·2 tons per square inch. Wöhler also gave the following figures, showing the stresses and range of stresses which would produce fracture in iron and steel only after an indefinite number of applications in bars subjected to simple tension, compression, or bending.

	Maximum Stress	Minimum Stress.	Range of Stress.
	Tons/in.^{2}	Tons/in.^{2}	Tons/in.^{2}
Wrought iron	7·65	− 7·65	15·3
	15·80	0	15·8
	21·0	11·50	9·5
Cast steel ..	13·38	−13·38	26·76
	23·0	0	23

The complete reversal limit of stress varies from one-fourth of the ultimate strength in harder steels to one-third of that in more ductile iron and steel. The following formula has been devised to show the relation between the maximum stress (f max.) which can be applied an indefinite number of times, the range of stress (R), and the ultimate static strength (f_1).

$$f \text{ max.} = \frac{R}{2} + \sqrt{f_1 + nRf_1},$$

where n is a constant depending on the material, and is 1·5 for ductile iron and steel and 2 for harder qualities. The harder high-carbon steels show a higher limiting range of stress than the milder steels, although not so large a proportion of the ultimate static strength.

The formula given below enables the actual working stress to be determined when a part is subjected to repeated stresses.

$$\text{Actual working load} = \frac{f}{1{\cdot}5}\left(1 + \frac{\text{minimum load}}{2 \times \text{maximum load}}\right),$$

where f is the dead load working stress.

The number of repetitions of stress which a material such as iron or steel will stand generally diminishes as the speed increases, i.e. over an experimental range of 60 to 2400 reversals per minute. It should be mentioned, however, that some experimenters have not found a diminution in limiting range of stress with speed.

Cast steel of about 58 tons per square inch tenacity and 2·5 per cent elongation does not show at high speeds a higher reversal limit of stress than 25-ton mild steel.

Machines have been devised for applying reversals of simple tension and compression of test pieces, thus exposing the material to simple homogeneous stresses (fig. 31). Among these may be mentioned Dr. J. H. Smith's reversal testing machine for taking four specimens simultaneously, and used by

Dr. Stanton at the National Physical Laboratory. The results are said to be little different from Wöhler's tests if carried out under correct conditions. Several testing machines have been constructed in which the force of an electromagnet is used to apply stress to the test piece. These machines are capable of high frequency of alternation.

Dr. B. P. Haigh's machine applies directly the alternating pull produced by a magnetic flux in a laminated iron armature. The inertia forces of all masses attached to and moving with the test piece are compensated for by springs, the force of which when strained counteracts the moving masses.

By hanging a heavy load near the fixed end of the rotating bar, the Wöhler test can be converted into an alternating shearing test. Machines for alternating torsion tests have also been devised. A repeated torsional inertia load may be applied by giving a rotary oscillation to fly-wheels.

The alternating tests described above occupy a long time, to obviate which tests have been devised for bringing about fracture after comparatively

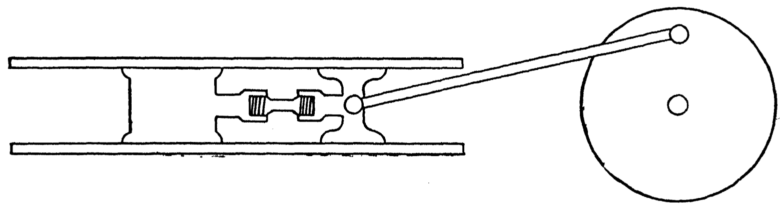


Fig. 31.—Diagram of Testing Machine for applying Reversals of Direct Stress

few reversals, or even at one blow. For practical purposes these possess a special utility and will now be described.

Impact Tests.—The failure of materials used in high-speed machinery under repeated forces of an impulsive character has led to many attempts to devise a test which will indicate the imperfection in a material likely to fracture under shock. To this end impact tests are coming into greater prominence in specifications, as the demands on materials get more exacting. This test reveals the property of brittleness or its inverse, toughness, i.e. the capacity of metals to withstand suddenly-applied load. The importance of brittleness in machine construction is apparent, it having been found that a larger number of failures accompany steel with a low impact value than steel with a high impact figure. Unfortunately the results of different machines have not been correlated, i.e. if a number of steels are compared, using one machine, another machine might place them in a different order. It should be mentioned, however, that a committee of the British Engineering Standards Association is considering the standardization of the impact test, and has already reported on the standard form of notched bar test pieces. For the reason given above, impact testing machines are chiefly used for showing whether the heat treatment of materials has been satisfactory. The results have no definite relation with either tensile or Brinell hardness tests. Other physical properties and the chemical composition must accompany the test before the usefulness of the material can be judged. The

general type of machine which has been developed on account of its great practical convenience is one in which the test specimen is broken by a single blow. The quantity measured is the specific work of rupture. It is customary to disregard any variation due to the velocity of application of the load. The effect of variation of velocity of impact is a point still in dispute, but it is apparently not serious if the velocity does not exceed a moderate value. On the other hand, some metallurgists maintain that a high velocity

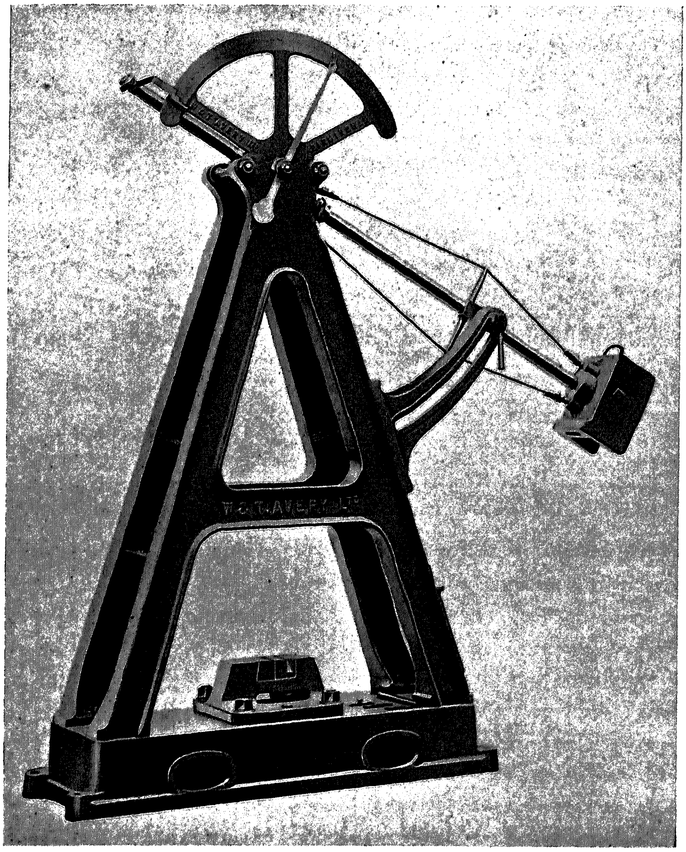


Fig. 32.—Izod Impact Testing Machine

is necessary to ensure a discriminating test, and that if the velocity is sufficiently increased even tough materials may be made to show a brittle fracture.

The test pieces used are usually notched to localize the stress and cause fracture to ensue in a definite manner.

One of the objections to machines of this type is the impossibility of calculating the proportion of the energy absorbed by the test piece, when part of the energy is necessarily spent in deforming the falling weight, the frame or anvil of the machine, and the foundations.

Single-blow Impact Testing Machines.—The *Izod Machine* made by Messrs. Avery (fig. 32) consists of a heavy pendulum swinging on ball

bearings in a vertical plane, and usually developing an energy of 120 ft.-lb. The specimen is held in a vice fixed on the bed of the machine. A pointer rigidly fixed to the upper end of the pendulum rod moves over a graduated arc, and indicates the angular position of the pendulum. The latter, having a striking edge at its centre of percussion, is swung up through a predetermined angle and held in a clamp with the axis of the notch set by gauge in the plane of the vice jaws. The specimen is fixed with its notched side towards the pendulum, which when released strikes the specimen when at the lowest point of its fall with a velocity of 3·5 m. per second. The fracture of the specimen absorbs a certain portion of the energy of the pendulum, so that, although continuing its swing, the pendulum does not rise to its original height. The end of its swing is recorded by having an idle pointer moved over the graduated scale by the upper end of the pendulum rod. The greater the resistance of the specimen to fracture, the shorter will be the distance which the pointer is carried over the graduated scale. The difference in the height of the centre of gravity of the pendulum at the starting-point and at the end of its swing gives the energy absorbed by the blow. The scale is graduated to record this quantity directly in foot-pounds, and would give the energy absorbed in fracturing the test piece if the base and other parts were absolutely rigid; for this reason it is important to have the machine bolted down securely. Specimens are usually made three in one bar to economize material. Rounded test pieces are generally used instead of square ones, as the machining can be more quickly performed. The specimens recommended by the Standards Committee are 10 mm. × 10 mm. in section with 45° V-shaped notches at intervals of 28 mm. Rounded specimens are 0·45 in. in diameter. The use of the notch is to concentrate the effect of the blow at one point; soft and ductile materials could otherwise bend over.

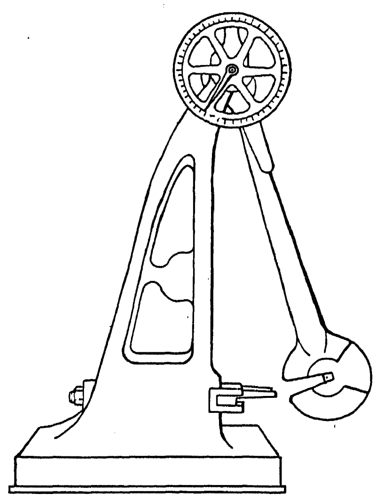


Fig. 33.—Charpy Impact Testing Machine

The Olsen machine is the American counterpart of the Izod.

The *Charpy Machine* (fig. 33), which is widely used on the Continent, is similar in some respects to the Izod, the energy absorbed in fracturing the test piece being measured by the reduction in the angle of swing of a pendulum. The test piece is placed in a horizontal position across a short span and resting against the vertical face of an anvil, the notched side being away from the pendulum. The pendulum strikes it immediately behind the notch, which is keyhole in shape, and cut at the middle of the specimen. These machines are made in two different sizes, of 25 and 200 Kgm. striking energy, the larger size being fitted with electric elevating control. It is important to keep the size of the specimens uniform and the notch exactly

to shape. The test piece used in the large machine is 160 mm. × 3(× 30 mm., and the smaller specimen 53·3 mm. × 10 mm. × 10 mm using test pieces of various sizes it has been shown that the law of sim does not generally hold. The results obtained with this machine are what irregular, the causes of inexactness being the give of the fixin tortion of the test piece as a whole, and indentation by the knife.

Frémont Machine.—This was one of the first machines devised for notched specimens under impact. Though not much used in this co its indications are considered of high value by some metallurgist engineers. The rectangular test piece 30 mm. × 10 mm × 8 mm. a 1 mm. square notch in the middle of its under side, rests on a hollo the distance between the supports being 22 mm. A tup weighing 5 Kgm., fitted with a rounded knife-edge, falls down vertical guides 4 m and strikes the specimen centrally, breaking it transversely through the The weight then falls on a small copper cylinder or crusher gauge, w compresses. The amount of compression is a measure of the residual of the tup, and this is subtracted from the known energy which the possesses before striking (20 or 60 Kgm.). The energy absorbed in br the test piece is thus obtained. An alternative method of measuri residual energy, though not as sensitive as the crusher gauge, cons allowing the weight to fall on a plate, which compresses supporting s The weight forces down a very lightly clamped hollow steel tube, the ment being magnified with a cranked indicator lever which moves previously calibrated scale.

The velocity of impact of the tup is 8·85 m. per second (29 ft. per se and is greater than in the pendulum type of impact machines. N the energy of rupture but the extent of deformation of the specin angle through which it has bent, is recorded.

Guillery Machine or Dynamometric Tup.—This machine (fig. 34) is de to have the conditions of impact as identical as possible to those ob in the Frémont machine, and in addition to possess the advantage o pactness. A rectangular specimen, 60 mm. × 10 mm. × 10 mm a V notch 1·5 mm. wide and root radius 0·75 mm., is placed in p on two supports 40 mm. apart, and the apparatus closed by mea fool-proof movable door. A fly-wheel is rotated by a handle until a speed is attained, as indicated by the rise of a coloured liquid in a which is virtually a fluid tachymeter. A knife in the edge of the fly is then released and strikes the centre of the test piece on the oppos to the notch. The specimen being broken, the speed of the rotati wheel is reduced corresponding to the amount of energy absorbed i ture. The scale is calibrated so that the work done is read off dire noting the drop in the height of the liquid column. The resilience material is measured by dividing the number of kilogram-metres ne to rupture the specimen, by the section in square centimetres at the of the nick.

When turning the flywheel with a velocity of 302 revolutions per

the linear velocity of impact of the knife is that of a body after it has fallen freely from a height of 4 metres (8·85 m. per second or 29 ft. per second), the moment of inertia of the wheel about its axis at this speed being equal to an accumulated energy of 60 Kgm.

The results obtained, however, are not quite identical to those obtained on the same material with the Frémont machine, probably due among other causes to the circular path of the knife and the less rigidity of the anvil.

Impact Tensile Machine.—Machines have been devised to break test pieces in direct tension at a single blow. The one generally used is of the

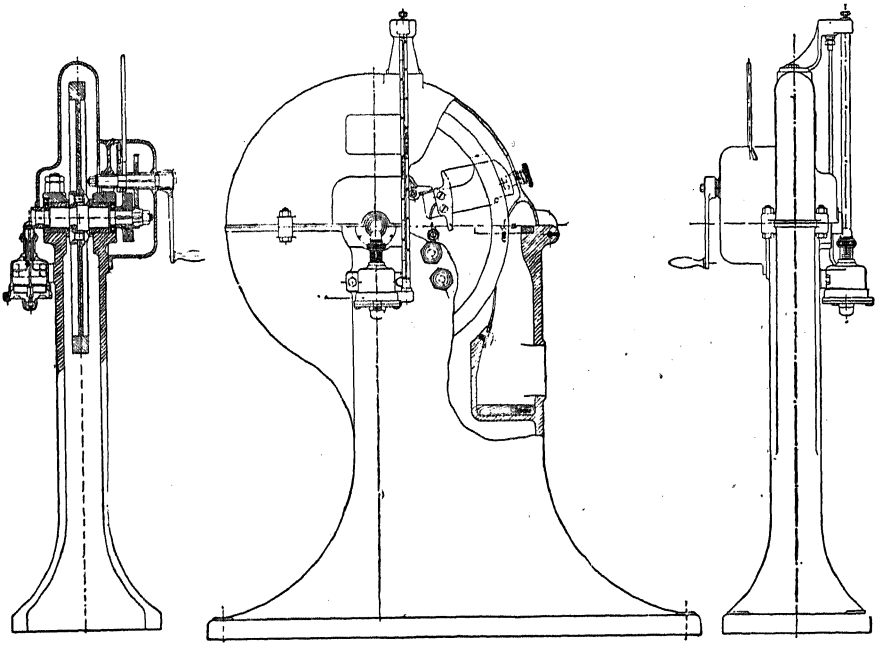


Fig. 34.—Guillery Machine

Charpy type, the blow being struck by means of a heavy pendulum. The specimen may be either rigidly attached to the pendulum itself or held by suitable supports and so arranged that, by means of heavy yokes, the test piece is broken in direct tension. The power absorbed in breaking the test piece is measured in terms of retardation of the pendulum. In a modification of this apparatus the test piece is itself fixed in a heavy anvil pendulum, which is set in motion by the blow.

Drop Testing for Car Wheels, &c.—Mention should be made of drop tests, which are generally made by subjecting a specimen to one or more blows from a weight dropped from a given height, and carefully observing the deformation and any signs of fracture. These tests are not strictly quantitative, but represent the only type of shock test in use before the introduction by Frémont of the modern method of shock testing in which the residual energy is measured after fracture of a specimen.

Repeated Blow Impact Tests.—These are made by repeated application of the load, so as to throw a given portion of the test bar alternately into tension and compression. In other words, the method combines shock with reversal of stress.

In the *Stanton Machine* (fig. 35) a hammer of known weight is arranged to fall upon the test piece from a known height, the specimen being rotated by a series of linkages through 180° between the blows. The test piece is a rod with a V-groove where the blows are struck. The revolutions of the test piece are recorded by a counter, and the number of blows is obtained by multiplying the record by two. When the test piece breaks, the machine is arranged to stop automatically.

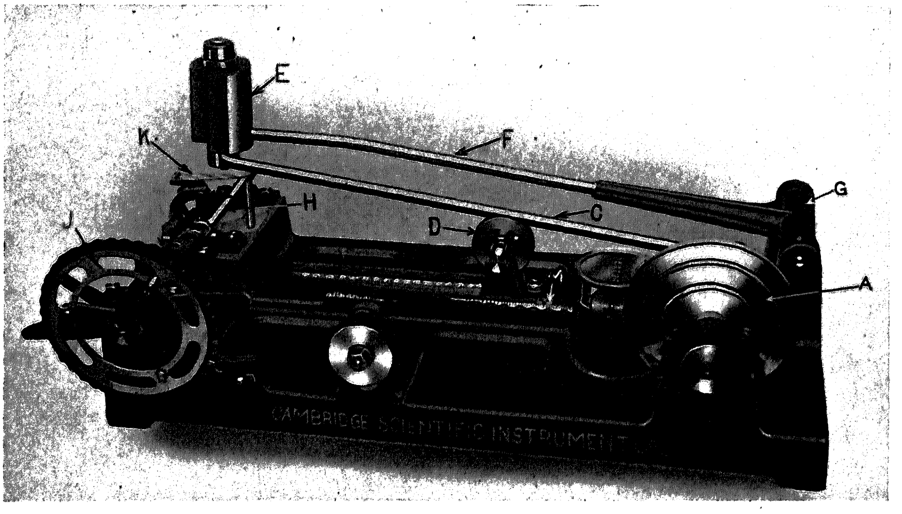


Fig. 35.—Stanton's Repeated Impact Testing Machine

The *Eden-Foster Machine* (fig. 36) is another machine which combines the effect of the pure fatigue test with impact or shock. Some workers have found it exhibits a consistency previously lacking in these two methods. The test consists in repeatedly dropping a hammer of known mass from a given height on to a specimen supported at either end in a horizontal position, the specimen being rotated through 180° about its own axis between each blow. The weight, height of drop, and speed of revolution may be varied to suit conditions. As before the stress reversals to fracture are automatically recorded.

A *Repeated Impact Testing Machine* has also been devised by Messrs. Krupp, and has been applied by them to the study of fractures of axles, crankshafts, &c. A cylindrical specimen is used with a nick made round it at the middle. A tup is allowed to fall on the bar at the position of the nick at about 85 times per minute. Between each blow the bar is turned either 180° or $\frac{1}{25}$ of a rotation.

Alternating Tests.—A common workshop test of the quality of a

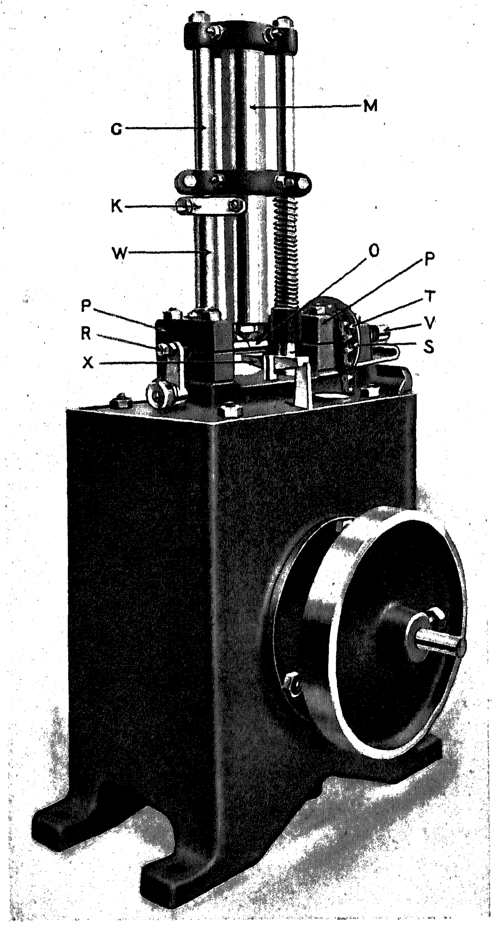


Fig. 36.—Eden-Foster Repeated Impact Machine

material is to bend a piece to and fro through a definite angle till fracture occurs. The *Sankey Machine* (fig. 37) has been devised to carry out this test whilst recording various points of information. The Sankey test is performed as follows: The test piece, $\frac{3}{8}$ in. in diameter and 4 in. long, is clamped at one end in a grip carried at one end of a stiff cantilever spring. At the other end of the test piece a long handle or lever is fixed, leaving a free length of the piece of $1\frac{3}{4}$ in. This is bent backwards and forwards, through a standard angle, which is about a radian, on each side of the straight, until the piece breaks. The slight movement of the spring actuates a mechanism which records the value of the bending moment put on the piece at each reversal. The bending moment of the first bend may be taken as a measure of the yield stress. From the autographic diagram obtained the amount of energy expended in fracture is determined, which is

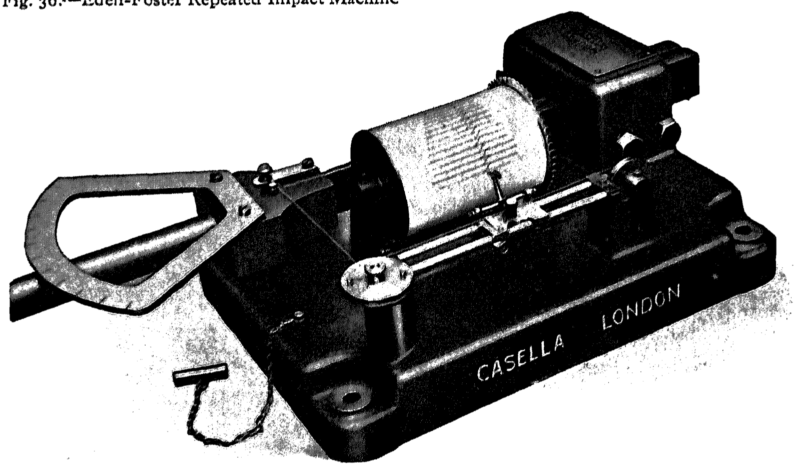


Fig. 37.—Sankey Alternating Bend-testing Machine

some measure of the quality of the material. The machine can be worked by hand, and is useful for comparing batches of material intended to be identical in quality. The number of bends may be taken as an indication of ductility. Calibration of the machine is effected by reversing the handle and applying known pulls at a definite distance by means of a spring balance.

In the *Arnold Alternating Test* (fig. 38) a bar is forcibly bent backwards and forwards through a distance equal to about one-eighth of its length, alternating bending being accompanied by a good deal of shock and vibration. The test has been applied among other purposes to demonstrate the difference in quality of different parts of large forgings. The specimen, $\frac{3}{8}$ in. in diameter and 5 to 6 in. long, is rigidly clamped at one end in a vertical position in the vice of the machine. A collar or slotted plunger fits loosely over the top of the test piece, at a height of 3 in. above the face of the dies, and is given a reciprocating motion, causing it to bend the piece alternately backwards and forwards through a distance of $\frac{3}{8}$ in. on either side of the vertical. The standard speed adopted is 650 alternations per minute. The number of alternations required to fracture the specimen is recorded. This number never reaches 2000, and the time taken is less than 3 minutes per specimen. As in this test the elastic limit is exceeded, the intensity of stress is unknown. The Sankey and Arnold tests give a measure of the same specific properties of the material, but are not fatigue tests, owing to the material being strained about its elastic limit.

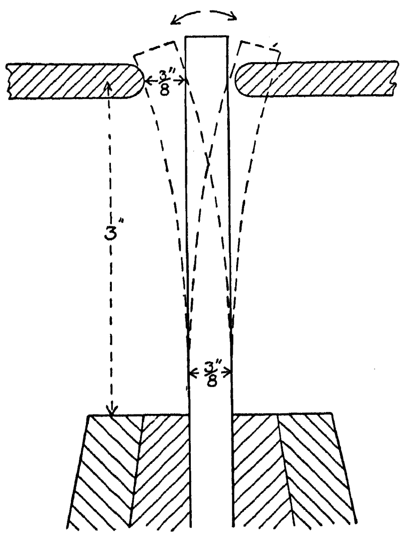


Fig. 38.—Diagram of Arnold's Repeated-impact Testing Machine

Factors of Safety.—The greatest calculated stress to which a part of a machine or structure will ever be subjected is called the *working stress*, and the ratio of the ultimate strength to the working stress is called the *factor of safety*.

The various experiments on alternating stress, as well as the results of general experience in the design and use of machines and structures, point to the use of different working stresses according to the nature of the straining efforts to be endured. In other words, the number by which the breaking strength of a material should be divided to give its safe strength depends on the conditions under which the material will be used.

The factor of safety may be considered as the product of four primary factors.

1. The ratio of the ultimate strength of the material to the elastic limit. This may be taken as 2 for ordinary material, $1\frac{1}{2}$ for forgings and nickel steel.

2. A factor depending on the nature of the stress. The value for dead loads is 1, for alternating stresses of one kind only is 2, and for alternating reversed stresses is 3.

3. A factor depending on the manner in which the load is applied. If applied gradually, the factor is taken as 1, if suddenly applied, 2.

4. A factor of ignorance. This must be sufficient to cover accidental and uncalculated straining action, deterioration, and such contingencies.

General factors of safety for different materials under various types of stress are given in the following table:

Material.	Factors of Safety for			
	Dead Load.	Live Load.		
		Stresses of One Kind only.	Reversed Stresses.	Varying Load and Shocks.
Cast iron	4 to 6	6 to 10	10 to 15	15 to 20
Wrought iron	3 to 4	5 to 6	8	12
Steel	4 to 5.	5 to 6	8	12
Timber	7 to 8	10	15	20
Brick	15	20	25	30
Stone	15	20	25	30

Minimum Factors of Safety under Dead Load.		Factor of Safety for	
Cast iron	4	Boilers	5
Wrought iron	3	Shaft carrying fly-wheel or armature	8
Mild steel	3	Steel work in buildings	4
Nickel steel	2·5	Steel work in bridges	5
Hardened steel	3	Steel wheel rims	8
Bronze, rolled	3	Cast-iron wheel rims	20
Brass, rolled	3	Steel castings	8

Effect of Temperature on the Strength of Metals and Alloys.—The tenacity, ductility, and elasticity do not vary much over ordinary atmospheric range of temperature but high temperature appreciably reduces the strength.

For steel and wrought iron the tenacity increases up to about 570° F. and then falls off continuously with further increase of temperature. The elastic limit falls continuously with increase of temperature. The elongation falls with increase of temperature, with a minimum towards 570° F., and then rises again continuously. The modulus of elasticity decreases steadily with rise of temperature.

The strength of cast iron begins to fall off at about 212° F., but at 390° F. is the same as at atmospheric temperature. At 750° F. it is appreciably weaker, and at 1112° F. less than half its original strength.

The strength of brass and gun-metal falls off quickly above 392° F.,

and they should not be used in valves and steam fittings when the steam is superheated.

The strength of copper begins to fall off at about 300° F. and at 840° F. is about half the original breaking strength.

The effect of low temperatures on mild steel is a progressive increase of tenacity, while the elongation practically vanishes.

Stress due to Change of Temperature.—Metals change their dimensions with temperature according to the approximate formula:

$$L_t = L_o (1 + at),$$

where a is the coefficient of expansion and t the change of temperature. If such change is resisted, the stress induced in the metal is proportional to the strain prevented, the pull on the constraint being Eat, where E is Young's Modulus.

For steel, the tensile strain per degree Fahrenheit is 0·000006, and if contraction is prevented, the corresponding intensity of stress is 0·145 tons per square inch. The cooling necessary to cause a stress of 1 ton per square inch is 12·15° F.

The different amounts of expansion in different metals in a machine may cause serious stress to be set up by a change of temperature.

CHAPTER VIII

Engineering Materials

The materials principally used in engineering work are cast iron, wrought iron, steel, copper and its alloys, bronze and brass, aluminium, timber, concrete, cement, brick, and stone.

Cast Iron

Cast iron is generally understood to be iron containing so much carbon or its equivalent that it is not malleable at any temperature. The amount of carbon varies ordinarily from a minimum of 2·20 per cent to about 5 per cent, depending on the amount of silicon, sulphur, phosphorus, and manganese present. Cast iron usually contains 3 to 4 per cent of carbon, none or all of which may be present in the combined form, and from $1\frac{1}{2}$ to $2\frac{1}{2}$ per cent of silicon. If the carbon is combined, it exists as the carbide, Fe_3C, giving the iron a hard white fracture. If the carbon is free, it is present in the form of graphite, and the iron has a grey fracture. Cast iron is granular and of a brittle nature; it cannot be bent, sheared, or punched, but can be cast sound, i.e. free from blow-holes and shrinkage faults, into intricate shapes.

Manufacture.—When iron ore is charged into a blast furnace, mixed

with limestone as a flux, and melted down with either charcoal, coke, or anthracite, the resulting metal is known as pig iron. When required by the engineer, the "pigs" are broken up and melted with a certain proportion of scrap in a *cupola*. This consists of an upright wrought-iron casing lined with fireclay brick and ganister (fig. 39). A fire is kindled and the cupola charged with alternate layers of coke and pig-iron, a blast of air from a blower being driven through. The iron melts and trickles down through the coke to the bottom. It is drawn off in the liquid state as required and cast into sand moulds of the desired shape.

Pig-iron is obtained in different degrees of hardness and strength varying from a soft, easily machinable iron of low strength, and porous to a certain degree, through a number of harder, tougher, and closer grained grades, to mottled and white iron. The amount of graphite present decreases, the combined carbon increases, and the silicon also generally decreases.

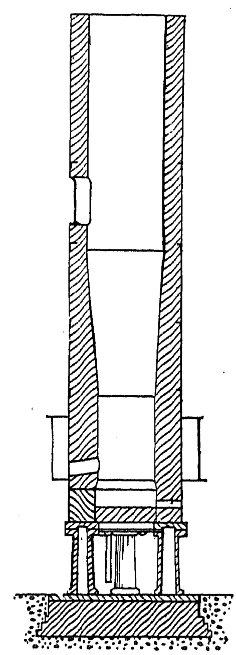


Fig. 39.—Section of Foundry Cupola with Drop Bottom

Formerly pig-iron was graded by breaking the pig and examining the fracture; the modern method of grading is by chemical analysis.

Properties.—In considering the properties of cast iron, it must be remembered that they are influenced by the composition, casting temperature, size of casting, &c., so that they may vary over a considerable range.

A peculiar property of cast iron which has been known for many years is that it *grows after repeated heatings*. Grey iron expands more rapidly than white iron, and the growth has been found to increase with the silicon content.

The *Sliding Friction* between two pieces of cast iron is less than between similar pieces of any other metal.

The *Shrinkage* allowance in castings is from $\frac{3}{32}$ in. to $\frac{1}{8}$ in. per foot. *Keep's test*, suggested for a mechanical analysis of the properties of pig-iron, is based on the variations in the amount of shrinkage caused by different amounts of silicon present.

The *Specific Gravity* varies from 6·9 to 7·5 according to the composition. It is usually taken as 7·22, corresponding to a weight of 450 lb. per cubic foot. The weight of a cubic inch is ·26 to ·27 lb.

The *Coefficient of Expansion* is 0·0000061 per degree Fahrenheit. As mentioned above, if continually exposed to heat, cast iron becomes permanently expanded. The growth amounts to $1\frac{1}{2}$ to 3 per cent, a fact for which due allowance must be made in installing grate bars, &c.

Mechanical Properties.—Great differences are found in test pieces from different parts of a casting, and the properties are much modified by the rate of cooling. Test bars cast separately give higher results than those

cast on a casting. Machining off the skin of a bar will reduce the ultimate strength.

The *Ultimate Strength* in tensile may be taken as: 7 to 9 tons per square inch for ordinary castings, and 10 to 14 tons for better grades. Commercially it is possible to maintain a standard of 12 tons. The maximum strength depends on the condition of the carbon. The larger the section the weaker the iron becomes. It has been found that round bars are probably the best test pieces.

Effect of Silicon.—Cast iron reaches its highest tensile strength with 1·8 per cent of silicon, and its highest compression strength at 0·75 per cent Si.

The *Elastic Limit* is not clearly defined either in tension or compression. It is quite low, for slight permanent sets may be detected under very low stresses, i.e. little if any part of the stress-strain curve is straight. If a specimen is loaded gradually a second time, the permanent set resulting is much less than in a piece not previously strained. Cast iron is a brittle material, i.e. it breaks with very little elongation or lateral contraction.

The *Modulus of Elasticity* varies according as it is measured on the first ton per square inch or over the whole range of stress. The following are average figures.

Ordinary commercial cast iron ..	5,500 to 6,500 tons per square inch.
Special grades	7,500 to 8,500 ,, ,,

Compressive Strength.—This is one of the most valuable properties of cast iron. A fair result is 40 tons per square inch, but figures as high as 60 tons have been obtained. It has been found that the size and shape of the test piece materially influences the result. The fracture generally takes place by shearing at an oblique plane, making an angle of 45° to 55° to the direction of application of stress. The presence of manganese raises the compressive strength. Cast iron stands up well under either steam or water pressure.

Shear and Torsion.—The ultimate resistance to shear is about 9½ tons per square inch. In torsion cast iron fails by tensile strain on an inclined section.

Transverse Strength.—The transverse test is the simplest and perhaps the one on which cast iron is most generally judged. Tested on a bar 2 in. deep by 1 in. wide and 42 in. long, with supports 36 in. apart and loaded at the centre, good average cast iron gives 26 to 30 cwt. breaking load. Readings have been obtained exceeding 40 cwt.

The *Ball Hardness* of cast iron ranges from just over 100 in the softest grey iron to over 400 in white iron.

Owing to liability to porosity, initial stress in cooling, &c., the *Safety Working Stresses* are rather low. The following are average figures:

In tension	1¼ to 1½ tons per square inch
In alternating tension	1 to 1¼ tons ,, ,,
In alternating tension and compression ..	½ ton ,, ,,
In compression	6 to 8 tons ,, ,,

Not much work has apparently been done on the resistance of cast iron to impact.

Every time cast iron is remelted it becomes harder and stronger, but also more brittle. Small castings may be made very hard by pouring into iron moulds or chills. The outer skin, which cools first, is made very hard. This principle is applied to the production of cast-iron rolls and parts subjected to wear which must be made harder than is necessary for parts not subjected to wear. The ball hardness of the hard face of a chilled casting may be as high as 445, while the grey back of the casting is only about 200 Brinell number. The intermediate mottled portion will average about 350.

The most common *defects* in cast iron are blow-holes, sand-holes, roughness of surface, cold shorts, and cracks resulting from uneven shrinkage.

Cast iron is used for steam cylinders, bed plates, frames of various machines, fly-wheels, large water-pipes, railway chairs, &c. Iron castings for pistons have approximately the following composition. Total carbon, 2·7 to 3·5 per cent; combined carbon, 0·5 to 0·8 per cent; Si 1·2 to 1·8 per cent; Mn 0·35 to 0·80 per cent; S $<$ 0·12 per cent; P $<$ 0·80 per cent.

Malleable Cast Iron is not so brittle as ordinary cast iron. Properly made, it is malleable, ductile, and tough. Malleable castings are made of best Cumberland white or mottled iron, and used in cases where shocks may have to be withstood.

There are two types of malleable iron—European or Réaumur, and American or Blackheart, named according to their method of production.

In the former method ordinary castings are surrounded with a substance that will extract some of the carbon, such as crushed red hematite, black oxide of iron or iron scale, and placed in a furnace at about 1650° F. for several days, the length of time depending on the size. The average properties are as follows:

Elastic Limit	12 to 15 tons per square inch.
Ultimate strength ..	19 to 23 „ „
Elongation	6 to 4 per cent.
Reduction of area ..	6 to 4 „

Bars 1-in. × $\frac{3}{8}$-in. bend from 45° to 90° over a 1-in. radius.

American blackheart is produced by converting the combined carbon of white cast iron into an amorphous uncombined condition by heating to a temperature between 1380° F. and 2100° F., generally 1470°–1560° F.

The castings are packed in retorts or annealing pots with an oxide of iron, generally hematite ore, and are kept red hot for several days in an annealing furnace.

The average results for tensile strength are as follows:

Elastic limit	12 to 16 tons per square inch.
Ultimate strength ..	19 to 23 „ „
Elongation	15 to 10 per cent.
Reduction of area ..	17 to 7 „

Bars 1-in. × $\frac{3}{8}$-in. bend from 90° to 180° over a 1-in. radius.

In compression the material rarely gives actual failure, but flows cut indefinitely with increase of load.

The Brinell hardness is from 101 to 145, and varies with the tensile strength and inversely as the carbon present. The electric resistance is 0·000041 ohm per cubic centimetre at 76° F.

Wrought Iron

Wrought iron differs from cast iron principally in the fact that there is less carbon and silicon in the former. In practice the distinction is so marked that they might almost be considered two distinct metals. Wrought iron is a typical ductile metal, and contains over 99 per cent of iron. In other words, it practically consists of ferrite (pure iron), with slag inclusions and a small amount of impurities. The carbon is usually less than 0·2 per cent, and it is desirable to keep the phosphorus below 0·25 per cent and the sulphur below 0·5 per cent.

When a bar is notched and broken, it breaks along the slag laminations, giving a grey fibrous structure. Under similar conditions steel shows bright fibrous spots.

The method of manufacture from pig-iron is known as the *puddling process*. Impurities are reduced in the various stages of the operation, and form the puddle cinder. The temperature in the puddling furnace is high enough to melt pig-iron, but not wrought iron, which comes from the furnace in a spongy, pasty state. It is removed in lumps of about 200 lb. each, formed into elongated blooms in a rotary squeezer, and rolled while still hot into " muck bars ". These, after cutting to length, are reheated in piles, and when white hot rerolled and welded together into bars, sheets, plates, and structural shapes. If the operations of cutting up, piling, reheating, and rerolling or rehammering are repeated, the quality of the iron is further improved, and the iron is called doubly refined. The purest grades are made from charcoal pig, and known as charcoal iron. In practice the various grades are distinguished by the marks, B, BB, BBB.

Wrought iron can be considerably bent without breaking, and can be sheared and punched, worked hot under the hammer, and forged into various shapes. Pieces can be welded by heating to about 2598° F and hammering together.

The surface of wrought iron can be made hard by case hardening. This process consists in heating for some hours in a carbonaceous mixture, or applying potassium cyanide when bright red, heating again, and quenching out in water. Wrought iron has been almost superseded in mechanical engineering work by mild steel, but is still used in cases where welding is required, as in chains, tubes, eyebolts, &c. It is also superior in cases where shocks have to be withstood, as in the coupling links of railway wagons. By being passed through shaped rolls when hot, wrought iron is given the many forms in which it is used

commercially, as round, square, and flat bars, sheets, T, angle, and channel iron.

Its chief uses are as spikes, nails, bolts, nuts, wire chain, horseshoes, sheets, plates, pipes and tubing, third rails, armatures, and electromagnets. The defects to which wrought iron is liable are blisters, excess of slag, spongy or spotted parts, rough edges.

Tensile Test.—The mechanical strength of wrought iron is affected by chemical composition, mechanical work, and heat treatment. Increase in strength results from reduction of cross-section. The following figures were obtained on best Yorkshire bars.

Yield point .. .. 14 tons per square inch.

The Y. P. is well defined and from 1 to 2 tons higher than the proportional and elastic limit.

Ultimate strength	..	22 to 24 tons per square inch.
Elongation ..	..	21 to 29 per cent.
Reduction of area	..	30 ,,

The tenacity and ductility are greater in the direction of the fibres than across them.

Compression Test.—The ultimate compressive strength is not well defined, but may be taken as 18 to 20 tons per square inch. The yield point is about the same as the yield point in tension.

The modulus of elasticity is 12,600 tons per square inch.

The Brinell ball hardness of pure Swedish wrought iron is 87.

Shear and Torsion.—The ultimate strength in single shear is about 19 tons per square inch. The elastic limit in torsion is 9 tons per square inch, and the modulus of elasticity in torsion is 5,700 tons per square inch. The melting-point of wrought iron varies with decreasing carbon from 2696–2894° F. The specific gravity varies from 7·4 to 7·9, and is generally taken as 7·85. The weight of a cubic inch is 0·28 lb. or 490 lb. per cubic foot.

The coefficient of expansion is 0·00000618 per degree Fahrenheit. The electrical conductivity is 16 if the conductivity of copper is taken as 100.

The specific heat of wrought iron is 0·1138.

The coefficient of friction of wrought iron on wrought iron is 0·44. The effect of temperature on the ultimate strength of wrought iron is to increase it up to about 570° F., after which temperature it falls off continuously as shown by the following figures:

	Degrees Fahrenheit.								
Temperature ..	68	212	370	570	750	930	1110	1290	1470
Relative strength ..	100	104	112	116	96	76	42	25	15

Safety working stresses for average qualities of wrought iron under various types of stress are as follows:

Type of Stress.	Steady Stress.	Variable Stress.	Shocks.
	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.
Tension ..	5 to $6\frac{1}{4}$	$3\frac{1}{2}$ to $4\frac{1}{2}$	2
Compression ..	4 to $5\frac{3}{4}$	2 to 4	$1\frac{1}{2}$
Shear	$4\frac{1}{2}$	3	$1\frac{3}{4}$
Flexure ..	$5\frac{1}{2}$	$3\frac{3}{4}$	$1\frac{3}{4}$
Torsion ..	$2\frac{1}{4}$	1	$\frac{3}{4}$

Steel

Steel is the name applied to numerous alloys, differing widely in physical properties, the principal constituent of which is the metal iron. The element which exerts the most influence on steel is carbon, and unless qualified by the prefix special or alloy, steel is usually inferred to be straight carbon steel.

Carbon Steel is a compound of iron and carbon, generally intermediate in composition between cast iron and wrought iron, but having a higher specific gravity than either, as shown in the following table:

Material.	Carbon Per Cent.	Specific Gravity.	Properties.
Cast iron ..	2 to 5	7·2	Not malleable, not temperable.
Steel	0·10 to 1·50	7·8	Malleable and temperable.
Wrought iron..	0·05 to 0·30	7·7	Malleable, not temperable.

Manufacture of Steel.—Steel is made from pig-iron by removing a large proportion of its carbon and as much of the silicon, sulphur, and phosphorus as possible, or by adding carbon to wrought iron, as in the cementation and crucible processes.

Crucible Process.—High-grade low-phosphorus wrought iron is used, and the carbon added to it. The original method consisted in cementing the wrought-iron bars by packing them in retorts with powdered charcoal and heating to a red heat for several days. These "blister steel" bars were cut into small pieces, remelted in a crucible, and poured into moulds. A later method consists in putting pieces of wrought iron into an air-tight crucible with the proper amount of powdered charcoal and melting it down. Other ingredients, such as chromium and tungsten, may be added in the crucible.

Bessemer Process.—Air is blown through the molten pig-iron placed in a pear-shaped vessel, called a converter. The silicon is first burned out, and forms slag, which is removed. The carbon then burns, giving off carbon monoxide gas. When all the carbon is burned out,

the flame drops and the blow is over. A certain amount of ferro-manganese or spiegeleisen is added to give the required amount of carbon to the steel. The liquid steel is then poured into the ingot moulds, and the ingots, while still hot, are rolled into blooms, billets, or rails, plates,. &c. The process is generally used for making high carbon steels:

Open-hearth Process.—By this process low carbon steels are generally produced. In the Siemens-Marten process, pig-iron and wrought-iron scrap are melted together, with a proportion of hematite added, on the hearth of a reverberatory furnace. A mixture of preheated air and producer gas is burned above the charge. The amount of silicon is not so great as if pig-iron were used alone, and the sulphur and carbon present are removed by the oxidation caused by the burning gases aided by the oxygen of the hematite. Carbon can always be added if required by the addition of ferro-manganese, as in the Bessemer process. In the Siemens process pure pig-iron is used and not scrap. The usual capacity of open-hearth furnaces is 30 to 50 tons, though much larger ones have been built.

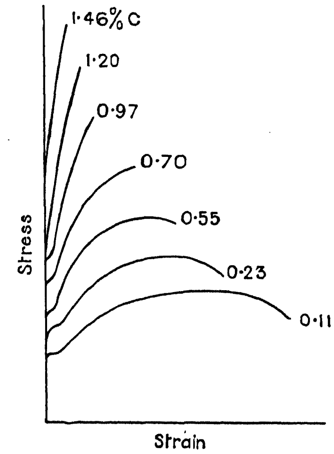


Fig. 40—Effect of Carbon on the Stress-strain Diagram of Steel

Electric Furnace Process.—Pig-iron, ore, and scrap are charged in various proportions, with steel scrap practically always added. The greater part of the steel made in the electric furnace, however, is from steel scrap, as scrap in mechanical forms, which would work badly in other furnaces, e.g. borings and turnings, can readily be melted. In arc furnaces the high temperature necessary for melting is derived from the electric arc between the electrodes, the heat from which is largely reflected or reverberated from the furnace roof. In induction furnaces, the temperature is induced by resistance to the electric current passing through the metal in the furnace. Steel of the highest quality is made in the electric furnace, which has been remarkably developed of recent times. No deleterious element, such as sulphur, is introduced by the heating agent, and no hot gases are introduced during the refining. A neutral or reducing atmosphere is maintained, and a pure metal almost free from occluded gases is produced.

Acid and Basic Steel.—These terms mean that the lining of the open-hearth furnace, converter, or electric furnace is made either of an acid or basic material. The lining has a certain effect in removing impurities from the steel. A basic lining, such as burned magnesite or dolomite, permits the refining of phosphorus (and sulphur). Acid linings, in which silica predominates, do not, and only those pig-irons which contain small quantities of phosphorus (and sulphur) can be converted into good steel.

Acid steel is slightly stronger than basic steel, partly owing to the higher

phosphorus in the acid steel, and partly to the effect of the lime used in the basic process in forming slag in the steel.

Physical Properties.—The physical properties of steel depend largely on the chemical composition. Carbon is the controlling element as regards strength; the higher the carbon the greater the strength and the less the elongation (fig. 40).

Silicon somewhat increases the hardness.

Sulphur causes red shortness or a tendency to be brittle while hot; phosphorus increases the strength but promotes brittleness, and is particularly deleterious when the steel is subjected to alternating stress. Manganese increases the strength somewhat, and promotes malleability. Formulæ have been derived for the calculation of the strength of steel from the analysis, but the strength is considerably influenced by the method of manufacture, and the heat treatment to which the steel has been subjected.

Carbon steel may be classified as follows:

Soft ..	.. 0·05 to 0·15 per cent C.	Not temperable, easily welded.
Medium	.. 0·15 to 0·35 ,, ,, C.	Poor temper, weldable.
Hard ..	.. 0·35 to 0·65 ,, ,, C.	Temperable, welded with difficulty.
Very hard	.. 0·65 to 1·00 ,, ,, C.	Highly temperable, not weldable.

The average properties of the classes of carbon steel shown above may be taken to be approximately the following when normalized, expressed in tons per square inch.

	Soft.	Medium.	Hard.	Very Hard.
Elastic limit (tension and compression)	13·5	15·5	22	31
Ultimate strength	22	27	40	54
Elastic limit (shear)	3	9	12·5	18
Ultimate shearing strength	18	22	31	40
Elastic modulus (tension)	13,000			
Elastic modulus (shear)	5,400			

It will be noticed that the modulus of elasticity does not vary with the carbon content, and that the shear modulus is about two-fifths that in tension and compression. The elastic limit in tension is a little higher than half the ultimate strength.

Mild Steel.—The steels termed soft and medium in the above tables are sometimes classed together as mild steel. Mild steel has many of the properties of wrought iron, but is stronger. It resembles wrought iron in having a yield point from 1 to 2 tons per square inch above the elastic limit. It can be welded, bent, sheared, and forged. Holes should, however, be drilled and not punched, as punching injures the surrounding metal. Mild steel does not appreciably harden if heated and quenched in water, and can readily be machined when cold by tool steel. It is more uniform and more ductile than wrought iron, and no fibre is produced in rolling or forging the ingot. The addition of a small percentage of nickel to mild steel renders

it stronger but more difficult to machine. The safety working stresses for mild steel of about 30 tons ultimate strength and 20 to 25 per cent elongation are

Simple tension	6	tons per square inch.
Simple compression	6	,, ,,
Alternating tension	4	,, ,,
Alternating tension and compression ..	$2\frac{1}{2}$	,, ,,

Mild steel used by the Admiralty for shipbuilding is required to have an ultimate strength of not less than 26 tons, and not more than 30 tons per square inch with an elongation of 20 per cent on 8 in.

Dead mild steel containing 0·1 per cent carbon is used for seamless tubing, pressed steel frames, rivets, &c. 0·2 per cent carbon steel is used for forged, machined, and case-hardened parts. Heat treatment does not increase the strength, but only the toughness. 0·25 per cent carbon steel is sometimes known as structural steel, and employed in the construction of boilers and bridges.

Steel with 0·3 per cent carbon responds to heat treatment in regards to strength as well as toughness, and is used for axles, driving shafts, &c. 0·4 per cent carbon steel is confined to parts requiring a high degree of strength and considerable toughness, such as crankshafts, propeller shafts, &c. Steels with higher percentages of carbon are stronger and more difficult to machine 0·8 per cent carbon steel is used for springs.

The ultimate tensile strength of some of the above steels is given below:

Structural steel for rivets ..	24·5	tons per square inch.
Structural steel for beams ..	27	,, ,
Boiler steel for rivets ..	22	,, ,,
Boiler steel for plates ..	27	,, ,,
Machine steel	33	,, ,,
Axle steel	44	,, ,,
Spring steel	55	,, ,,

Mechanical and Heat Treatment of Steel.—Forging or pressing under a hammer or press renders the material more compact, and increases the specific gravity and strength. Drawing produces similar results.

Heat treatment has a considerable influence on mechanical properties. ' Normalizing " means heating a steel to a temperature exceeding its upper critical range by not more than 122° F., and allowing it to cool slowly.

Annealing consists in reheating followed by slow cooling to remove internal stresses or induce softness. If in addition it is desired to refine the crystalline structure, the temperature must exceed the upper critical range. The effect of annealing is to reduce the ultimate strength and increase the ductility.

Hardening means heating a steel to its normalizing temperature, followed by cooling more or less rapidly in water, oil, or air. The hardness attained depends on the amount of carbon present, and the rate at which the steel is cooled. Hardened steel has its tenacity raised, but

does not show a marked yield point, becomes brittle, and has little ductility.

Tempering is heating a previously hardened steel to a temperature below its change point with a view to decreasing its hardness and increasing its toughness. If a pyrometer is not used, the desired temper is recognized by the colours which appear on a clean polished surface of the steel, due to the formation of a film of oxide. Though necessarily only of an approximate nature, this is the readiest method of determining tempering temperatures of steel. Tempering colours and the temperatures corresponding to them are as follows:

	Degrees F.		Degrees F
Pale Yellow	428	Pale Blue	569
Straw	446	Dark Blue	604
Golden Yellow	470	Red in the dark	752
Brown	491	Red—indirect sunlight ..	976
Purple	531	Red in sunlight	1076
Bright Blue..	550	Dark Red	1292

Cementing or Case Hardening is the process of hardening the surface of low carbon steel or iron by carbonizing the surface. The parts are packed in an iron box containing a carbonizing material such as 60 per cent charcoal and 40 per cent barium carbonate. The box and contents are heated to about 1652° F. for a length of time depending on the depth of hardened surface desired, then withdrawn from the furnace and allowed to cool down. The articles are then replaced in a muffle furnace and reheated to about 1652° F., after which they are quenched in cold water or other suitable quenching medium, a double treatment, viz. heating to about 1382° F. and again quenching, being necessary to refine the "case" i.e. the outer skin Of recent years the gas carbonizing process has come into prominence owing to the satisfactory results obtained with this process, which consists in forcing a current of carbonaceous gas over the articles instead of the use of charcoal.

Steel Castings.—Cast steel is harder and stronger than cast iron and will bend before breaking, and is thus used when more or less complicated designs are required to withstand heavy or variable loads. The metal gives off gases on cooling, and piping is caused in the top of ingots by the contraction during solidification, the action proceeding inwards from the portions in contact with the walls of the mould. Cast steel is not so fluid as cast iron, and castings are sometimes unsound unless special precautions are taken. With the object of removing this, steel is sometimes cast under pressure. The ingots, after being run into moulds of special construction, are subjected to great pressure in a hydraulic press, the effect of which is to compress the metal and so close up the cavities, at the same time making the steel denser and of greater strength.

The properties of steel castings containing about 0·25 per cent carbon may be taken approximately as follows:

Ultimate strength ..	20 to 30 tons/in.^{2}
Elongation	25 to 15 per cent.

For fluid compressed steel

Ultimate strength ..	40 tons/in.^{2}
Elongation	32 per cent.

The safety stresses are 4 to 6 tons per square inch in simple tension and compression.

Steel castings are superior to those of iron, both as regards strength and ductility, and are used among others in the following applications: toothed wheels, hydraulic cylinders, high pressure valve bodies, arms of hydraulic riveters, &c.

Steam-pipes to carry high-pressure or superheated steam are almost invariably made of cast steel in preference to cast iron.

In castings, whether of steel or non-ferrous metals and alloys, it is essential to avoid sharp corners and sudden variation in thickness. Pulleys and small wheels usually have the arms cast in the form of an S to provide a certain amount of give while the metal is cooling.

Tool Steel.—Carbon tool steel contains about 1 per cent carbon. It is received into the engineer's stores in square, hexagonal, or round bars, which are cut and forged to the required shape. The tool is then hardened and tempered to the required temper, generally that suitable for cutting mild steel, cast iron, or wrought iron. The hardening temperature is generally between 1382° F. and 1562° F., the tempering temperature being from 428° F. to 570° F. The hardening and tempering may be done in one operation by not quenching quite out.

When a hardened carbon steel tool is reheated and allowed to cool, it becomes soft. By adding tungsten and a small percentage of manganese and chromium it was found that the steel became self-hardening, i.e. it did not become soft when heated and allowed to cool. The earliest example of this type of steel is Mushet steel, which contains 1·5 to 2 per cent carbon, 6 per cent tungsten, 0·5 per cent chromium, and 2 per cent manganese. By increasing the percentage of tungsten and chromium, and lowering the carbon to about 0·65 per cent, a steel is produced which maintains its cutting edge at a high temperature. High-speed steel contains about 18 per cent tungsten and 3½ per cent chromium, and is hardened by heating almost to the melting-point and then either cooling in an air blast or quenching out in a bath of molten salt or in oil.

Alloy Steels

Of recent years many alloy steels have been developed, having certain properties which render them specially suitable for some particular purpose.

Nickel Steel is used for constructional purposes and in America for rails. It usually contains about 3 per cent of nickel, and about 0·3 per cent of carbon. It must be suitably heat-treated, and it is found that wide variations may be obtained in the elastic limit by varying the quenching medium.

When annealed the elastic limit is about 21 tons per square inch, and the ultimate strength 40 tons per square inch. The elastic limit is about 15 per

cent higher, and the ultimate strength about 25 per cent higher than ordinary structural steel. The average shearing strength is also roughly 16 per cent over that of carbon steel. Owing to its greater strength it is possible to build very long span bridges more cheaply when nickel steel is used.

Axles, crankshafts, driving and transmission shafts requiring strength and toughness are made of nickel steel. It is also used in the manufacture of boiler tubes, which will last nearly twice as long as mild steel tubes, and in addition a lighter gauge of tube may be employed with safety.

The following working stresses may be allowed:

Tension in eyebars	13	tons per square inch.
Stress in plates and shapes	12	,, ,,
Bending stress in pins	22	,, ,,
Bending stress in rivets	13	,, ,,
Shearing stress on pins	11	,, ,,
Shearing stress on rivets	6	,, ,,

The coefficient of expansion of steels with a high percentage of nickel is small, as shown in the following table:

Nickel per cent.	Coefficient of Expansion, deg. F.
27	0·00000612
30	0·00000306
32	0·00000194
34	0·00000139
36	0·00000083
38	0·00000022

Steel containing 36 to 38 per cent of nickel is known as Invar and used for measuring-tapes and scales. High nickel steel is also resistant to many types of corrosion.

Chromium Steels.—The influence of chromium is primarily to impart hardness to steel. Low carbon steel with about 0·5 per cent chromium is used for structural purposes.

Ball bearings are generally made with steel containing about 1 per cent carbon, with the addition of chromium up to 1 per cent or more. The crushing strength is tested by placing three balls in a rather close-fitting tube, and applying the crushing stress by means of hardened steel plates. The middle ball will ordinarily break first.

The ultimate crushing strength depends upon the diameter, as shown in the following figures:

Diameter of Ball.	Ultimate Strength, lb./in.^{2}
$\frac{1}{16}$ in.	400
$\frac{1}{8}$,, ...	2,500
$\frac{1}{4}$,, ...	6,000
$\frac{1}{2}$,, ...	25,000
$\frac{3}{4}$,, ...	56,000
1 ,, ...	100,000

A factor of safety of about 10 should be allowed.

Steel with a high percentage of chromium (about 13 per cent) is known as stainless steel owing to its resistance to corrosion and attack by some acids. It finds a wide application in cutlery, valves for internal combustion engines, &c. Steels with a lower percentage of chromium are used in the construction of safes, and for castings subjected to very severe stresses, such as rock crushing machinery.

Nickel-chromium Steels have a wide application on account of their resistance to fatigue. They are sometimes divided into low, medium, and high, referring to the percentage of these elements and carbon present. The first type is used for case hardening purposes. Medium nickel-chromium steels are used for constructional purposes, machine parts, gears, axles and shafts, &c. An ultimate strength as high as 70 tons per square inch has been obtained on these steels in conjunction with 15 per cent elongation.

Chrome-vanadium Steels are resistant to fatigue. A steel of the following composition: C 0·40 per cent; Mn 0·77 per cent; Cr 1·22 per cent; Va 0·2 per cent, is used for springs, and has the following properties:

Condition.	Elastic Limit.	Ultimate Strength.
Annealed ..	30 tons per square inch	45 tons per square inch
Oil quenched ..	87 ,, ,, ,,	93 ,, ,, ,,

Other applications include axles, gears, driving shafts, &c.

A steel containing C 0·32 per cent; Mn 0·60 per cent; Cr 0·87 per cent; and Va 0·18 per cent, gave the following:

Condition.	Elastic Limit.	Ultimate Strength.	Elongation.	Reduction of Area.
	Tons per sq. in.	Tons per sq. in.	Per Cent.	Per Cent.
Annealed ..	43	60	21	57
Oil quenched and tempered ..	85	94	12	43

Manganese Steel.—The addition of 1 to 2 per cent of manganese to steel is found to impart useful properties, but when the amount is increased to from 4 to 6 per cent, the steel becomes very brittle indeed.

The manganese steel of commerce contains about 1·2 per cent of carbon, and 12 to 13 per cent of manganese. It has a breaking strength of about 70 tons per square inch, but a low elastic limit. Elongation of as much as 70 per cent has been obtained, the test pieces stretching uniformly with no necking. It is very tough when quenched, and has a high degree of resistance to wear. It is only about 200 Brinell ball hardness, but hardens on the slightest deformation, so that it cannot be cut by tools, but has to be ground. It is used for points and crossings, crusher jaws, teeth on steam shovels, and other parts subject to extreme conditions of wear. It is practi-

cally non-magnetic, and many useful applications of this property have been found.

Tungsten Steel.—The addition of tungsten to steel has already been mentioned in connection with high-speed steels which contain about 16 to 18 per cent of tungsten, and possess the property of retaining their cutting edge at high temperatures. Tungsten imparts the property of air hardening to steels. Steel with 6 to 7 per cent of tungsten, with or without a small percentage of chromium, is used for permanent magnets. These are usually quenched either in oil or water to render them as hard as possible, as this is necessary for high retentivity.

Molybdenum Steel has properties similar to those of tungsten steel, except that a smaller quantity of molybdenum than tungsten is required to secure corresponding results.

Silicon Steel.—Steel with about 0·35 per cent carbon, 0·8 per cent manganese, and 0·4 per cent silicon is used for structural purposes. Transformer steel in which the losses due to hysteresis and eddy currents are low contains about 3 per cent silicon. Steel with about 14 to 16 per cent silicon is resistant to attack by acids. Silico-manganese steel is also used for springs.

Copper Steel, containing from 1 to 4 per cent of copper, is about equal in strength to nickel steel. 1 to 2 per cent of copper, or even less, is claimed to render steel resistant to atmospheric corrosion, but resistance to attack by alkaline solutions is not so marked.

Non-ferrous Metals

In addition to iron and steel, several of the non-ferrous metals and alloys are of importance to the mechanical engineer and are widely used by him.

Aluminium is a white, malleable metal which is used in cases where lightness is required. It can be cast into intricate shapes. The specific gravity is from 2·55 to 2·75 when hammered and rolled, and generally taken as 2·58. It is almost non-corrodible. The tensile strength is as follows:

As cast, about 8 tons, with elastic limit about 4 tons per square inch. As rolled, 14 to 17·5 tons. As drawn into wire, 22 tons.

In practice it is not usual to load above 1 ton per square inch. On account of its softness aluminium is usually alloyed in practice, the addition of zinc yielding a stronger alloy. The melting-point of aluminium is 1172° F., and the metal weighs 0·09 lb. per cubic inch. It is applied in the manufacture of steel as a deoxidizer.

The *thermit process* for welding iron and steel depends on the affinity of powdered aluminium and iron oxide, which are finely divided and ignited, a temperature approaching 5432° F. being attained. The process is very useful for welding breakages in large parts.

Copper is a reddish, malleable, and ductile metal. It is fibrous and very soft, and can be bent, sheared, and hammered into various shapes when cold. After hammering it requires to be annealed, i.e. raised to a fairly high temperature and quenched. The specific gravity varies slightly: for electro-

lytic copper S.G. = 8·94, as cast 8·6, and 8·95 as drawn or rolled; the melting-point is 1920° F.

The tensile strengths are: As cast, 9 to 12 tons/in.^{2}, with elastic limit of 4 tons; rolled into plates, 15 to 16 tons/in.^{2}; drawn into wire, 23 to 30 tons/in.^{2}, with elastic limit of 18 tons.

The strength of copper decreases with the temperature as follows: As rolled, the ultimate strength at 300° F. is 14 tons per square inch, and at 600° F. is 9 tons per square inch. The metal will safely stand a tensile stress of $1\frac{1}{2}$ tons per square inch below 300° F.

Copper is one of the best conductors of heat and electricity, and does not corrode in dry air. It is used for steam-pipes and locomotive fire-boxes, also for small pipes which require to be bent cold, and bolts used under conditions that would make them rust if of iron or steel. Two pieces of copper can be joined together by brazing. A properly brazed joint is as strong as the metal itself.

Bronze.—*Aluminium Bronze* is a strong and ductile alloy consisting of 90 per cent of copper and 10 per cent of aluminium.

The ultimate strength is as follows: As cast, 30 tons per square inch, with elongation of 20 per cent on 2 in. Rolled, 38 tons per square inch, with elongation of 29 per cent on 2 in. The strength of the alloy falls off at 572° F., and it becomes brittle. At 752° F. it is so brittle that it is of little use. A cubic inch weighs 0·273 lb.

Ordinary *bronze* is an alloy of copper and tin, the tin being added to harden the copper. The average strength of bronze is given in the attached table.

STRENGTH OF BRONZE

Copper.	Tin.	Yield Point.	Ultimate Strength.	Elongation.	Compressive Strength.
Per Cent.	Per Cent.	Tons per sq. in.	Tons per sq. in.	Per Cent.	Tons per sq. in.
100	0	6·25	12	7	18
95	5	7·5	13·5	10	20·5
90	10	9·5	13	4	24
85	15	11·5	14·75	1·6	33
80	20	12·5	14·25	0·5	55

Gun-metal contains about 10 per cent of tin, and is used for strong castings, being tough and of high tensile strength. A small amount of zinc increases the fluidity. It is softer than cast iron, and will bend slightly before breaking. It cannot be forged or welded, but can be cast into intricate shapes, and is soft and easy to machine. It is not porous, and does not readily corrode. Gun-metal is used for internal parts of steam- and water-valves, cocks, and engine fittings. Another use is for bearings in which steel shafts revolve, as the friction between this alloy and steel is not excessive, and the gun-metal, being softer, wears away before the steel.

The ultimate strength is from 10 to 16 tons per square inch. With an

elongation of 10 to $7\frac{1}{2}$ per cent. It will safely bear a stress of $1\frac{1}{2}$ to 2 tons per square inch. Gun-metal weighs 0·3 lb. per cubic inch. The melting-point is 1800° F.

Phosphor Bronze.—Phosphorus is added to bronze as a deoxidizer, and also to give a harder and a stronger bronze. The percentage of tin varies from 5 to 10; the higher the tin the harder the castings. The phosphorus analyzes about 0·30 per cent. For high malleability and ductility, the tin is made less than 5 per cent, and the phosphorus about 0·1 per cent. A small percentage of lead is sometimes added for ease in machining. Phosphor bronze is remarkable for complete fluidity, so that perfect castings can be made. It has all the properties of gun-metal, but is stronger, and, when hot, can be forged and rolled into rods. The tensile strength of castings is about 17 tons, and of forgings 23 to 25 tons per square inch. The safety tensile stress for castings is from 2 to 3 tons per square inch, and for forgings 4 to 5 tons. It is used for valve spindles, pump rods (where steel would corrode), shaft bearings, small castings where strength is required, boiler fittings, tubes, &c.; 1 cubic inch of phosphor bronze weighs 0·3 lb.

Manganese Bronze is obtained by adding ferromanganese to bronze, the influence of the manganese being to harden the metal. It possesses great strength and power to resist the action of sea water, so that it is largely employed for propeller blades.

Silicon Bronze.—A small quantity of silicon in bronze increases its strength and ductility without reducing its electrical conductivity.

Brass is the name given to alloys of copper and zinc. The mechanical properties vary greatly according to composition and heat treatment. The effect of increasing zinc content is shown in the following table:

Copper.	Zinc.	Yield Point.	Ultimate Strength.	Elongation.	Compressive Strength.
Per Cent.	Per Cent.	Tons per sq. in.	Tons per sq. in.	Per Cent.	Tons per sq. in.
100	0	6·25	12	7	18
95	5	5·4	12·5	12	12·5
90	10	4·5	13·5	18	13
85	15	4·0	14·25	25	14·5
80	20	3·5	15	33	17·5

The most valuable brasses lie between the limits of 60 to 85 per cent of copper. Yellow brass contains 70 per cent copper and 30 per cent zinc. Admiralty metal is 70/30 brass with 1 to 1·5 per cent tin. This small percentage of tin is added to make the metal break up more readily under the action of a cutting tool. The effect of a small quantity of lead is also to make the metal easier to machine.

Red brass contains 85 per cent copper and 15 per cent zinc.

The tensile strength of 70/30 brass is 13·5 to 15·5 tons per square inch as cast, with an elongation of 56 per cent. It is used for smoke tubes for locomotive type boilers and, to some extent, for condenser tubes, where corrosion trouble is not very great.

Brass for cold working contains 27 to 35 per cent zinc, and has a high tensile strength and maximum ductility.

Delta Metal is the name applied to several alloys of copper and zinc with small amounts of lead, tin, or iron. Both manganese bronze and delta metal are stronger than phosphor bronze, and can be made stronger than mild steel, but at the expense of ductility. They make sound, fine castings, and are largely applied in hydraulics on account of their non-corrosive properties. The ultimate strength is from 29 to 35 tons per square inch, with an elongation of 35 to 25 per cent on 2 in. The safety stress for castings is 3 to 7 tons per square inch, and for rods 5 to 7 tons per square inch.

Muntz Metal, or Naval Brass contains a higher percentage of zinc, viz. 38 to 40 per cent. It has a higher tensile strength than ordinary brass, and is more ductile, but not so strong as bronze.

The ultimate strength of rods is 26 tons per square inch, with 45 per cent elongation on 2 in. The safety tensile stress is $5\frac{1}{4}$ tons per square inch. It is used for condenser tubes and bolts which have to resist the corrosive action of sea water.

The alloys or *spelters* used for brazing are composed of copper-zinc alloys. As the proportion of zinc increases the melting-point is lowered. An easily fusible spelter may be made of 2 parts of zinc and 1 part of copper, but the joint will be weaker than when an alloy more difficult to fuse is employed.

Miscellaneous Alloys

White Metal is an alloy of 90 per cent tin, 3 to 5 per cent copper, and 3 to 8 per cent antimony, with less than 1 per cent of lead. It is sometimes called babbit metal, though the name covers a wide range of compositions. It is very soft, and has valuable anti-friction properties, i.e. remains cool when subjected to rubbing contact under heavy pressure. It is employed for lining bearings.

Admiralty proportions for white metal are: 85 to 89 per cent tin, 2 to 7 per cent copper, and 8 to 9 per cent antimony.

For the sake of cheapness white metal is sometimes made from lead and antimony instead of tin and antimony, but it is then not so good. The strength of white metal is $3\frac{1}{4}$ tons in tension, and $7\frac{1}{2}$ tons per square inch in compression.

Duralumin is a useful alloy as light as aluminium, and weight for weight as strong as steel. It is also remarkably resistant to corrosion. It has approximately the following analysis: 3·5 to 5·5 per cent copper, 0·5 to 0·8 per cent manganese, 0·5 per cent of magnesium, and the remainder aluminium. The specific gravity is 2·77 to 2·85, and the melting-point 1200° F.

It may be forged or drawn in either the cold or hot state (752°–842° F.). For further treatment in the cast state it should be annealed at 752° F. A final treatment after pressing or rolling is given by heating in a salt bath and quenching in water. Finished parts should not be heated above 300° F.

The mechanical properties are as follows:

Yield point	10 to 16 tons per square inch.
Ultimate strength ..	18 to 28 „ „ „
Elongation	20 to 15 per cent.
Reduction of area ..	34 to 25 per cent.
Hardness	98 to 125 Brinell.

The effect of treatment on the ultimate strength is shown by the following:

As cast	15·5 tons per square inch.
Quenched from 752° F. to 932° F. and aged	26 „ „
Sheets and tubes	22 „ „
Hard rolled	38 „ „

Nichrome is an alloy containing approximately 60 per cent nickel and 14 per cent chromium, with the remainder chiefly iron. It is used for parts exposed to high temperatures owing to its comparative freedom from oxidation and change of form.

TIMBER

Wood as a building material is produced by the seed-bearing trees, which may be divided into three groups: conifers, broad-leaved trees, and tropical trees. The first two groups provide the structurally valuable materials.

Timber is roughly classed into two varieties: softwoods, such as pine, spruce, cedar, cypress, larch, and fir; hardwoods, such as oak, walnut, maple, chestnut, ash, boxwood, and whitewood.

The cross-section of a tree trunk consists of two parts, an inner and darker core of heartwood and an outer portion of sapwood. Both consist of tubes, shown in section as rings, each representing one year's growth of the tree. Slow growth is indicated by closeness of the rings, and is associated with greater strength than quick growth. Flat sawing consists in cutting the timber tangentially to the annular rings. Rift sawing consists in cutting the rings as nearly as possible in a radial direction.

Timber is subjected to natural seasoning to expel the moisture from the pores of the wood by exposing it to a free circulation of air.

The greatest shrinkage takes place in the cross-section of the fibres or tangential to the annular rings.

Some average values of shrinkage are as follows:

Light conifers	3 per cent.
Heavy conifers	4 „
Ash, elm, walnut, poplar, beech, sycamore ..	5 „
Birch, chestnut	6 „
Young oak, hickory	up to 10 „

Timber is subject, both in the growing and converted states, to decomposition and attack by animal and vegetable life, e.g. dry rot, wet rot, worms, &c.

The chief defects are knots, pitch pockets and shakes, splits or checks in the timber.

The life of timber can often be extended 50 per cent by a chemical impregnation of the wood cells or an exterior application of a preservative coating which will penetrate the fibres.

Strength of Timber.—Special attention had to be given during the war to the testing of timber, particularly in its application to aircraft construction. This material is peculiar in being anisotropic, having three principal axes, along the grain, radially across the grain, and tangentially across the grain.

The strength in tension and compression along the grain is greater than across it. The superiority of riven wood over cut wood has long been recognized. In plywood alternate layers are glued together to give a product of approximately equal strength in all directions.

Tests of strength have not been standardized to an extent which allows their inclusion in specifications for timber. As a general rule the heaviest is the strongest timber, the comparison being made on different woods in the same state of dryness.

As timber is a variable material, it is necessary to test a large number of similar pieces in order to get representative results. Small test pieces are not satisfactory on account of the proportion of local defects. A large section has, however, the same strength per square inch as a small one if both are free from defects.

Effect of Moisture.—The maximum strength of timber is attained when all but 4 to 5 per cent of its own weight of water has been dried out of it. Fresh-cut timber has about one-half of its maximum strength. From containing 60 per cent of its own weight of water downwards the strength rises steadily. After good air-drying the amount of moisture retained is 12 to 15 per cent.

Tension Tests.—These are not of much practical importance, for in structures timber usually fractures by shearing or splitting.

Very large ends have to be used for gripping the test pieces. Along the grain the tensile strength is a maximum.

The following are some average figures for tensile strength along the grain.

Material.	Tenacity along the Grain.	Young's Modulus.
	Tons per sq. in.	Tons per sq. in.
Oak	4 to 8	about 650
Ash	2 to 7	,, 700
Elm	2 to 6	450 to 500
Teak	2 to 7	about 1000
Yellow pine ..	1 to 2	,, 700
Red pine ..	2 to 6	,, 700
Spruce	2 to 5	,, 700
Beech	2 to 5	,, 600
Mahogany ..	2 to 5·5	,, 550

Compression Tests are made both along and across the grain on rectangular blocks. In compression along the grain there is a well-defined ultimate strength, but wood has a badly defined proportional limit.

Compression tests on short blocks in the direction of the grain give the most reliable characteristic of quality of timber.

The fibres usually buckle over the whole of an internal surface inclined to the direction of pressure. The crushing strength across the grain is much smaller. A few average values of the crushing strength along the fibre are given.

Material.	Crushing Strength along the Fibre.
	Tons per sq. in.
Oak	2 to 4·5
Ash	3·8 to 4·2
Yellow pine ..	2 to $2\frac{1}{2}$
Red pine ..	2·5 to 5
Beech	3 to 4
Spruce	2·5 to 3
Elm	3 to 5
Mahogany ..	3 to 3·5

Bending Tests are the commonest type of test on timber.

Flexure test are made on (1) small specimens, (2) large beams. Failure may be by shear along the neutral axis, by compression along the upper side of the beam, or by tension along the lower side.

Rectangular specimens and central loads on beams supported at each end are usually employed, local indentation by supports and loading being prevented by metal plates. If the length of the specimen is ten times the depth, failure is by longitudinal shear.

The factor of strength generally measured is the modulus of rupture $f = \frac{3}{2} \frac{Wl}{bd^2}$. This is a good indication of the value for structural purposes. The limit of proportionality of deflection to load is at a considerable proportion of the total load.

Pine and fir were found in flexure tests to have an elastic limit of about 4000 lb. per square inch, and a maximum strength of 4900 lb. per square inch.

Under heavy loads timber deforms continuously, permanent resistance being offered to about one-half the breaking load of an ordinary test.

Shearing Tests along the grain are of great importance. They are made by the use of shearing blocks placed in the testing machine. Shearing always takes place along the grain, separating but not rupturing the fibres.

The following are some average values for ultimate shear along the neutral axis.

Oak	730 to 1000 lb. per square inch.
Ash	480 to 700 ,, ,,
Fir	300 to 350 ,, ,,
Yellow pine	290 to 410 ,, ,,
Spruce	250 to 375 ,, ,,

Impact Flexure Tests show the shock-resisting quality of timbers. Recently impact testing machines for timber have been developed on the lines of the Izod machine, which have yielded valuable information (fig. 41).

Besides structural purposes, timber is used for pulleys, pulley block

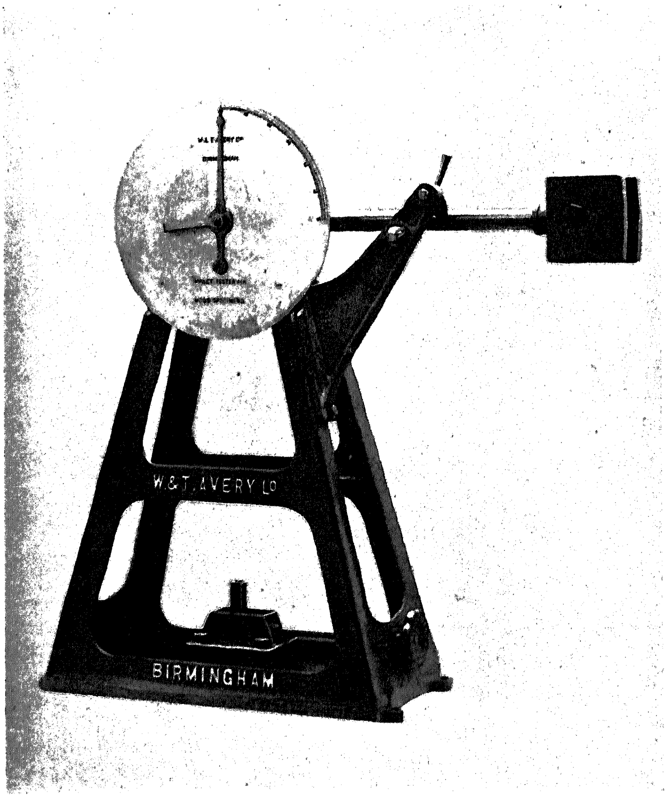


Fig. 41.—Impact Testing Machine for Wood

sheaves, and occasionally for shaft bearings. Lignum-vitæ is suitable for bearings which have to work under water.

Cement, Stone, and Brick

Cements are produced by roasting limestone with various amounts of clay, the product being subsequently finely ground. Portland cement is the most important cement used by the engineer, and is made from a mixture of about three parts of limestone or chalk to one part of clay. Tests show that it can be made with remarkable uniformity. When mixed with water it combines chemically with a certain quantity, and sets in a solid mass impervious to water. The effect of gypsum in cement is to increase the time taken to set hard; beyond 2 per cent in amount, it is injurious.

Tensile Tests.—Although Portland cement is not usually employed

"neat", but in a mixture with inert material such as sand, broken brick, or stone, the tensile test of neat cement is found to be a good index to quality. The neat cement is mixed with water, usually 18 to 25 per cent, and allowed to set in a mould to form briquettes. The form of briquette (fig. 42) has been standardized by the Engineering Standards Committee, as are also the clips or jaws by which the briquette is held. The age and treatment of briquettes after mixing must be specified. The tensile breaking load increases with the speed of loading, the standard rate specified being 500 lb. per minute.

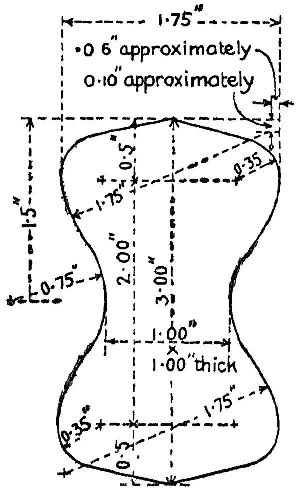


Fig. 42.—Standard Briquette for Tensile Test of Neat Cement

The tensile testing machines (fig. 43) are of special design, and either of the single or compound lever type. The application of the load at a uniform rate may be accomplished by running in water or fine shot through a controllable opening into a pan, hanging at one end of the lever, the fracture of the briquette automatically cutting off the supply. In another type a travelling weight is caused to move along a graduated lever at a uniform speed.

The permissible amounts of moisture and calcium sulphate are often fixed. Excess in the amount of lime causes crumbling of the cement after setting.

As coarse grains have a weakening effect, it is usually specified that the residue on a sieve with 76×76 wires per square inch must not exceed 3 per cent, and on a sieve with 180×180 wires per square inch shall not exceed 22·5 per cent, the diameter of the wires being 0·0044 in. and 0·002 in. respectively.

A briquette made under standard conditions should have the following tensile strength.

7	days after	gauging	— 400	lb. per	square	inch.
28	,,	,,	— 500	,,	,,	

To accord more nearly with practice, sand mixture tests are made to test the adhesion of the cement and sand, the briquettes being moulded from a mixture of 3 parts by weight of British standard sand, and 1 part of cement, with enough water to thoroughly wet the mixture. The tensile strength should be 120 lb. per square inch seven days after gauging, and 225 lb. per square inch 28 days after gauging.

Compression Tests are made on 3 or 4 in. cubes in an ordinary testing machine, care being taken to obtain satisfactory bedding of the test pieces.

A *Compression Testing Machine for Cement* (capacity 50 tons) is illustrated in fig. 44.

Six-inch cubes of cement can be placed in this machine between two platens, the upper one of which is adjustable. The lower platen is seated

upon a hydraulic ram, which is forced upwards by means of a screw compressor; the pressure of the fluid in this cylinder is conveyed to a small subsidiary cylinder and ram, at the end of which latter is fixed a knife-edge, and this communicates the strain to the steelyard, where it is balanced off by the movement of a poise traversing suitable graduations.

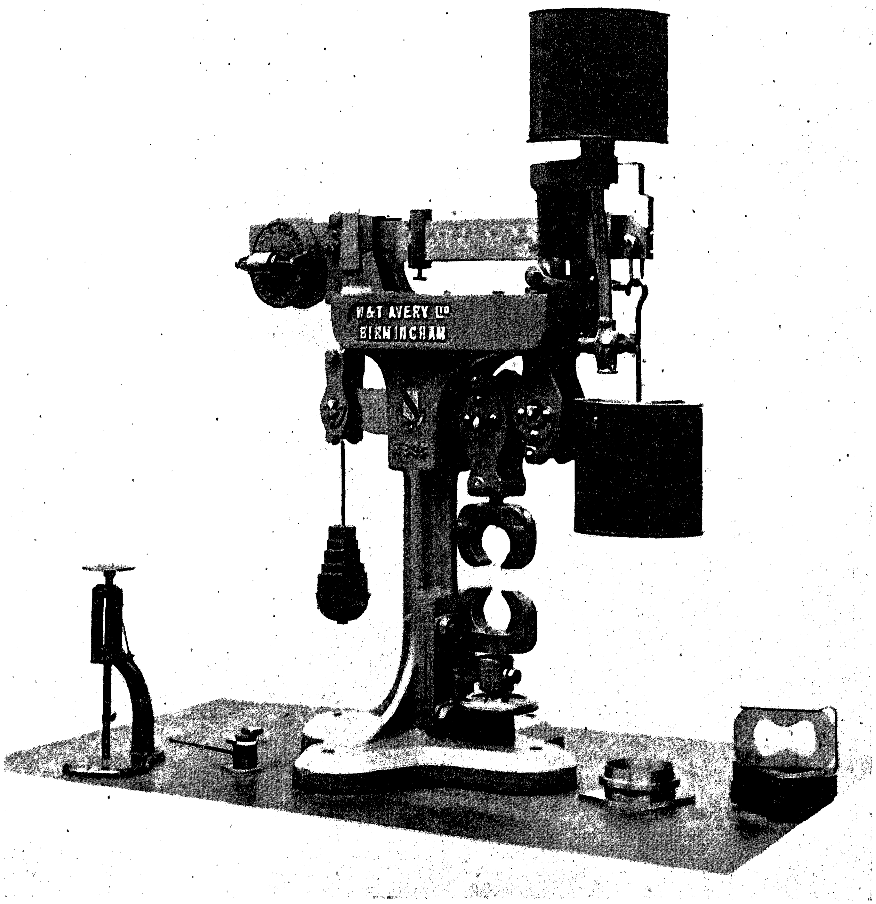


Fig. 43.—Cement Testing Machine

The crushing strength is from 5 to 11 times the tensile strength, the ratio increasing with age. Fracture takes place in the characteristic manner for brittle materials by shearing at about an angle of 45° to the direction of compression. The average compressive strength of Portland cement one month old is 2000 lb. per square inch, and one year old is 3000 lb. per square inch.

Specific Gravity.—Imperfectly calcined cement is lighter than cement of good quality. A loss in specific gravity accompanies the de-

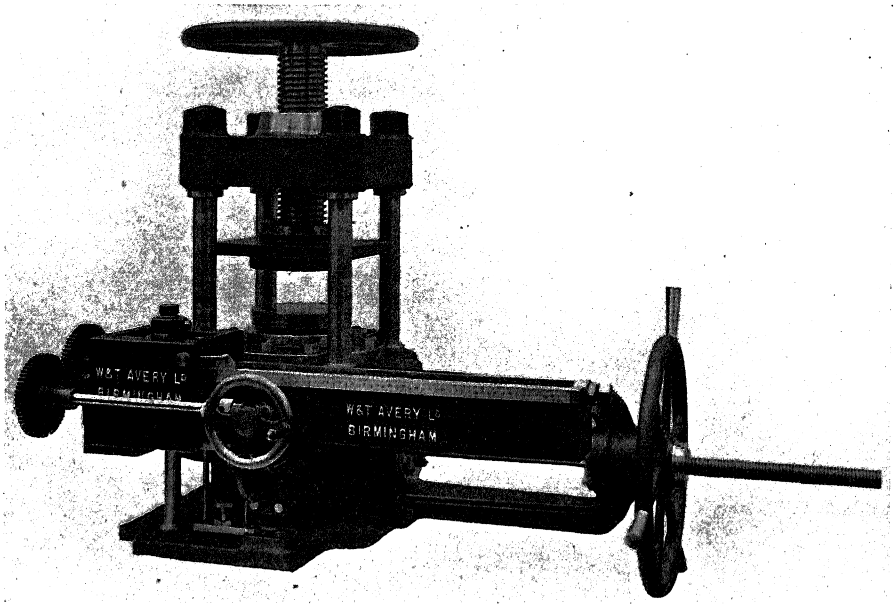
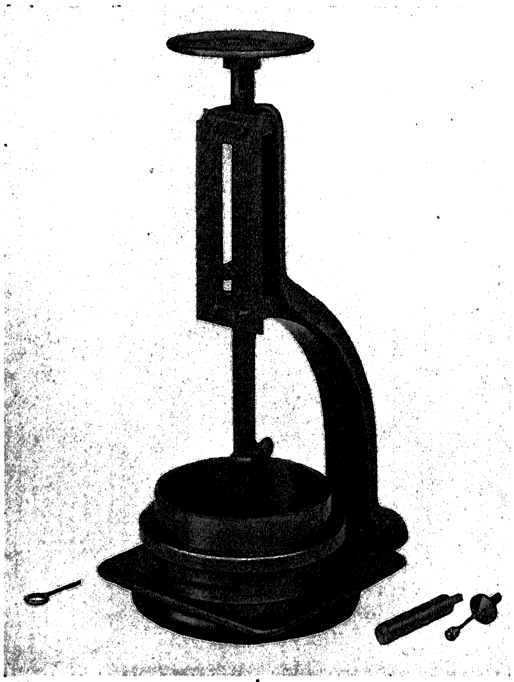
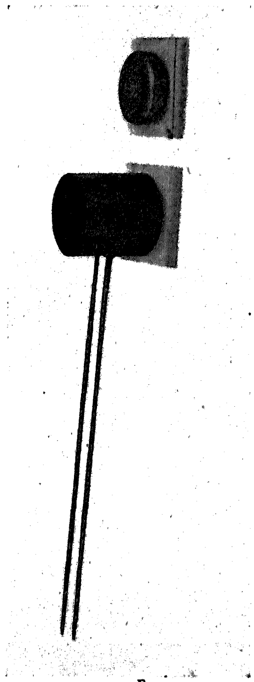


Fig. 44.—Compression Testing Machine for Cement



A

Vicat Needle and Mould for ascertaining the Time of Setting of Cement



B

Le Chatelier Test for the Soundness of Cement

Fig. 45

terioration of cement which has been left exposed to the atmosphere. It is a good practice to specify a specific gravity of 3·10 after delivery. It may readily be determined by the usual specific gravity bottle method.

Time of Setting.—The time of setting depends upon the requirements of the work for which the cement is intended, and should be between ten minutes and half an hour for quick-setting cements, and two to five hours for slow-setting cements. The time of setting is tested by the indentation of a weighted needle (fig. 45, A) which has a flat point $\frac{1}{18}$ in. square and weighs $2\frac{1}{2}$ lb. If the needle fails to make an impression when its point is gently applied to the surface, the cement is considered to be set.

A soundness test devised by Le Chatelier is sometimes applied by forming a briquette in a mould 30 mm. long by 30 mm. diameter, formed by a thin split brass cylinder 0·5 mm. thick, to which two pointers 165 mm. long are attached, one on either side of the split (fig. 45, B). After the cement is set by being placed for 24 hours in water at 58° F., the mould is placed in water, which is boiled for six hours. A perfectly sound cement should not expand more than 10 mm. If the cement shows any material increase on this, its soundness may be regarded as doubtful, and a further examination is necessary.

Concrete.—The strength of concrete varies with the nature and proportions of the inert materials mixed with the cement, and increases with age after the time of setting. Crushing tests are the chief commercial tests. They are made on cubic specimens of 4 in. to 9 in. sides, which are sawn out, and opposite faces ground parallel. In crushing tests the compression plate of the testing machine should have a spherical seating, and the bedding should be such as to evenly distribute the pressure. The load at which the specimen cracks and the ultimate strength are recorded. Fracture takes place at angles of about 45° to the direction of compression. The crushing strength of Portland concrete is 1000 lb. per square inch, rising to 2000 lb. one year old.

Stone.—The crushing strength of stone varies with its nature. For sandstone and limestone it may be taken as 300 to 400 tons per square foot, while for granite it is often 1500 tons per square foot. Porosity is determined by weighing the stone when dried, and again after saturation by immersion in water. Building stone is chosen from considerations of durability and appearance.

Brick.—The strength of bricks varies with the composition of the clays from which they are made, the method of manufacture, &c.

The average compressive strength of common bricks may be taken as 80 to 150 tons per square foot, but best hard bricks may withstand up to 800 tons per square foot. Blue Staffordshire bricks have a strength of about 400 to 500 tons per square foot.

The following tests are also occasionally carried out on stone and bricks: transverse test, absorption test, freezing test, quenching test, and acid test, according to the exigencies of the application.

Concrete is an ideal material for machinery and hammer foundations,

and it does not deteriorate. It should not be used, however, more than twenty minutes after mixing. Stone in addition to being strong and durable has great vibration absorbing power, but is costly.

Brick is not so durable as stone, but is cheaper.

Concrete is also used for machine- and forge-shop floors partly on account of its fire resisting qualities.

Of recent years great developments in the uses of concrete for constructional purposes have taken place. The strength of concrete is reinforced by embedding steel bars in it, so that the steel work takes the tensile stresses, and the concrete the compressive stresses. Beams and columns of this reinforced concrete are now being largely used in the construction of buildings.

ULTIMATE STRENGTH OF METALS USED IN ENGINEERING

Metal.	Tension.	Compression.	Shear.
	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.
Cast iron	8 to 14	40 to 50	9 to 11
Malleable cast iron	19 to 25	40 to 50	—
Wrought-iron plates with the fibre	21 to 23	20	16 to 17·5
Wrought-iron plates across the fibre	19	—	14
Wrought-iron bars	20 to 24	—	15 to 18
Iron wire as drawn	35	—	—
„ annealed	26	—	—
Mild steel for rivets	26 to 29	25	20 to 23
„ structural	28 to 32	27	21 to 24
Carbon steel rails	30 to 40	—	—
„ railway axles ..	35 to 40	—	—
„ castings	20 to 31	31	27
„ forgings	25 to 35	—	—
„ wire	70 to 90	—	—
Carbon tool steel	70	—	45
Copper, cast	9 to 11	18 to 20	13·5
„ annealed	13 to 15	8 to 10	6 to 9
„ hard drawn	20 to 22	—	—
„ wire as drawn	26	—	—
„ wire annealed	16	—	—
Brass	13 to 14	13·5	10 to 16
Gun-metal	14 to 17	9	15
Phosphor bronze	22 to 26	—	24
Manganese bronze	29 to 40	53	22 to 30
Aluminium, cast	6 to 8	30	3 to 5
„ rolled	11 to 18	5·5	5·5 to 6
„ bronze	40 to 43	—	25
Lead	1	—	—
Tin, cast	1·5 to 2	2·5	—
Zinc, cast	2·2 to 2·5	9	—

Young's Modulus for Metals used in Engineering

Metal.	Coefficient or Modulus of Elasticity in Tension and Compression.
	Tons per sq. in.
Cast iron	6000 to 9000
Wrought iron	12,000 to 13,000
Malleable cast iron	11,500 to 12,000
Mild steel, structural	13,000
Carbon steel castings	13,500
,, wire	14,000
Copper, cast	4500 to 6500
,, annealed	6000 to 7000
,, wire as drawn ..	8000
Brass	4000 to 5000
Gun-metal	4500 to 5500
Aluminium	4500 to 5000
Phosphor bronze	6000 to 6500

Note.—The shearing modulus or modulus of rigidity is almost exactly two-fifths of Young's modulus except in the case of cast iron (5400 tons per square inch).

Elastic Limit and Safety Working Stress of various Metals used in Engineering

Metal.	Elastic Limit.	Safety Stress.	
		Tension.	Compression.
	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.
Wrought iron (with the fibre)	10 to 15	5	4
Cast iron	8 to 14	$1\frac{1}{2}$	6
Mild steel	14 to 18	6	6
Steel castings	11 to 16	5	6 to 8
Aluminium	4 to 7	1	1
Copper rods or sheets ..	2 to 4	$1\frac{1}{2}$	$1\frac{1}{4}$
Gun-metal	5 to 8	$1\frac{1}{2}$	$1\frac{1}{2}$
Phosphor bronze .. Manganese bronze Delta metal .. } cast	10 to 20	2 to 7	6
Phosphor bronze .. Manganese bronze Delta metal .. } rolled	13 to 25	4 to 7	5
Brass	4	1	1

SPECIFIC GRAVITY AND WEIGHT PER CUBIC INCH, &c., OF SOME METALS

Metal.	Specific Gravity.	Weight per Cubic Inch.	Weight per Cubic Foot.
		Pounds.	Pounds.
Cast iron	7·2 to 7·5	0·26	449·2
Wrought iron ..	7·8	0·28	489·8
Mild steel	7·8	0·28	489·8
Aluminium ..	2·56	0·09	159·7
Copper	8·82	0·32	550·4
Bronze	8·85	0·32	552·2
Brass, 70/30 ..	8·4	0·30	524·1
Lead	11·37	0·41	709·5
Tin	7·29	0·26	454·8
Zinc, rolled ..	7·15	0·26	446·1
Chromium ..	6·50	0·23	405·6
Nickel	8·80	0·32	549·1
Tungsten	18·77	0·68	1171·2
Molybdenum ..	8·56	0·31	534·2
Manganese ..	7·42	0·27	463·0

TABLE FOR THE CONVERSION OF BRINELL HARDNESS NUMBERS TO TENACITY

AVERAGE VALUES FOR IRON AND STEEL

Brinell Hardness Numbers.	Factor to Conversion to Tons per Square Inch.
150 to 200	0·235
200 „ 250	0·227
250 „ 300	0·222
300 „ 350	0·218
350 „ 400	0·216
400 „ 450	0·214
450 „ 500	0·212
500 „ 550	0·211
550 „ 600	0·210
600 „ 650	0·209

TABLE SHOWING THE RELATION BETWEEN HARDNESS AND TENACITY

AVERAGE VALUES FOR IRON AND STEEL

Brinell Ball Hardness Number.	Scleroscope Hardness Number.	Yield Point.	Ultimate Strength.
		Tons per sq. in.	Tons per sq. in.
150	32	20	36
200	39	32	46
300	56	56	66
400	72	79	86
500	83	102	106
600	88	124	126

Safety Stresses for Some Timbers

Timber.	Tension.	Compression.
	Tons per square inch.	Tons per square inch.
Oak	·60	·42
Fir	·42	·36
Larch	·42	·33
Pine	·54	·36
Red pine	·42	·33

Strength of Representative American Timbers

Kind of Timber.	Weight.	Bending Test.			Compression Test Parallel to Grain.		Shearing Strength Parallel to Grain.	Tensile Strength Perpendicular to Grain.
		Elastic Limit.	Modulus of Rupture.	Modulus of Elasticity.	Elastic Limit.	Ultimate Strength.		
	Lb. per c. ft.	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.	Tons per sq. in.
Ash	49	2·5	4·4	665	1·6	1·9	0·65	0·31
Cedar	27	1·5	2·3	425	1·1	1·25	0·32	0·09
Chestnut	55	1·4	2·5	415	0·9	1·1	0·36	0·19
Cypress	41	1·7	2·9	475	1·2	1·4	0·37	0·12
Elm	52	1·6	3·1	460	1·0	1·3	0·41	0·25
Fir	38	2·2	3·5	700	1·5	1·7	0·40	0·09
Hickory	64	2·5	5·3	650	1·6	2·0	0·41	0·30
Maple	67	1·9	3·3	550	1·1	1·4	0·51	0·28
Oak	62	2·1	3·7	560	1·35	1·6	0·56	0·34
Pine, Yellow	50	2·4	3·9	730	1·7	2·0	0·48	0·13
,, Red	42	1·6	2·8	615	1·1	1·4	0·35	0·09
Spruce	33	1·5	2·4	450	1·0	1·1	0·30	0·09
Larch	47	2·0	3·2	580	1·4	1·6	0·40	0·11
Walnut	58	2·4	4·2	630	1·6	1·9	0·55	0·25

Weight and Representative Strength of Non-Metallic Materials

Material.	Specific Gravity.	Weight in Pounds per Cubic Foot.	Tensile Strength.	Compressive Strength.
			Pounds per sq. in.	Pounds per sq. in.
Portland cement*	3·1	194	400	2,000
Concrete	2·2	137	120	1,000
Bricks, hard	2·0	123	350	8,000
,, common	1·8	112	40	1,500
Brickwork (best)	1·6 to 1·8	100 to 112	300	2,000

* 7 days old.

CRUSHING STRENGTH OF STONE AND BRICK

Material.	Weight per Cubic Foot. Pounds.	Crushing Strength. Tons per Square Foot.
Granite, hard fine grain	166	950 to 1350
„ coarse grain		600 to 900
Limestone, hard ..	141 to 172	450 to 550
„ soft ..		75 to 200
Sandstone, hard ..	127 to 168	300 to 600
„ tender ..		75 to 275
Red Brick	125 to 135	50
Yellow Brick ..		65 to 90
Staffordshire Blue Brick		450 to 475

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